

Program : M.A./M.Sc. (Mathematics)

M.A./M.Sc. (Final)

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Mathematical Programming

Section – A

(Very Short Answers Questions)

1. Define a Hyperplanes?

A The equation $c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n = Z$ or $\bar{C} \bar{X} = \bar{Z}$ defines a hyperplane in n-dimensional space E^n Here not all C_1 's are zero simultenuously.

2. Write the set of line segment.

A $S = \{x : x \in E^n \text{ and } x = \lambda x_1 + (1 - \lambda)x_2, 0 \leq \lambda \leq 1\}$

3. Define a convex set ?

A Subset $S \subset R^n$ is convex If and only if

$$x_1x_2 \in S \implies \lambda x_1 + (1 - \lambda)x_2 \in S : 0 < \lambda \leq 1$$

4. What do you mean the Intersection of two convex sets?

A Convex set.

5. Define extreme point

A An extreme point (Vertex) of a convex set is a point of the set which does not lie on any segment point two other points of the set.

6. Define a convex functions?

A Let S be a non empty convex subset of R^n . A function $f(x)$ on S is said to be convex if for any two vectors x_1 and x_2 in S.

$$f[\lambda x_1 + (1 - \lambda)x_2] \leq \lambda f(x_1) + (1 - \lambda)f(x_2). \quad 0 \leq \lambda \leq 1$$

7. Write the table of the revised simplex method standard form-I?

A Starting Table in Standard Form-I (Revised Simplex Method)

Variable in B.F.S.	Solution $X_B^{(1)}$	B_1^{-1}					$Y_k^{(1)} = B_1^{-1}\alpha_k^{(1)}$	$\frac{X_{Bi}}{Y_{ik}} \cdot y_{ik}^{<0}$
		$v_1^{(1)}$	$v_2^{(2)}$	$v_{mM}^{(1)}$	$v_{MH}^{(1)}$		
x_1	X_{B_1}	Y_{1K}
x_2	X_{B_2}	Y_{2K}
.....

.....
.....
x_m	X_{BM}	Y_{MK}
Z	Z	$Z_K - C_K$	θ $= \text{Min} \left(\frac{X_{BI}}{Y_{IK}} \right)$

Here $V_1^{(1)}, V_2^{(2)}, \dots, V_{m+1}^{(1)}$ are the respective columns of the inverse of basis B_1^{-1} . In the column $X_B^{(1)}$ we write values of the variables. In the first table:

$$B_1 = I_m, B_1^{-1} = I_m, X_B^{(1)} = b^{(1)} \text{ and } V_{m+1}^{(1)} = l_{m+1}$$

8. Define a Initial B.F.S. (Only Formula)?

A Initial B.F.S. $X_B^{(1)} = B_1^{-1}b^{(1)}$

9. Write the notation of bounded variable linear programming problem (BVLPP)

A Max or Min $Z = CX$

S.t. $AX \leq, =, \geq b$

$l_j \leq x_j \leq u_j, \quad \forall j = 1, 2, 3, \dots, n$

10. The following l.p.p. by standard form-II of revised simplex method.

Min $Z = x_1 + 2x_2$

S.t. $2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$

and $x_1, x_2 \geq 0$

Write the Introducing surplus variables in the given problem.

A Max $Z = -x_1 - 2x_2 + 0x_3 + 0x_4$

S.t. $2x_1 + 5x_2 - x_3 + 0x_4 = 6$

$x_1 + x_2 + 0x_3 - x_4 = 2$

& $x_1, x_2, x_3, x_4 \geq 0$

11. The following l.p.p. by revised simplex method (Standard form-I)

Max $Z = 3x_1 + x_2 + 2x_3 + 7x_4$

S.t. $2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$

$-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$

$x_1 + x_2 - 2x_3 + 3x_4 \leq 100$

& $x_1 \geq 2, x_2 \geq 1, x_3 \geq 3, x_4 \geq 4$

Write the Introducing slack variables.

