Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final)
Paper Code:MT-10
Mathematical Programming
Section –B
(Short Answers Questions)

1. To show that A hyperplane is a closed set.

A (P.No. 4, The. 1.1)

2. A hyperplane is given by the equation:

$$3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$$

Find in which half spaces do the points (-6, 1, 7, 2) and (1, 2, 4, 1) lie.

A (P.No. 9, Example 1.1)

3. Show that $f(x) = x^2$ is a convex function.

A (P.No. 11, Example 1.2)

4. Toshow that the hyperplane is a convex set.

A (P.No. 5, The. 1.2)

5. To prove that the closed half spaces $H_1 = \{X : X\,X \geq Z\} and\ H_2 = \{X : X \leq Z\}$ are convex sets.

A (P.No. 5, The. 1.3)

6. To find the nature of quadratic form $Q(X) = X'\,AX$ where

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

A (P.No. 15, Example 1.6)

7. Explain the algorithm of revised simplex method (Standard fom-I)

A (P.No. 25, Example 2.3)

8. Solve the L.P.P. using standard form-I or II of revised simplex method.

Max $z = x_1 + x_2$

S.t. $3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

& $x_1, x_2 \geq 0$

A (P.No. 49, Que. 7)

9. Explain the basis and its inverse in standard form-II?

A (P.No.37)

10. Solve the following linear programming problem by revised simplex method.

Max $z = 2x_1 + x_2$

S.t. $3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3$

and $x_1, x_2 \geq 0$

A (P.No. 26, Example 2.1)

11. Explain the mathematical form revised simplex method (Standard-II)?

A (P.No. 37)

12. Explain the bounded variable simplex algorithm?

A (P.No. 50)

13. Explain All I.P.P. algorithm or cutting plane algorithm?

A (P.No. 67)

14. Explain the geometrical interpretation of Gomory's cutting plane method?

A (P.No. 83)

15. Explain Gomory's mixed I.P.P. method (Fractional cut method)?

A (P.No. 90)

16. Solve the integer programming problem :

Max $z = 7x_1 + 9x_2$

S.t. $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 3$

$x_1 \geq 0,\ x_2 \geq 0$

and x_1, x_2 are integers

A (P.No. 84, Example 5)

17. Solve the following mixed I.P.P. problem?

Maximize $Z = -3x_1 + x_2 + 3x_3$

S.t. $-x_1 + 2x_2 + 3 \leq 4$

$4x_2 - 3x_2 \leq 2$

$x_1 - 3x_2 + 2x_3 \leq 3$

$x_1\ and\ x_2$ are integers and $x_1, x_2, x_3 \geq 0$

A (P.No. 94, Que. 5)

18. Explain the fractional cut?

A (P.No. 90)

19. Explain the algorithm of branch and bound?

A (P. No. 96)

20. Solve the following I.P.P. by branch and bound technique?

Max $Z = x_1 + x_2$

s.t. $3x_1 + 2x_2 \leq 12$

$x_2 \leq 2$

$x_1, x_2 \geq 0$ *and integers*

A (P. No. 97)

21. Explain the Geometrical Interpretation of branch and bound method with example?

A (P.No. 108, Example 5)

22. Distinguish between pure and mixed programming?

A (P.No. 111)

23. Use Branch and bound method to solve the following integer linear programming problems.

Max $Z = x_1 + 5x_2$

s.t. $x_1 + 10x_2 \leq 20$

$x_2 \leq 2$

$x_1, x_2 \geq 0$ *and integers*

A (P.No. 112, Que. 8)

24. Difference between continuous and integer programming?

A (P.No. 111)

25. Write the quadratic form :

$Q_{(x)} = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$ in matrix form.

A (P.No. 114, Exa. 2)

26. In each of the following cases write the objective function in the form:

$$Z = X^T AX + q^T X$$

(i) $z = x_1^2 - 4x_1x_2 + 6x_1x_3 + 5x_2^2 - 10x_2x_3 + 8x_3^2$

(ii) $z = 5x_1^2 - 12x_1x_2 - 16x_1x_3 + 10x_2^2 - 26x_2x_3 + 17x_3^2 - 2x_1 - 4x_2 - 6x_3$

A (P.No. 115, Exa. 4)

27. Determine the sign of definiteness for each of the following matrices :

(a) $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 5 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & -3 & 3 \\ 2 & 0 & -5 \end{bmatrix}$

A (P.No. 117, Exa.5)

28. Test the definiteness of the quadratic form:

$$X^T AX = (x_1 x_2 x_3) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

A (P.No. 117, Exa. 6)

29. Explain the positive and negative definiteness of quadratic froms?

A (P.No. 116)

