Question Bank

B. Sc. (Part III)

Elementary Quantum Mechanics & Spectroscopy

PH09

Section-A

Very Short Answer Type Questions

Q.1 Define Compton shift.

Q.2 Calculate the de Broglie wavelength of a 100 keV electron.

Q.3 Explain the physical significance of the wavefunction $\psi(q)$ and $\psi^{*}\psi\, dq$.

Q.4 The wave function of a particle is given by $\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$, where $0 < x < a$. What is the probability of finding the particle in the region $0 < x < a/2$?

Q.5 Write the time dependent and time independent Schrodinger equation for the wave function ψ.

Q.6 The wave function of a free particle is defined by $A\, e^{ikx}$. Calculate the probability current density.

Q.7 Write the equation of continuity connecting probability density and probability current density. Give its physical significance.

Q.8 The wave function for a particle in one-dimensional motion in the range $0 < x < a$ is described by $\psi(x) = A \sin\left(\frac{n \pi x}{a}\right)$. For the normalized wave function $\psi(x)$, find the value of the constant A.

Q.9 Is the wave function $\psi(x) = e^{x}$ well behaved ? if not, then why?

Q.10 What are the basic properties of a well behaved Schrodinger wave function ?

Q.11 Evaluate the commutator $\left[x, \frac{d}{dx}\right]$ by operating on a wave function

Q.12 Find the value of the commutator $(\hat{x}\,\hat{p}_x - \hat{p}_x\,\hat{x})$.

Q.13 Define the expectation value of an observable in quantum mechanics.

Q.14 What is the physical significance of an eigenvalue equation.

Q.15 Differentiate between expectation value and eigenvalue of an observable.

Q.16 What do you mean by the degeneracy of an eigenvalue.

Q.17 What is a linear operator?

Q.18 What mathematical conditions a well behaved quantum mechanical wave function have to satisfy?

Q.19 Is $\psi(x) = \exp\left(-x^2\right)$ a well behaved Schrodinger wave function? Answer 'Yes' or 'No' with your reasons.

Q.20 If you double the width of a one-dimensional infinite potential well, is the energy of the ground state of the trapped electron multiplied by 4, 2, ½, ¼, or some other number ?

Q.21 The unnormalized ground-state wave function of a particle is given as

$$\psi_0(x) = \exp\left(-\alpha^4\, x^4 / 4\right)$$

with eigenvalue $E_0 = \hbar^2\, \alpha^2 / m$. What is the potential in which the particle moves?

Q.22 A particle in an impenetrable potential box with walls at $x = 0$ and $x = a$ has the following wave function at some initial time :

$$\psi(x) = \frac{1}{\sqrt{5\,a}} \sin\frac{\pi\,x}{a} + \frac{1}{\sqrt{5\,a}} \sin\frac{3\,\pi\,x}{a}$$

What are the possible results of the measurement of energy on the system and with what probability would they occur?

Q.23 An electron is bound in a one-dimensional potential field of width 10 Fermi. What is the ground state energy of the electron in this field?

Q.24 Consider a proton as a bound oscillator with a natural frequency of 3 x 10^{21} Hz. What is the energy of its ground and first excited states?

Q.25 Write stationary state time dependent wave function.

Q.26 What is the energy eigenvalue of the nth state of one-dimensional harmonic oscillator ? also what is th parity of the ground state wave function of harmonic oscillator ?

Q.27 What conclusion one draws from Stern and Gerlach experiment ?

Q.28 What are the energy eigenvalues of a rigid rotator ?

Q.29 Determine the rotational energy levels of HCl . Assume that the interatomic distance to be a constant 1.27 x 10^{-8} cm.

Q.30 A particle is in an angular momentum state of $l = 2$, $m = 1$. What is the probability of finding it at the position $\theta = \pi / 4$ and $\phi = \pi / 4$ within $d\,\theta = 0.01$ radians and $d\,\phi$ the same.

Section B

Short Answer Type Questions

Q.1 Determine the lower limit of the possible values of the energy of an oscillator , using the uncertainty relation.

Q.2 Why do spectral lines have finite width?

Q.3 Find the probability current for the plane wave $\exp\left[i\left(k\,x - \omega\,t\right)\right]$

Q.4 Use the ground-state harmonic oscillator eigenfunction to show by direct integration that :

$$\langle p \rangle = 0$$
$$\langle p^2 \rangle = \hbar\, m\, \omega - m^2\, \omega^2 \langle x^2 \rangle .$$

Q.5 What is a Hermitian operator ? Prove that momentum operator $\left(-i\,\hbar\,\frac{\partial}{\partial x}\right)$ is Hermitian.

Q.6 If C and D are two hermitian operators, work out which, if any , of the following combinations (i) CD (ii) D^2 (iii) $CD - DC$ (iv) $CD + DC$ are hermitian .