A Max $Z^* = Z - 41 = 3u_1 + u_2 + 2u_3 + 7u_4$

$$\begin{aligned} \text{S.t. } & 2u_1 + 3u_2 - u_3 + 4u_4 \leq 20 \\ & -2u_1 + 2u_2 + 5u_3 + u_4 \leq 26 \\ & u_1 + u_2 - 2u_3 + 3u_4 \leq 91 \end{aligned}$$

$$\& \quad u_1, u_2, u_3, u_4 \geq 0$$

Introducing slack variable U_5, U_6, U_7 in the given problem

$$\begin{aligned} 2u_1 + 3u_2 - u_3 + 4u_4 + u_5 &= 20 \\ -2u_1 + 2u_2 + 5u_3 + 0u_5 + u_6 &= 26 \\ u_1 + u_2 - 2u_3 + 3u_4 + 0u_5 + 0u_6 + u_7 &= 91 \\ -3u_1 - u_2 - 2u_3 - 7u_4 + 0u_5 + 0u_6 + 0u_7 + Z^* &= 0 \end{aligned}$$

$$\& \quad u_1, u_2, u_3, u_4, u_5, u_6, u_7 \geq 0$$

12. Write the formula of departing vector in BVLPP.

$$\text{A } \theta_1 = \min_i \left\{ \frac{X_{Bi}}{Y_{ir}} \cdot y_{ir} > 0 \right\}$$

$$\theta_2 = \min_i \left\{ \frac{U_1 - X_{Bi}}{-Y_{ir}} \cdot y_{ir} > 0 \right\}$$

$$\text{i.e. } \theta = \min \{ \theta_1 \theta_2 U_r \}$$

Where U_r is the upper bound of variable x clearly when $Y_{ir} > 0, \theta_2 \rightarrow \infty$

13. Define a Integer programming problem (I.P.P.)?

A A linear programming problem:

$$\text{Max } Z = cx$$

$$\text{Subject to } A\bar{X} = b,$$

$$\bar{X} \geq 0$$

And some $x_j \in X$ are integers.

Where $C, X \in R^n, b \in R^m$ and A is an $m \times n$ real matrix is called integer programming problem (I.P.P.)

14. What do you mean by mixed Integer programming problem (Mixed I.P.P.)?

A An Integer programming problem is said to be "Mixed Integer programming problem" If not all $x_j \in X$ are integers.

15. How can you construct Gemory's constraint?

A The consider I.p.p. required modification by Introducing Secondary constraints also called Gemory's constraints.

16. How can you represent the "all Integer programming problem"?

A R.E. Geory, In 1956

17. The following (integer programming problem) simplex table :

			C_j	7	9	0	0
C_B	B	X_B	B	y_1	y_2	y_s	y_4

9	α_2	x_2	$3\frac{1}{2}$	0	1	7/22	1/22
7	α_1	x_1	$4\frac{1}{2}$	1	0	-1/22	3/22
$Z_j - C_j$				/	!	?	?

Write $Z_j - C_j = ?$

A

$Z_j - C_j$	0	0	28/11	15/11
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18. How can solving the “branch and bound technique?”

A The Integer and the mixed Integer programming problems [Both]

19. Write Branch and Bound Method divided the feasible region into smaller parts.

A Branching

20. The following I.P.P. (Use Branch and bound Technique)

$$\text{Max } Z = 3x_1 + 3x_2 + 13x_3$$

$$\text{s.t. } -3x_1 + 6x_2 + 7x_3 \leq 8$$

$$6x_1 - 3x_2 + 7x_3 \leq 8$$

$$0 \leq x_j \leq 5 \text{ and } x_j \text{ are integers } j = 1, 2, 3.$$

Write the Initial B.F.S.

A Initial B.F.S.

$$x_4 = 8$$

$$x_5 = 8, x_1 = x_2 = x_3 = 0$$

21. While solving an I.P.P. any non-integer variable in the solution is picked up to.

A Obtain the cut constant.

22. What do you mean by Branch and Bound method?

A This technique is applicable to both the L.B.P. pure as well as mixed. In this method first we solve the continuous I.P.P. ignoring the integer valued restrictions.

