Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-05
Mechanics
Section – B
(Short Answers Questions)

Note :- Each question carries 8 marks and maximum word limit will be 100 words.

1. Find the M.I. of a rectangular parallelepiped of mass M and length of edges 2a, 2b, 2c about an axis through the centre and parallel to edge of length 2a, 2b and 2c.
(Pg. 6)
2. Sate and prove D' Alembert's principle.
(Pg. 10)
3. Explain the theorem of Parallel axes for moment of Inertia and for product of Inertia.
(Pg. 9)
4. Deduce the general equations of Motion of a rigid body from D' Alembert's principle. (Where forces are finite)
(Pg. 12)
5. A circular board is placed on a smooth horizontal plane and a boy runs round the edge of it at a uniform rate. What is the motion of the board?
(Pg. 17)
6. Find the kinetic Energy of a body about a fixed axis.
(Pg. 31)
7. Find the Moment of momentum about the axis of rotation.
(Pg. 32)
8. Prove that the period of oscillation in simple pendulum depends only on the length of the string and not upon the quantity of the mass attached.
(Pg. 43)
9. Show that the centre of suspension and centre of oscillation are convertible (or interchangeable).
(Pg. 47)
10. A rectangular plate swings in a vertical plate about one of its corner. It its period is one second. Find the length of the diagonal.
(Pg. 63)
11. Explain the rolling and sliding friction.
(Pg. 89)

12. A uniform solid cylinder is placed with its axis horizontal on a plane. Whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane, so that is may roll and not slide is $\frac{1}{3}\tan\alpha$.
(Pg. 100)
13. A uniform rod of mass m is placed at right angles to a smooth plane of inclination α with one end in contact with it. The rod is then released. Show that when the inclination to the plane is ϕ, the reaction of the plane will be $\left\{\frac{3\,(1-sin\ \phi)^2+1}{(1+3\ cos^2\phi)^2}\right\}\ mg\cos\alpha.$
(Pg. 104)
14. A cylinder rolls down a smooth plane whose inclination to the horizon is α, unwrapping as it goes a fine string fixed to the highest point of the plane find its acceleration and the tension of the string.
(Pg. 106)
15. Two equal uniform rods, AB and BC are freely joined at B and turn about a smooth joint at A. When the rods are in a straight line, w being the angular velocity of AB and x. the velocity of the centre of mass of BC : BC implings on a fixed inelastic obstate at a point d; show that the rods are instantaneous brought to rest if $BD = 2a\left(\frac{2u-aw}{34+2aw}\right)$, where 2a is the length of either rod.
(Pg. 136)
16. A body having an axis of symmetry OC, moves about a fixed point O under no forces except a constant retarding couple KC about the axis OC. If A, A, C are the moments of inertia and w_1, w_2, w_3 the angular velocities about the principal axes OA, OB, OC. Show that at time y, $w_1 = \pi\cos\left[\lambda t\left(\pi - \frac{1}{2}\ kt\right)\right], w_2 = \pi\sin\left[\lambda t\left(\pi - \frac{1}{2}\ kt\right)\right],\ w_3 = \pi$ −kt where $\lambda = \frac{A-C}{A}$, the initial values of w_1, w_2, w_3 being $\pi, 0, \pi$ respectively.
(Pg. 142)
17. Find the kinetic energy of a body with one point fixed.
(Pg. 147)
18. Find the Angular momentum about a fixed point.
(Pg. 148)
19. Show that in the free motion of body with an axis of symmetry (c) about its C.G. If n denote the spin about the axis c and ϕ denotes the Euler's third angle then: $A\ \dot{\phi} = (A-C)n$
(Pg. 152)
20. Prove that for a rigid body moving about a fixed point $\frac{dT}{dt} = \overrightarrow{W}.\overrightarrow{G}$, where G is the moment of external forces about fixed point and $T = \frac{1}{2}\overrightarrow{H}.\overrightarrow{W}$, where H is the angular momentum about the fixed point.
(Pg. 152)
21. Find the Integrals of Energy and Angular Momentum.
(Pg. 156)

22. Prove that if rectangular parallelepiped (edges 2a, 2a, 2b) rotates about its centre of gravity its angular velocity about one principal axis is constant and about the other principal axes is periodic, the period being to the period about he first mentioned principal axis as $(b^2 + a^2):(b^2 - a^2)$
(Pg. 157)
23. Show that for a body of revolution the maximum value of the angle between the axis of the impulsive couple acting on it and the instantaneous axis of initial motion set up by the couple in the body if $sin^{-1}\left(\frac{C-A}{C+A}\right)$.
(Pg. 176)
24. A dice in the form of a portion of parabola bounded by its latus rectum and its axis has its vertex A fixed and is stuck by a blow through the end of its latus rectum perpendicular to its plane. Show that the dice starts revolving about a line through A inclined at an angle $tan^{-1}\left(\frac{14}{25}\right)$ to the axis.
(Pg. 177)
25. A small insect moves along a uniform bar of equal to itself and of length 2a, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$, If the insect starts from the middle point of the bar and move along the bar with relative velocity v, show that the bar in time t will turn through angular $\frac{1}{\sqrt{3}}\ tan^{-1}\frac{Vt}{a}$.
(Pg. 182)
26. A uniform circular board of mass M and radius a, is placed on a perfectly smooth horizontal plane and free to rotate about a vertical axis through its centre; a man of mass M', walks round the edge of the board whose upper surface is rough enough to prevent his slipping: when he has walked completely round the board, to his starting point, show that board has turned through an angle $\frac{M'}{M+2M'}\ 4\pi$.
(Pg. 185)
27. A circular plate is turning in its own plane about a point A on its circumference. Suddenly A is fixed and point b, also on the circumference, fixed. Show that the plate will be reduced to rest if the arc AB is one third of the circumference.
(Pg. 192)
28. Three equal uniform rods placed in a straight line are freely joined and move with velocity v perpendicular to their lengths. If the middle point of the middle rod be suddenly fixed, show that the ends of the two rods will meet in time $\frac{4\pi a}{9v}$, where a is the length of each rod.
(Pg. 193)
29. Two link rods AB and BC each of length 2a are freely joined at B, AB can turn round the rod A and C can more freely on a vertical straight line through A. Initially the rods are held in a horizontal line, c being in coincidence with A and they are then released. Show that when the rods are inclined at an angle θ to the horizontal the angular velocity of either is L

