Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final)
Paper Code:MT-10
Mathematical Programming
Section – C
(Long Answers Questions)

1. Prove that $f(x) = \frac{1}{x}$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

A (P.No. 12, Exa. 1.3)

2. If $f(x)$ is continuous, $f(x) \geq 0, -\infty < x < \infty$ then to show that the function
   $\phi(x) = \int_x^\infty (y - x) f(y) dy$ is a convex function provided the integral converges.

A (P.No. 13, Example 1.5)

3. To prove that every supporting huperplane of a closed convex set which is bounded from below contains at last one extreme point of the set.

A (P.No. 7, The. 1.6)

4. The sum of convex functions is convex and if atleast one of the functions is strictly convex then to show that the sum is strictly convex.

A (P.No. 11, The. 1.8)

5. To prove that A positive semi definite quadratic form $f(X) = X^T AX$ is a convex function over R^n

A (P.No. 16, The. 1.10)

6. Show that $f(x) \begin{bmatrix} b(x-\alpha). b < 0, x < \alpha\alpha \\ 0, \quad x \geq \alpha \end{bmatrix}$ is a convex set for all x.

A (P.No. 19, Que. 4)

7. Solve the following L.P.P. using revised simplex method.
   Max $z = 3x_1 + 6x_2 + 2x_3$
   S.t. $3x_1 + 4x_2 + x_3 \leq 2$
   $x_1 + 3x_2 + 2x_3 \leq 1$
   & $x_1, x_2, x_3 \geq 0$

A (P.No. 29, Example 2)

8. Solve the following L.P.P. using revised simplex method :
   Max $z = 3x_1 + x_2 + 2x_3 + 7x_4$
   S.t. $2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$
   $-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$

$x_1 + x_2 - 2x_3 + 3x_4 \leq 100$

& $x_1 \geq 2, x_2 \geq 1, x_3 \geq 3, x_4 \geq 4$

A (P.No. 32, Que. 3)

9. Solve the following L.P.P. by standard form-II of revised simplex method.

$2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$

$x_1, x_1 \geq 0$

Min Z = $x_1 \geq 2x_2$

A (P.No.40, Exa. 2.4)

10. Solve the following L.P.P. with the help of revised simplex method but without use of artificial variables.

Max z = $2x_1 + 6x_2$

S.t. $x_1 + 3x_2 \leq 6$

$2x_1 + 4x_2 \geq 8$

$-x_1 + 3x_2 \leq 6$

and $x_1, x_2 \geq 0$

A (P.No. 45, Example 5)

11. Using bounded variable technique solve the following L.P.P.

Max z = $x_1 + 3x_2$

S.t. $x_1 + x_2 + x_3 \leq 10$

$x_1 - 2x_3 \geq 0$

$2x_2 - x_3 \leq 10$

and $0 \leq x_1 \leq 8,\ 0 \leq x_2 \leq 4\ x_3 \geq 0$

A (P.No. 51, Exa. 6)

12. Using the bounded variable technique. Solve the following linear programming problem :

Max z = $2x_1 + x_2$

S.t. $x_1 + 2x_2 \leq 10$

$x_1 + x_3 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

and $0 \leq x_1 \leq 3,\ 0 \leq x_2 \leq 2$

A (P.No. 57, Exa. 8)

13. Using the bounded variable technique. Solve the following L.P.P.

Max z = $3x_1 + 5x_2 + 2x_3$

S.t. $x_1 + 2x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 23$

and $0 \leq x_1 \leq 4,\ 2 \leq x_2 \leq 5, 0 \leq x_3 \leq 3$

A (P.No. 53, Exa. 7)

14. Solve the following mixed integer programming problem :

Max z = $4x_1 + 6x_2 + 2x_3$

S.t. $4x_1 - 4x_2 \leq 5$

$-x_1 + 6x_2 \leq 5$
$-x_1 + x_2 + x_3 \leq 5$
$x_1, x_2, x_3 \geq 0$
x_1, x_2 are integers.