23. The following I.P. table :

			C_j	3	3	13	0	0	$\theta = \frac{X_{BU}}{Y_{ik}} \cdot Y_{im} > 0$
C_B	B	X_B	b	y_1	y_2	y_3	y_4	y_5	
0	α_4	x_4	8	-3	6	7	1	0	8/7
0	α_5	x_5	8	6	-3	7	0	1	8/7

$(Z_j - C_j)$?	?	?	?	?	
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Write the $(Z_j - C_j)$ and Min θ

A

$Z_j - C_j$	-3	-3	-13	0	0
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24. What do you mean by developed by branch and bound algorithm.

A "Land and Doing"

(To Solve all – integer and mixed integer programming problems.)

25. Define a Quadratic Form?

A Quadratic Form is a function of n-variables

I.C. $Q(X) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j$ Where a_{ij} are constants. It can also be written as $Q(X) = A^T A X$ where $X = [x_1, x_2, \dots, x_n]$ and $A = [a_{ij}]$ is a $n \times n$ symmetric matrix.

26. Write the quadratic from:

$Q(X) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$ in matrix form.

A $Q(X) = x_1^2 + 2x_2^2 - 7x_3^2 - (2 - 2)x_1x_2 + (3 + 3)x_1x_3 - \left(\frac{5}{2} - \frac{5}{2}\right)x_2x_3$

$$= (x_1 x_2 x_3) \begin{bmatrix} 1 & -2 & 3 \\ -2 & 2 & -\frac{5}{2} \\ 3 & -\frac{5}{2} & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

27. Write the objectives Function

$$Z = x_1^2 + 2x_1x_2 + 46x_1x_3 + 3x_2^2 + 2x_2x_3 + 5x_3^2 + 4x_1 - 2x_2 + 3x_3$$

In the form

$$Z = X^T \cdot A X + q^T X$$

$$A \quad Z = (x_1 x_2 x_3) \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (4, -2, 3) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{Here } A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}, q = \begin{bmatrix} 4 \\ -2 \\ \end{bmatrix}$$

28. Write the definiteness of the quadratic from :

$$X^T A X = (x_1 x_2 x_3) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

A The given quadratic from is indefinite.

29. Define a matrix notation of GNLPP (General Non-Linear- Programming Problem)

A Maximize

OR $Z = f(X)$ Determine $T \in R^n$

Minimize

s.t. $g^i(x) \leq, = \text{ or } \geq b_i, x \geq 0, i = 1, 2, \dots, m$

Where $f(x)$ or some $g^i(x)$ or both are non-linear in X .

30. What do you mean by Lagrange's Function.

A $L(x_1, x_2, \lambda) = f(x_1, x_2) - \lambda h(x_1, x_2)$ Here λ is an unknown constant called Lagrange's Multiplier and the function $L(x_1, x_2, \lambda)$ is called Lagrange's function.

31. Define a mathematical programming problem?

A A general mathematical programming problem (MPP) can be stated as given below:

Minimize $f(X)$

Subject to $g_i(X) \geq 0$

$h_j(X) = 0$

$X \in S$

Where $X = (X_1, X_2, \dots, X_n)^T$ is a vector of decision variables and $f_i, g_i (i = 1, 2, \dots, m)$ and $h_j (j = 1, 2, \dots, P)$ are the real valued functions of variable x_1, x_2, \dots, x_n

32. Define a relation b/w saddle point of $F(X, \lambda)$ and Minimal point of $F(X)$.

A Minimize $z = f(x)$

Subject to $G(X) \leq 0$

And $F(X, \lambda) = f(X) + \lambda G(X)$

Where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$

& $\lambda \geq 0$

33. Write the necessary conditions for the following nonlinear programming problem.

Minimize $f(X) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$

Subject to $2x_1 - x_2 = 4$

$x_1, x_2 \geq 0$

A The necessary conditions for the minimum of $f(x)$ are :

$$6x_1 + 2x_2 + 6 + 2\lambda = 0$$

$$2x_2 + 2x_1 + 2 - \lambda = 0$$

$$2x_1 - x_2 - 4 = 0$$

34. Write the Hessian Matrix?

A Hessian matrix : $H^B = \begin{bmatrix} O & U \\ U^T & V \end{bmatrix}$

35. Write the $\lambda_0^T G(X_0) (\because GX_0 \leq 0)$

If $f(x, \lambda)$ has a saddle point (X_0, λ_0) for each $\lambda \geq 0$

A $\therefore \lambda_0^T G(X_0) = 0$

36. What do you mean by Saddle point?

A A point (X_0, λ_0) is said to be a saddle point of the Lagrangian function $f(x, \lambda)$

$$F(X_0, \lambda) \leq F(X_0, \lambda_0) \leq F(X, \lambda_0)$$

37. Define convex programming problem?

A A convex programming problem can be stated as follows :

Minimize $f(X), X = (X_1, X_2, \dots, X_n) \in E^2$

Subject to : $g_1(X) \leq 0 ; i = 1, 2, \dots, m$

$$X \geq 0$$

Where $f(X)$ and $g_i(X)$ are all convex functions.

