M.A./M.Sc. (Final) Mathematics

Paper-I : Analysis and Advanced Calculus

Unit 1 : Normed linear spaces, Quotient space of normed linear spaces and its completeness. Banach spaces and examples.

Unit 2 : Bounded linear transformations. Normed linear space of bounded linear transformations. Weak convergence of a sequence of bounded linear transformations. Equivalent norms. Basic properties of finite dimensional normed linear spaces and compactness, Reisz Lemma.

Unit 3 : Multilinear mapping. Open mapping theorem, Closed graph theorem. Uniform boundness theorem.

Unit 4 : Continuous linear functionals. Hahn-Banach theorem and its consequences. Embedding and Reflexivity of normed spaces. Dual spaces with examples.

Unit 5 : Inner product spaces. Hilbert space and its properties.

Unit 6 : Orthogonality and Functionals in Hilbert Spaces. Phytagoren theorem. Projection theorem. Orthonormal sets. Besel's inequality, Complete orthonormal sets. Parseval's identity. Structure of a Hilbert space. Riesz representation theorem. Reflexivity of Hilbert spaces.

Unit 7 : Adjoint of an operator on a Hilbert space. Self-adjoint. Positive. Normal and Unitary operators and their properties.

Unit 8 : Projection on a Hilbert space, Invariance. Reducibility, Orthogonal projections. Eigen-values and eign-vectors of an operator, Spectrum of an operator, Spectral theorem.

Unit 9 : Derivatives of a continuous map from an open subset of Banach space to a Banach space. Rules of derivation. Derivative of a composite. Directional derivative. Mean value theorem and its applications.

Unit 10 : Partial derivatives and Jacobian Matrix. Continuously differentiable maps. Higher derivatives. Taylor's formula, Inverse function theorem, Implicit function theorem.

Unit 11 : Step function. Regulated function, primitives and integrals. Differentiation under the integral sign, Riemann integral of function of real variable with values in normal linear space.

Unit 12 : Existence and uniqueness of solutions of ordinary differential equation of the type $x' = f(t, x)$.