Q.7 Prove that eigenvalues of a Hermitian operator are real.

Q.8 Prove that eigenfunctions of a Hermitian operator belonging to different eigenvalues are orthogonal.

Q.9 Find the value of the operator $\hat{L}_X \hat{L}_Y - \hat{L}_Y \hat{L}_X$.

Q.10 Find the value of the commutator $[L^2 , L_z]$.

Q.11 Explain the meaning of stationary state. Write the complete Schrodinger wave function for the stationary state of a free particle.

Q.12 derive Bohr's angular momentum quantization condition for the Bohr atom from de Broglie's relation.

Q.13 X-rays of wavelength 1 A^0 are scattered by a carbon block. The scattered radiations are viewed at an angle of 90^0 to the direction of incidence. Calculate the Compton shift and the energy imparted to the recoil electron in Joules . Given rest mass of the electron m_0 = 9.1 x 10^{-31} kg, velocity of light c = 3 x 10^8 m/s, planck's constant h = 6-6 x 10^{-34} J. s.

Q.14 A certain system is described by the Hamiltonian operator

$$H = -\frac{d^2}{dx^2} + x^2 \quad .$$

Q.15 Show that $Ax\, e^{-(x^2/2)}$ is an eigenfunction of H and determine the eigenvalue.

Q.16 Using Schrodinger wave equation deduce the equation of continuity

$\frac{\partial \rho}{\partial t} + div\, \vec{j} = 0$, where ρ is the probability density and $\vec{j}$ is the probability current density.

Q.17 Determine the energy levels and normalized wave functions of the stationary states of a particle in infinite potential well.

Q.18 Describe Stern-Gerlach experiment to show the quantization of spin angular momentum.

Q.20 State the principle of superposition of states in quantum mechanics.

Q.21 The wave function for a particle in one-dimensional motion in the range $0 < x < 2a$ is described by $\psi(x) = \frac{1}{\sqrt{2a}} \exp^{\left[\frac{i}{\hbar}(px - Et)\right]}$ and at all other places $\psi(x) = 0$. Calculate the probability for finding the particle in the region defined by $0 < x < a/2$.

Q.22 The eigenfunction of the operator $\left(\frac{\partial^2}{\partial x^2} - x^2\right)$ is $e^{-\frac{x^2}{2}}$. Calculate the corresponding eigenvalue.

Q.23 Find the eigenfunction of the operator $\left(\frac{\partial^2}{\partial x^2} - x^2\right)$.

Section C

Long Answer Type Questions

Q.1 Determine the transmission coefficient of a particle having energy $E < V_0$ for a rectangular potential barrier defined by

$V(x) = 0$ for $x < 0$ and for $x > a$,

$V(x) = V_0$ for $0 < x < a$.

Explain Tunnel effect.

Q.2 A Gaussian wave packet is described by the wave function

$$\psi(x) = \left[\frac{1}{\pi^{1/4}\sqrt{d}}\right] \exp\left[ikx' - \frac{x'^2}{2d^2}\right].$$

Compute the expectation values of x, x^2, p, p^2. Hence prove that in this case the uncertainty product is given by $\left\langle(\Delta x)^2\right\rangle\left\langle(\Delta p)^2\right\rangle = \frac{\hbar^2}{4}$. Interpret the result physically.

Q.3 Write down Schrodinger's equation for a particle in a one dimensional box. Solve it to obtain normalized eigenfunctions and show that eigenvalues are discrete.

Q.4 Determine the transmission coefficient of a particle for a rectangular potential barrier defined by

$$U(x) = U_0 \qquad 0 \le x \le a \ ;$$

$$\text{and} \quad U(x) = 0 \qquad \text{for } x < 0 \text{ and also for } x > 0 .$$

Consider the case when the energy E of the incident particle is (i) greater than U_0 (ii) less than U_0 .

Q.5 Solve Schrodinger's equation for a one-dimensional simple harmonic oscillator to determine energy eigen values and eigen functions. Plot the wave functions for the ground state, first and second excited state.

Q.6 Write the Schrodinger equation for the motion of a particle in a spherically symmetric field in spherical polar coordinates. Solve it by the method separating the variables.

Q.7 Determine the energy eigenvalues and eigen functions of a rigid rotator and explain rotational spectra of diatomic molecule

Q.8 Find the energy eigenvalues and normalized eigenfunctions of a particle bound in a rectangular three-dimensional potential box of sides of length l_x, l_y, and l_z. What is the degree of degeneracy of the ground-state and first excited energy level ?

Q. 9 Determine the reflection coefficient of a particle from a rectangular potential wall defined by

$$U(x) = 0, \qquad x < 0$$

$$U(x) = U_0, \qquad x \ge 0 \ ;$$

the energy of the particle is $E > U_0$.