A (P.No. 90, Exa. 7)

15. Find the optimum integer solution to the following I.P.P.
Max $z = 4x_1 + 4x_2$
S.t. $2x_1 + 4x_2 \leq 7$
$5x_1 + 3x_2 \leq 15$
$x_1, x_2, \geq 0$ and are integers

A (P.No. 87, Exa. 6)

16. Solve the following Integer programming problem.
Max $z = 2x_1 + 10x_2 - 10x_3$
S.t. $2x_1 + 20x_2 + 4x_3 \leq 15$
$6x_1 + 20x_2 + 4x_3 = 20$
And $x_1, x_2, x_2 \geq 0$ and are integers

A (P.No. 76, Example 3)

17. Find the optimum integer solution to the l.p.p.
Maximize $Z = 3x_1 + 4x_2$
S.t. $3x_1 + 2x_2 \leq 3$
$x_1 + 4x_2 \leq 10$
& $x_1, x_2, \geq 0$ and are integers

A (P.No. 71, Eax. 2)

18. Find the optimum integer solution to the l.p.p.
Maximize $Z = x_1 + 2x_2$
S.t. $2x_2 \leq 7$
$x_1 + x_2 \leq 7$
$2x_1 \leq 11$
& $x_1, x_2, \geq 0$ and are integers

A (P.No. 68, Exa. 1)

19. Use branch and bound method to solve following L.P.P.
Maximize $Z = 7x_1 + 9x_2$
S.t. $-x_1 + 3x_2 \leq 6$
$7x_1 + x_2 \leq 35$
$x_2 \geq 7$

A (P. No. 99, Exa. 2)

20. Use branch and bound method to solve the following I.P.P.
Minimize $Z = 4x_1 + 3x_2$
s.t. $5x_1 + 3x_2 \geq 30$
$x_1 \leq 4$
$x_2 \leq 6$
$x_1, x_2 \geq 0$ *and are integers*

A (P. No. 101, Exa. 3)

21. Use Branch and bound technique to solve the following problem :

Maximize $Z = 3x_1 + 3x_2 + 13x_3$

s.t. $-3x_1 + 6x_2 + 7x_3 \leq 8$

$6x_1 - 3x_2 + 7x_3 \leq 8$

$0 \leq x_j \leq 5$

and x_j *are integers for* $j = 1, 2, 3 \ldots$

A (P.No. 103, Example 4)

22. Use branch and bound method to solve the following I.P.P.

Maximize $Z = x_1 + 2x_2$

s.t. $x_1 + x_2 \leq 7$

$2x_1 \leq 11$

$2x_2 \leq 7$

$x_1, x_2 \geq 0$ *and are integers*

A (P.No. 111, Que. 6)

23. Write the objective function in the form $Z = X^T AX + q^T X$

(i) $Z = 2x_1^2 + x_1x_2 + 9x_1x_2 + 3x_2^2 + x_2x_3 + 2x_2$

(ii) $Z = x_1^2 - 6x_1x_2 + x_3^2 + 9x_3$

A (P.No. 120, Que. 7)

24. Determine whether each of the following quadratic form is positive definite or negative definite.

(a) $2x_1^2 + 6x_2^2 - 6x_1x_2$

(b) $-x_1^2 - x_2^2 - 4x_3^2 + x_1x_2 - 2x_2x_3$

A (P.No. 120, Que. 10)

25. Obtain the necessary and sufficient conditions for the following NLPP.

Minimize $Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

A (P.No. 124, Exa. 10)

26. Find the dimension of a rectangular parallelopied with largest volume whose sides are parallel to the coordinate planets to be inscribed in the ellpsoide.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

A (P.No. 129, Exa. 13)

27. Write short notes on :

(i) Necessary conditions for general NLPP.