30. Determine whether or not the quadratic forms $A^T\ AX$ are positive definite, where:

(i) $A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 1 & 1 \\ 3 & 5 \end{bmatrix}$

A (P.No. 118, Exa. 7)

31. Obtain a set of necessary condition for the non-linear programming problem :

Maximize $\quad Z = x_1^2 + 3x_2^2 + 5x_3^2$

S.t. $\quad 5x_1 + 2x_2 + x_3 = 5$

& $\quad x_1, x_2, x_3 \geq 0$

A (P.No. 128, Exa.12)

32. If $f(X,\ \lambda)$ has a saddle point (X_0, λ_0) for each $\lambda \geq 0$ then to prove that

$$G(X_0) \leq 0 and\ \lambda_0^T G(X_0) = 0$$

A (P.No 140, The. 1)

33. Obtain the necessary conditions for the following nonlinear programming problem:

Minimize $f(X) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$

Subject to $2x_1 - x_2 = 4$

$x_1, x_2 \geq 0$

A (P.No. 149, Exa.3)

34. Use Lagrangian function to find the optimal solution of the following non linear programming problem:

Minimize $f(X) = -3x_1^2 - 4x_2^2 - 5x_3^2$

Subject to $x_1 + x_2 + x_3 = 10$

$x_1, x_2, x_3 \geq 0$

A (P.No. 152, Exa. 5)

35. Solve the following nonlinear programming problem graphically:

Maximize $\quad f(x_1, x_2) = x_1 + 2x_2$

Subject to $\quad x_1^2 + x_2^2 \leq 1$

$2x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

A (P.No. 157, Exa. 9)

36. Write the Kuhn-Tucker necessary and sufficient conditions for the following non linear programming problem to have on optimal solution.

Min $\quad f(x_1, x_2) = x_1^2 - 2x_1 - x_2$

Subject to $\quad 2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

& $\quad x_1, x_2 \geq 0$

A (P.No. 170, Exa. 1)

37. What are the Kuhn-Tucker conditions and how are they of fundamental importance in the theory of nonlinear programming.

A (P.No. 163)

38. Formulate the Kuhn-Tucker necessary conditions for the following prolem:
Maximize $f(X)$
Subject to $g_i(X) \geq 0; \ i = 1, 2, \dots \dots m$
$g_i(X) \leq 0; \ i = m+1, m+2, \dots \dots p$
$h_i(X) = 0 \ ; \ j = 1, 2, \dots . q$
& $X \geq 0$

A (P.No. 182)

39. Use Kuhn-Tucker conditions to determine x_1, x_2, x_3 so as to minimize
Min. $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 6x_2$
S.t. $x_1 + 3x_2 \leq 2$
$2x_1 + 3x_2 \geq 12$
$x_1, x_2 \geq 0$

A (P.No. 174, Exa.5)

40. Explain the general non-linera programming problem (NLPP)?

A (P.No. 162)

41. Determine the optimal solution of the following non linear programming problem using the Kuhn-Tucker conditions:
Min. $f(x_1, x_2) = x_1^2 + x_2^2 - x_1x_2$
S.t. $x_1 + x_2 \leq 8$
$x_1, x_2 \geq 0$

A (P.No. 173, Exa. 4)

42. Explain the algorithm of Beale's method?

A (P. No. 207)

43. Solve the following quadratic programming problem by Beale's method:
Max. $f(x_1, x_2) = x_1 + x_2 - x_1^2 + x_1x_2 - 2x_2^2$
S.t. $2x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$

A (P.No. 216, Exa. 9)

44. Solve the following quadratic programming problem of Wolfe's method :
Min. $f(x_1, x_2) = 4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$
S.t. $2x_1 + x_2 \leq 6$
$x_1 - 4x_2 \leq 0$
& $x_1, x_2 \geq 0$

A (P.No. 201, Exa. 5)

45. Use Wilfe's method to solve the following quadratic programming problem:
Min. $f(x_1, x_2) = x_1^2 + x_2^2 - 2x_1 - 4x_2$
S.t. $x_1 + 4x_2 \leq 5$

$2x_1 + 3x_2 \le 6$

& $\quad x_1, x_2 \ge 0$

A (P.No. 204, Exa. 6)

46. Explain the Interstice procedure by Wolfe's method?

A (P.No. 188)

47. Explain the quadratic programming problems?

A (P.No. 184)

48. Drive the dual of the quadratic programming problem:
Min $\quad f(X) = C^T X + \frac{1}{2} X^T G X$

Subject to $\quad AX \ge b$

Where A is an $m \times n$ real matrix and G is $n \times n$ real positive semidefinite a symmetric matrix.