38. Write the Hessian matrix for the function $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$

A $H^B = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

39. Write the Lagrangian function in the following nonlinear problem.

Min. $f(x_1, x_2) = x_1^2 - 2x_1 - x_2$

S.t. $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

A The Lagrangian Function for the given minimization problem is:

$$F(X, \lambda) = x_1^2 - 2x_1 - x_2 + \lambda_1(2x_1 + 3x_2 - 6) + \lambda_2(2x_1 + x_2 - 4)$$

40. Write the Kuhn-Tucker conditions in the following non linear programming problem :

Max. $f(x) = 8x - x^2$

Subject to $x \leq 3$

$x \geq 0$

A The Kuhn-Tucker conditions are:

$$8 - 2x - \lambda \leq 0, x \geq 0, x(8 - 2x - \lambda) = 0$$

& $(3 - x) \geq 0, \lambda \leq 0, \lambda(3 - x) = 0$

41. Write the Hessian matrix for the Function :

$$f(x_1, x_2, x_3) = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$$

A Hessian matrix :

$$H^B = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 0 \\ 1 & 2 & -2 & 0 & 0 \\ 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

42. If a convex objective function is to be minimized then write the solution space.

A Convex set

43. What do you mean by quadratic programming problem?

A The problem of optimizing a quadratic function subject to a set of linear constraints is called a quadratic programming problem.

44. Define a mathematical form of general quadratic programming problem.

A General non linear programming problem:

$$\text{Maximize } f(X) = C^T X + \frac{1}{2} X^T G X$$

$$\text{Subject to } AX \leq 0$$

$$X \geq 0$$

Where X and $C \in E^n$, $b \in E^m$, G is $n \times n$ symmetric matrix and A is an $m \times n$ matrix is called general quadratic programming problem.

45. Define a Mathematical notation of Wolfe's method.

$$\text{A Maximize } f(x_1, x_2, \dots, x_n) = \sum_{j=1}^n C_j x_j + \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n x_j d_{jk} x_k$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} x_j \leq b, \quad i = 1, 2, \dots, m$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n$$

Where $d_{jk} = d_{kj}$ for all j and $k=1, 2, \dots, n$

And $b_j \geq 0$ for all $i = 1, 2, \dots, m$

$$46. \text{ Write the Min. } \left\{ \frac{\frac{1}{2}}{|-1/2|}, \frac{\frac{3}{2}}{|-11/2|} \right\}$$

A 3/11

47. How can you quadratic programming problem is a convex programming problem?

A True

$$48. \text{ Write the minimum } \left\{ \frac{3/2}{|1/2|}, \frac{1/2}{|-3/2|}, \frac{3}{|-3|} \right\} \text{ terms?}$$

A 1/3

49. Define a duality in Non-linear programming problem?

$$\text{A Maximize } f(x)$$

$$\text{(PI) Subject to } g_1(X) \geq 0 ; \quad i = 1, 2, \dots, m$$

$$h_j(X) = 0, \quad j = 1, 2, \dots, p$$

Where $X^T = (x_1, x_2, \dots, x_n)$ and the functions f, g_i and h_j are assumed to be continuously differentiable functions over some open set $S \subset E^n$

50. Write the dual function in the following primal Function.

$$L(X) = \text{Min} : (X, \lambda, H), \quad X \in E^n \quad (\lambda, H) \in \Lambda$$

A Dual function :

$$L^*(\lambda, H) = \text{Max} : (X, \lambda, H), (\lambda, H) \in \Lambda, \quad X \in E^n$$

51. Write the dual problem in the following quadratic programming problem?

$$\text{Max} \quad f(X) = C^T X + \frac{1}{2} X^T G X$$

$$\text{Subject to} \quad AX = b, \quad X \geq 0$$

$$\text{A Min} \quad L(X, \lambda) = \frac{1}{2} X^T G X + \lambda^T b$$

$$\text{Subject to} \quad -G X + A^T \lambda \geq C$$

$$X \geq 0$$

52. How can you the dual of the dual of the quadratic programming problem is the quadratic programme itself – true or false?