(ii) Sufficient conditions for GNLPP and

(iii) Lagrange's Multipler method

A (P.No. 121, 122, 123)

28. Obtain the necessary and sufficient conditions for the optimum solution of the following NLPP.

Minimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

Subject to $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

& $x_1, x_2, x_3 \geq 0$

A (P.No. 127, Exa. 11)

29. Determine the sign definiteness of each of the quadratic forms $X^T AX$:

(a) $A = \begin{bmatrix} 2 & 1 & 4 \\ 6 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & -2 & 1 \\ -4 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

A (P.No. 119, Que.6)

30. Which of the following quadratic form?
(a) $Z = x_1^2 + 2x_2^2$
(b) $Z = \frac{x_1}{x_2}$
(c) $Z = x_1^2 - x_2^2 + 4$
(d) $Z = x_1^2 + 2x_1x_2 + x_2^2 + 4x_1$

A (P.No. 119, Que. 5)

31. What is a general nonlinear programming problem ? Establish the relation between saddle point and the minimal point of the nonlinear programming problem.

A (P.No. 138, 140)

32. Solve the following programming problem graphically:
Minimize $f(x_1x_2) = x_1^2 + x_2^2$
Subject to $x_1 + x_2 \geq 4$
$2x_1 + x_2 \geq 5$
and $x_1, x_2 \geq 0$

A (P.No 158, Exa. 10)

33. Solve the following non linear programming problems using the method of lagrange multipliers:

Maximize $f(x, y, z) = ay\,z$

Subject to $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$x, y, z \geq 0$

A (P.No. 161, Exa.Que.4)

34. Solve the following nonlinear programming problem using the method of lagrangian multipliers:
Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$
Subject to $4x_1 + x_2^2 + 2x_3 = 14$

& $x_1, x_2, x_3 \geq 0$

A (P.No. 150, Exa. 4)

35. Let X_0 be a solution of the NLPP

Maniimize $f(X): X \in R^n$

Subject to $G(X) \leq 0$ where

$G(X) = (g_1(X), g_2(X), \dots \dots . g_m(X))^T$ and

$f(X), g_i(X)$; $i = 1,2 \dots \dots m$ are all convex functions.

Let the set of points X such that $G(X) < 0$ be normally. The there exists a vector $\lambda_0 \geq 0$ *in* R^n Such that

$$f(X) + \lambda_0^T F(X) \geq f(X_0)$$

A (P.No. 141, The. 3)

36. Use method of Lagrangian multiplies to solve the following non linear programming problem :

Optoimize $f(X) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to $x_1 + x_2 + x_3 = 20$

& $x_1, x_2, x_3 \geq 0$

A (P.No. 154, Exa. 7)

37. Write the Kuhn-Tucker necessary and sufficient conditions for the following non linear programming problem to have on optimal solution.

Min $f(x_1 x_2) = x_1^2 - 2x_1 - x_2$

S.T. $2x_1 + 3x_1 \leq 6$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

A (P.No. 170, Exa. 1)

38. Use Kuhn-Tucker conditions to solve the following non-linear programming problem :

Man $f(x) = 8x - x^2$

S.T. $x \leq 3$

& $x \geq 0$

A (P.No. 171, Exa. 2)

39. Solve the following non linear programming problem :

Min. $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$

S.t. $x_1^2 - x_2 \leq 0$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

A (P.No. 176, Exa.6)

40. Use Kuhn-Tucker conditions to solve the following non linear programming problem :

Optimize $f(x_1, x_2, x_3) = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$

Subject to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 6$

& $x_1 x_2 \geq 0$

A (P.No. 179, Exa. 8)

41. Use Kuhn-Tucker conditions to solve the following non linear programming problem :
    Max. $f(x_1, x_2) = 7x_1^2 - 6x_1 + 5x_2^2$
    S.t. $x_1 + 2x_2 \leq 10$
    $x_1 - 3x_2 \leq 9$
    $x_1, x_2 \geq 0$

A (P.No. 178, Exa. 7)

42. Solve the following nonlinear programming problem :
    Min. $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$
    S.t. $x_1^2 - x_2 \leq 0$
    $x_1 + x_2 \leq 2$
    $x_1, x_2 \geq 0$

A (P. No. 176, Exa. 6)

43. In the quadratic programming problem :
    Maximize $f(X) = C^T X + \frac{1}{2} X^T G X$
    Subject to $AX \leq 0$ & $X \geq 0$
    The function $f(X)$ cannot have an unbounded maximum if $X^T G X$ is negative definite or if $C = 0$ $If\ C \neq 0\ and\ X^T GX$ is negative semidifinte then $f(X)$ may have an unbounded maximum.