A (P.No. 233, Exa. 1)

49. Set G be a positive semi definite symmetric matrix then write the dual of the following quadratic programming problem:
Min $\quad f(X) = C^T X + \frac{1}{2} X^T G X$
Subject to $\quad AX \ge b$

& $\quad X \ge 0$

A (P.No. 234)

50. Explain the quadratic programming and duality?

A (P.No. 224)

51. Explain the convex programming problem with suitable example?

A (P.No. 236)

52. To Prove that every local maximum of the general convex programming problem is its global maximum.

A (P.No. 237, The.1)

53. To show that the set of all optimum solutions (global maximum) of the general convex programming problem is a convex set.

A (P.No. 238, The. 2)

54. Explain the piecewise linear approximation of a non linear continuous function.

A (P.No. 239)

55. Explain the algorithm of separable programming?

A (P.No. 240)

56. Solve the following convex separable programming problem:
Max. $\quad z = 2x_1 - x_1^2 + x_2$
S.t. $\quad 2x_1 + 3x_2 \le 6$
$2x_1 + x_2 \le 4$

& $x_1, x_2 \geq 0$

A (P.No. 247, Que. 2)

57. Explain the basic features of a dynamic programming problem?

A (P.No. 249)

58. Explain the solution procedure of dynamic programming?

A (P.No. 250)

59. Solve the following convex separable programming problem:
    Max $z = (x_1 - 2)^2 + (x_2 - 2)^2$
    Such that $x_1 + 2x_2 \leq 4$
    & $x_1, x_2 \geq 0$

A (P.No. 248, Que. 5)

60. Solve the following problems by suing dynamic programming:
    Min $z = \sum_{i=1}^{n} x_i$
    Subject to $\prod_{i=1}^{n} x_1 = b$
    & $x_i \geq 0, \quad i = 1, 2, \dots \dots n$

A (P.No. 257, Que. 2)

61. Explain the solution of Linear programming problem using dynamic programming?

A (P.No. 258)

62. Solve the following problems by using dynamic programming :
    Minimize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \quad c_1 < c_2 < \cdots .. < c_n$
    Subject to $x_1 + x_2 + \cdots .. + x_n = b$
    and $x_1, x_2, \dots . x_n \geq 0$

A (P.No. 257, Que.4)

63. Solve the following problems by using dynamic programming :
    $- \sum_{i=1}^{n} P_i \, log \, P_1$
    Subject to $\sum_{i=1}^{n} P_i = 1 \, is$
    Maximum when $P_1 = P_2 = \cdots \dots .. = P_n = 1/n$

A. (P.No. 257, Que. 3)

64. Use dynamic programming to solve the following L.P.P.
    Max $z = 2x_1 + 5x_2$
    Such that $2x_1 + x_2 \leq 43$
    $2x_2 \leq 46$
    & $x_1, x_2 \geq 0$

A (P.No. 259, Exa. 1)

65. Solve by dynamic programming :
    Max. $z = 3x_1 + 7x_2$
    S.T. $2x_1 + x_2 \leq 8$

$2x_1 + 2x_2 \leq 15$

& $\quad x_1, x_2 \geq 0$

A (P.No. 262, Exa. 3)

66. Solve by dynamic programming:

Max. $\quad z = x_1 + 9x_2$

S.T. $\quad 2x_1 + x_2 \leq 25$

$x_2 \leq 11$

& $\quad x_1, x_2 \geq 0$

A (P.No. 263, Exa. 4)

67. Solve by dynamic programming.

Max $\quad z = 3x_1 + x_2$

Subject to $\quad 2x_1 + x_2 \leq 6$

$x_1 \leq 2$

$x_2 \leq 4$

and $\quad x_1 \geq 0, x_2 \leq 0$

A (P.No. 264, Que. 5)

68. Write the quadratic form in matrix vector notation.

$$f(X) = x_1^2 - 2x_1x_2 + 4x_2^2$$

A (P.No. 119, Que. 3)

69. Determine whether of the following quadratic form:

$$x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$$

A (P.No. 120, Que. 9)

70. Write down the quadratic form whose associated matrices are :

(i) $\begin{bmatrix} 2 & -3 & 1 \\ -3 & 4 & 2 \\ 1 & 2 & -6 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 14 \end{bmatrix}$

A (P.No. 119, Que.4)

71. Solve the following L.P.P. using dynamic programming:

Max $\quad z = 3x_1 + 7x_2$

Subject to $\quad x_1 + 4x_2 \leq 8$

$x_2 \leq 8$

and $\quad x_1 \geq 0, x_2 \leq 0$

A (P.No. 264, Que. 1)