A False

53. What is Lagrangian Saddle point?

A The lagrangian function $L(X, \lambda, H)$ as a function of X and $L(\lambda, H)$. The point $L(X_0, \lambda_0, H_0)$ is called a Lagrangian saddle point of L where $X_0 \in E^n, (\lambda_0, H_0) \in \Lambda$ and

$$L(X, \lambda_0, H_0) \leq L(X_0, \lambda_0, H_0) \leq L(X_0, \lambda, H) \text{ for all } X \in E^n \text{ and } (\lambda, H) \in \Lambda$$

54. Write the dual programming problem in the following quadratic programming problem.

$$\text{Max} \quad f(X); \quad X \text{ is unrestricted in sign}$$

$$\text{Subject to} \quad g_1(X) = b_i, \quad i = 1, 2, \dots, m$$

A Dual programming problem is

$$\text{Min } L(X, \lambda)$$

$$\text{Subject to} \quad \frac{\partial L(X, \lambda)}{\partial x_j} = 0; \quad j = 1, 2, \dots, n$$

$$\text{Where } L(X, \lambda) = f(X) + \sum_{i=1}^m \lambda_i (b_i - g_i(X))$$

55. Define Convex programming problem?

A The problem of maximizing a concave function or minimizing a convex function over a convex set is called a convex programming problem.

A general convex programming problem (C.P.P.) can be defined as:

$$\text{Maximize} \quad f(x)$$

$$\text{Subject to} \quad x \in S \quad X^T D X$$

Where $x \in R^n, f(x)$ is a concave function on a convex set SCR^n

56. Define a separable programming problem?

A A non linear programming problem of the form

$$\text{Maximize } Z = \sum_{j=1}^n f_j(x_j)$$

$$\text{Subject to } \sum_{j=1}^n g_{ij}(X_j) \{ \leq, =, \geq \} b_i, \quad i = 1, 2, \dots, m$$

57. What do you mean by convex separable programming problem (CSPP) ?

A A convex programming problem in which all the functions (objective function and constraints) are separable is called a convex separable programming problem.

58. Define separable function?

A A function $f(x_1, x_2, \dots, x_n)$ is said to be separable if it can be expressed as the sum of n single valued functions.

$$f_1(x_1), f_2(x_2), \dots, f_n(x_n) : i. e.$$

$$f(x_1, x_2, \dots, x_n) = f_1(x_1) + f_2(x_2) + \dots + f_n(x_n)$$

For example linear function given by

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Where c' are constants is a separable function

59. Write the formula of straight line segment $[f(x)]$?

$$A \quad \bar{f}(x) = f_k + \frac{f_{k+1} - f_k}{x_{k+1} - x_k} (x - x_k); \quad x \in [x_k, x_{k+1}]$$

60. Write the $g_{11}(x)$ & $g_{12}(x_2)$ in the following points and functions.

x_1	0	1	2	-	-	$g_{11}(x_1) = 4x_1^2$
x_2	0	1	2	3	4	$g_{12}(x_2) = x_2^2$

A

x_1	$g_{11}(x_1) = 4x_1^2$	x_2	$g_{12}(x_2) = x_2^2$
0	0	0	0
1	4	1	1
2	16	2	4
		3	9
		4	16

61. Define a transition function?

A The total decision process is related to it's adjoining stage by a quantitative relationship called a transition function.

62. What do you mean by Bellman's principal of optimality?

A An Optimal policy (a sequence of decisions) has the property that whatever the initial state and decisions are, the remaining decisions must constitute an optimal policy will regard to the state resulting from the first decision.

