Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final) Question Bank-2015
Paper Code:MT-08
Section A (Very short answer type Questions)

1. (a). Derive the Newton–Raphson formula to find p^{th} root of a given number N.
(b) Write the formula for finding root of the equation by Muller's method what is rate of convergence of this method?

(c) Find Quotient and remainder on division of polynomial $x^4 - 5x^3 + 4x - 18$ by a linear factor $(n-2)$, using synthetic division method.

(d) Find the largest eigen value of A = $\begin{pmatrix} 12 \\ 34 \end{pmatrix}$ by power method

Ans. Let $X_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ be the eigen valve.

Then $AX_0 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 4(0.5) = 4X_1$

$AX_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 5.5 \end{pmatrix} = 5.5\begin{pmatrix} 0.45 \\ 1 \end{pmatrix}$

$\therefore$ The largest eigen value is 5.5

(e) Derive Hermitian matrix and unitary matrix

(f) What is Economization of the power series.

(g) Write the formula for Adams – Moultan Predictor.

(h) What do you mean by Absolute stable and relatively stable methods?

(i). State the Newton – Raphson formula and explain how it is
used to obtain real root ?

(j). Write the formula for finding a root of the equation by chebshe v method is faster than Newton- Raphson method.

(k). What is the advantage of Graeffe's root squaring method.

(l). For solving a live on system, compare Gaussian elimination method and Gauss Jordan method.

(m). Write the Rutishauser method to find the eigen valves of the matrix

(o). What is least – squares principle

(p). Write the Tayler series expansion of a function.

(q). What is Initial Value Problem (IVP) and Bandary Value Problem (BVP).

Section –B

1. Find a root of the equation $3x - \sqrt{1 + \sin x} = 0$ using iteration method.

2. Find complex root of the equation $Z^2 + 1 = 0$ by Newton – Raphson method. Use $Z_0 = \frac{1}{2}(1+i)$ as an initial approximation.
3. Solve the system of equation by LV Factorization method:-
$2x+3y+3=9,\ x+2y+3y=6\ ,\ 3x+y+2z=8$
4. Obtain a second degree polynomial approximation to the function $f(x)=x^3$, on the interval $[0,1]$, using least– squares principle. Take weight function $W(x)=1$
5. Using the chebyshev polynomials, obtain the least squares approximations of second degree for the function $f(x)=x^3+x^2+3\ ,\ x\in[-1,1]$
6. Use Picard's method to compute y(0.5) where y(+) is the sowtion to the given IVP dy/dt = 1+y, y(0) = 1
7. Solve the boundary value problem
$\frac{d^2y}{dx^2}=y$, Y(0) =0 , Y(1)=1.2
by employing shooting method; take $y'(0)=0.85, 0.95$ as initial guesses.
8. Solve the BVP by Numerov method
$\frac{d^2y}{dx^2}=x+y, \text{y}(0)=0;\ y(1)=0$
with step size $x=\frac{1}{4}$
9. Find the real root of the equation $x^3-2x-5=0$ Regular – falsi method.
10. Find a real root of the equation $x^4+7x^4+24x^2=15=0$ using Birge– vieta method. Perform two iterations.
11. Solve the following linear equation

$2x_1+8x_2+2x_3=14$

$6x_1+6x_2-x_3=13$

$2x_1+x_2-2x_3=5$

12. Using the Rutishavser method find all the eigen valves of the matrix.
13. Using the method of least – Squares find a straight line that list the following data. Also find the value of y at $x=68.5$

X	71	68	73	69	67	65	66	67
Y	69	72	70	70	68	67	68	64

14. Solve the initial value problem by Taylor's series method
$\frac{dy}{dt}=-[y+2t]\ ,\ t\in\{0,0.2\};\ \ y(0)=-1$
15. Evaluate y (1.5) by Adams - Basufarth method of order far, given that $\frac{dy}{dt}=\in^2(1+y)$
y(1.1)=1.233, y(1.2)=1.548 ; y(1.3)=1.979, y(1.4)= 2.575

16. Solve the BVP $\frac{d^2y}{dx^2} = xy,\ \ y(0) + y^1(0) = 1; y(1) = 1$

with stop size $h = \frac{1}{3}$.

Section –C

17. Find the root of the equation $x^3 - 2x - 5 = 0$ by Muller's method. Take 1, 2, and 3 as initial approximations.
18. Perform two iterations of Bairstow–method to find two roots of the equations $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ use (z, z) as initial approximation.
19. Find all the eigen valves and eigen vectors of the following matrix using Given's method. $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{pmatrix}$
20. (a) Solve the following initial value problem

    $\frac{d^2}{dt^2} + 2\frac{dy}{dt} + y = 0, \in [0, 0.1]$

    $y(0) = 0;\ y'(0) = 1$

    (b) Solve by Milne's method

    $\frac{dy}{dt} = \frac{t}{y}, y(1) = 2; \mathrm{t} \in [1, 1.4]$

    $[pg - 188, \mathrm{Ex}\,10.2]$
21. Find the root of the equation $x^3 - x^2 - x - 1 = 0$ using chebyshev method and Newton-Raphson method. Compare the results.
22. Using Jacobi's method of find all the eigenvalves and eigen vectors of the following matrix A (perform there itaions)
23. (a) Derive the gram- Schmidt Orthogonalizing process,

    (b) Obtain the chebyshow polynomial approximation of second degree (best minimax approximation) to $f(x) = x^3$ on the interval $[0,1]$
24. Solve the BVP, $\frac{d^2y}{dt^2} = y, y(0) = 0;\ y(1) = 1.1752$ by shooting method together with Range – Kutta method.