A (P.No. 185, The.1)

44. Explain Wolfe's algorithm?

A (P.No. 186)

45. Solve the following quadratic programming problem using Wolfe's method:
    Min. $f(x_1, x_2) = 8x_1 - 10x_2 + x_1^2 + 2x_2^2$
    S.t. $x_1 + x_2 \leq 5$
    $x_1 + 2x_2 \leq 8$
    & $x_1, x_2 \geq 0$

A (P.No. 194, Exa. 2)

46. Solve by Wolfe's method:
    Max. $f(x_1, x_2) = 2x_1 + x_2 - x_1^2$
    S.t. $2x_1 + 3x_2 \leq 6$
    $2x_1 + x_2 \leq 4$
    & $x_1, x_2 \geq 0$

A (P.No. 198, Exa. 4)

47. Solve the following quadratic programming problem by Beala's method:
    Max. $f(x_1, x_2) = 2x_1 + 3x_2 - 2x_1^2$
    S.t. $2x_1 + 4x_2 \leq 4$
    $x_1 + 2x_2 \leq 2$
    & $x_1, x_2 \geq 0$

A (P.No. 218, Exa. 10)

48. Use Beale's method to solve the quadratic programming problem:

Min. $f(x_1, x_2) = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

S.t. $x_1 + x_2 \leq 2$

& $x_1, x_2 \geq 0$

A (P.No. 209, Exa. 7)

49. Explain the Duality in Quadratic programming.

A (P.No. 228)

50. State and prove duality theorem for quadratic programming problem.

A (P.No. 230)

51. Drive the dual of the quadratic programming problem is :

Min $f(X) = C^T X + \frac{1}{2} X^T G X$

Subject to $AX \geq b$

Where A is an $m \times n$ real matrix and G is an $n \times n$ real [positive semidefinite a symmetric matrix.

A (P.No. 233, Exa.1)Type equation here.

52. If f(X) is a concave function then give the dual of the following quadratic programming problem:

Max $f(X) = C^T X + \frac{1}{2} X^T G X$

Subject to $AX \geq b$

$X \geq 0$

A (P.No. 234, Que.2)

53. Find an optimal solution of the following convex separable programming problem:

Max $z = 3x_1 + 2x_2$

Subject to $4x_1^2 + x_2^2 \leq 16$

& $x_1 x_2 \geq 0$

A (P.No. 241, Exa. 1)

54. Solve the following convex separable programming problem:

Min $Z = x_1^2 - 2x_1 - x_2$

Such that $2x_1^2 + 3x_2^2 \leq 6$

& $x_1 x_2 \geq 0$

A (P.No. 244, Exa. 2)

55. Solve the following convex separable programming problems:

Max $Z = (x_1 - 2)^2 + (x_2 - 2)^2$

Such that $2x_1^2 + 3x_2^2 \leq 6$

& $x_1, x_2 \geq 0$

A (P.No. 248, Que. 5)

56. To prove that :

(a) Every local maximum of the general convex programming problem is it's global maximum.
(b) The set of all optimum solutions (global maximum) of the general convex programming problem is a convex set.

A (P.No. 237, Que. 33)

57. Solve the following convex separable programming problems :

Max $z = x_1 + x_2^4$

Subject to $3x_1 + 2x_2^2 \leq 9$

& $x_1, x_2 \geq 0$

A (P.No. 247, Que. 1)

58. Use Bellmon's optimality principle to divide a positive quantity 'b' into n parts in such a way that's their product is maximum.