63. Write the mathematical form of Principal of optimality?

A $f_n(x) = \text{optimum} [r(d_n) \boxtimes f_{n-1}\{T(x \boxtimes d_n)\}] \quad d \in \{x\}$

Where, symbol \boxtimes denotes any mathematical relationship between x and d_n including addition, subtraction, multiplication and root operations.

$f_n(x)$: the optimal return from an n – stage process when initial state is x .

$\{x\}$: set of all admissible decisions.

$r(d_n)$: immediate return due to decision d_n

$T\{x \boxtimes d_n\}$: The transfer function which gives the resulting state.

64. Write the Bellman's principal of optimality in the following problem.

$$f_r(b) = \text{Max} (x_1, x_2, \dots, x_r), \quad r = 1, 2, \dots, n$$

$$x_1, x_2, x_r$$

Subject to $x_1 + x_2 + \dots + x_r = b, x_r \geq 0$

A. Bellman's principal of optimality, we have

$$f_x(b) = \max_{x_r} [x_r \max (x_1, x_2, \dots, x_{r-1})], \quad x_1 \dots x_{r-1}$$

$$= \max_{x_r} [x_r \cdot f_{r-1}(b - x_r)]$$

$$= \max [Z \cdot f_{r-1}(b, z)], \quad \text{if } x_r = z \text{ to be } 0 \leq z \leq b$$

decision variable.

65. Write the $\frac{ds}{dz}$ in the following function $S = z^2 + M \left(\frac{b}{z}\right)^{2/m}$

A $\frac{ds}{dz} = 2z - \frac{2b^{2/m}}{z^{m+1}}$

66. What is a number of variables in a problem.

A \therefore Number of variable in a problem = Number of stages

67. Define recurrence relation of Bellman's principle?

A $f_k(B_1^k, B_2^k, \dots, B_m^k)$

$$= \max_{x_k} [c_k x_k + f_{k-1}(B_1^k - \alpha_{1k} x_k, \dots, B_m^k - \alpha_{mk} x_k)]$$

We can determine x_j (Optimal value of r_k) at the stage $k, k = \overline{1, n}$

Which yields $f_k(\beta_1^k, \beta_2^k, \dots, \beta_m^k)$

Thus at the n th stage optimal value of i.e. x_n is determined.

68. Solve by dynamic programming.

$$\text{Max } z = x_1 + 9x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 25$$

$$x_2 \leq 11$$

$$\text{And } x_1 \geq 0, x_2 \leq 0$$

A Hint : $f_1(u, u_1) = \text{Max} (x_1)$ where $x_1 \geq 0, u_1 \geq 0, x_1 \leq \frac{u_1}{2}$

$$= \frac{u_1}{2}, \because 0 \leq x_1 \leq \frac{u_1}{2}$$

$$f_2(u_2, u_2) = \text{Max} [9x_2 + f_1(u_2 - x_2, v_2 - x_2)]$$

$$= \min_{x_2} \left[\frac{17}{2} x_2 + \frac{u_2}{2} \right]$$

Where $0 \leq x_2 \leq \text{Min}(u_2, v_2) = \text{Min}(25, 11)$

106 ay $x_2^* = 11$

Hence optimal solution is $x_1 = 7, x_2 = 1$ and $\text{Max } Z = 106$

69. How can solved by probabilsibistic problems?

A Fynamic programing

70. Writhe the subproblem (For stage -1) in the folloing L.P.P.

[Using Dyanming]

$$\text{Max } z = 3x_1 + 5x_2$$

$$\text{Subject to } x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 13$$

$$\text{and } x_1, x_2 \geq 0$$

A Sub problem are :

$$f_1(u_1, v_1, w_1) = \text{Max}(3x_1)$$

$$\text{Subject to } x_1 \leq u_1$$

$$0 x_2 \leq v_1$$

$$3x_1 \leq w_1$$

$$\text{And } x_1 \geq 0 \text{ for stage -1}$$

71. Solve the following L.P.P. using dynamic programming.

$$\text{Max } z = 3x_1 + x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 6$$

$$x_1 \leq 2$$

$$x_1 \leq 4$$

$$\& \quad x_1, x_2 \geq 0$$

A $x_1 = 2, x_2 = 2, \text{Max } z = 8$

72. How can solved by discrete & continuous, determisnistic problems?

A Dynamic programming