OR

Find maximum value of the product of $x_1, x_2, \dots . x_n$

A (P.No. 251, Exa.1)

59. Make use of dynamic programming to show that $\sum_{i=1}^{n} P_i \log P_i$ subject to $\sum_{i=1}^{n} P_i = 1,\ P_i > 0$ is minimum when $P_1 = P_2 \dots \dots . . = \frac{1}{n} (i\ in\ suffix)$

A (P.No. 253, Exa. 2)

60. Use dynamic programming to solve the following problem:

Min $(x_1^2 + x_2^2 + \cdots \dots . + x_n^2)$
Subject to $x_1, x_2, \dots \dots x_n = b$
& $x_1, x_2, \dots \dots . x_n \geq 0$

A (P.No. 255, Exa.3)

61. Solve the following problems by using dynamic programming:

Min $\sum_{i=1}^{n} x_1^2$ Subject to $\sum_{i=1}^{n} x_1 = b_i x_i \geq 0$

i=1,2,…..n

Hence or otherwise minimize $x_1^2 + x_2^2 + x_3^2$

Subject to $x_1, x_2, x_3 \geq 15$

& $x_1, x_2, x_3 \geq 0$

A (P.No. 257, Que.1)

62. Solve the following problems by using dynamic programming:

Maximize value of y_1, y_2, y_3 Subject to

$y_1 + y_2 + y_3 \leq 15\ and\ y_1, y_2, y_3 \geq 0$

A (P.No. 257, Que.5)

63. Use dynamic programming to solve the following L.P.P.

Max. $z = 2x_1 + 5x_2$
Such that $2x_1 + x_2 \leq 43$
$2x_2 \leq 46$

  & $x_1, x_2 \geq 0$

A. (P.No. 259, Exa. 1)

64. Solve the following L.P.P. using dynamic programming:

Max $z = 3x_1 + 5x_2$

Such that $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 0$

& $x_1, x_2 \geq 0$

A (P.No. 260, Exa. 2)

65. Solve the following L.P.P. using dynamic programming :

Max. $z = 3x_1 + 7x_2$

S.T. $x_1 + 4x_2 \leq 8$

$x_2 \leq 8$

& $x_1, x_2 \geq 0$

A (P.No. 264, Que.1)

66. Solve the following L.P.P. using dynamic programming :

Max. $z = 3x_1 + x_2$

S.T. $2x_1 + x_2 \leq 6$

$x_1 \leq 2$

$x_2 \leq 4$

& $x_1, x_2 \geq 0$

A (P.No. 264, Que.5)

67. Explain the solution of linear programming problem using dynamic programming?

A (P.No. 253)

68. Formulate the Kuhn-Tucker necessary conditions for the following problem

Maximize $f(X)$

Subject to $g_i(X) \geq 0 \;\; ; i = 1, 2, \dots \dots m$

$g_i(X) \leq 0, \;\; i = m + 1, m + 2, \dots . P$

$h_i(X) = 0, \;\; j = 1, 2, \dots \dots . . q$

$X_i \geq 0$

A (P.No. 132, Que. 3)

69. Solve the following L.P.P. using dynamic programming:

Max. $z = 10x_1 + 30x_2$

S.T. $3x_1 + 6x_2 \leq 168$

$12x_2 \leq 240$

& $x_1, x_2 \geq 0$

A (P.No. 264, Que. 3)

70. If f(x) is a convex function over the non negative or that of E^n then show that:

$S = \{X : f(x) \leq b, X \geq 0\}$ is a convex set.

A (P.No. 119, Que.3)

71. Show that the following function are convex:
   (a) $f(x) = |x|$
   (b) $f(x) = e^x$

A (P.No. 19, Que. 9)

72. A positive quantity b is to be divided in to n parts in such a way that the product of the n parts is maximum. Use Lagrange Multiplier technique to obtain the oprimal sub division.

A (P.No. 130, Exa. 14)