$$\sqrt{\left(\frac{3g}{a}\frac{\sin\theta}{1+3\,cos^2\theta}\right)}$$

(Pg. 198)

30. Show that the velocity V of its centre when at its lowest point is given by :

    $$V^2 = \frac{log}{7}\ (2a-b)$$

    (Pg. 200)

31. Deduce the principle of energy from the Lagrange's equations.

    (Pg. 209)

32. Use Lagrange's equations to find the equation of motion of a simple pendulum.

    (Pg. 210)

33. Use the Lagrange's equations in findings the small oscillations of a conservative system about a position of equilibrium.

    (Pg. 218)

34. Three equal uniform rods AB, BC, DC are smoothly jointed at B and C and the ends A and D are fastened to smooth fixed points whose distance a part is equal to the length of either rod. The frame being at rest in the from of a square. A blow I is given perpendicular to AB at its middle point and in the plane of the square. Show that the energy set up is $\frac{3I^2}{40\,m}$, where m is the mass of each rod. Find also the blows at the joints A and C.

    (Pg. 235)

35. Deduce the steady motion of a Top.

    (Pg. 242)

36. Find the stability condition for the motion of a top, when the axis of top is vertical.

    (Pg. 243)

37. Find the stability condition for the motion of a top when the axis of top is not vertical.

    (Pg. 244)

38. A circular disc of radius a has a thin rod pushed through its centre perpendicular to its plane the length of the rod being equal to the radius of a disc. Show that the system can not spin with the rod vertical unless the angular velocity is greater than $\sqrt{\frac{2\,og}{a}}$.

    (Pg. 247)

39. Deduce the Lagrange's equations from Hamilton's Principle.

    (Pg. 260)

40. A heavy bead of mass m is freely movable on a smooth circular wire of radius a, which is made to rotate about the vertical diameter with spin w, θ being the angle made by the radius through the bead at any time with the downwards vertical prove that the action A is :

$$A = \int_{\theta_1}^{\theta_2} ma^2 \left\{\frac{2H}{ma^2} + \frac{2g}{a}\cos\theta + w^2 sin^2\theta\right\}^{1/2} d\theta$$

Where H is the Hamiltonian of the system.

(Pg. 263)

41. A particle of unit mass is projected so that its total energy is h is a field of force of which the potential energy is ϕ(g) at a distance r from the origin. Deduce from the principle of energy and least action that the differential equation of the path is :

$$c^2\left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right] = r^4[h - \phi(r)]$$

(Pg. 267)

42. Find the equation of the stream lines passing through the point (1, 1, 1) for an incompressible flow $\vec{q} = 2x\hat{\imath} - y\hat{\jmath} - z\hat{k}$.

(Pg. 286)

43. Given $u = W_y,\ v = W_x\ and\ w = 0$, show that the surfaces intersecting the stream line orthogonally exist and are the planes through z-axis.

(Pg. 286)

44. Find the stream lines and paths of the particles when :

$$u = \frac{x}{(1+t)}, \qquad v = \frac{y}{(1+t)}, \quad w = \frac{z}{(1+t)}$$

(Pg. 289)

45. A velocity field is given by $\vec{q} = x\hat{\jmath} + (y+t)\hat{\jmath}$. Find the stream function and the stream line for this field at t =2.

(Pg. 291)

46. Find the stream function $\phi(x, y, t)$ for the given velocity field $u = Ut$.

(Pg. 292)

47. If $u = 2xy\ and\ v = (a^2 + x^2 - y^2)$ are the velocity components of a fluid motion, then find the stream function.

(Pg.292)

48. Deduce the velocity potential (Velocity function) in fluid flow.

(Pg. 295)

49. Find the equation of continuity due to cylindrical symmetry.

(Pg. 306)

50. Find the equation of continuity due to spherical symmetry.

(Pg. 307)

51. Show that $u = \frac{2xyz}{(x^2+y^2)^2}$; $v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}$ $and\ w = \frac{y}{(x^2+y^2)}$ are the velocity components of a possible fluid motion. Is this motion irrotational.

(Pg.309)

52. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3\,xz}{r^5}, \frac{3\,yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$ prove that the liquid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$.

(Pg. 310)

53. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is $\frac{\partial P}{\partial t} + \frac{\partial (Pw)}{\partial \theta} = 0.$
Where w be the angular velocity of a particle whose azimuthal angle is θ at time t.
(Pg. 323)
54. If σ is the cross sectional area of a stream filament, establish the equation of continuity in the form $\frac{\partial}{\partial t}\ (P\sigma) + \frac{\partial}{\partial s}\ (P\sigma q) = 0$ where s is measured along the filament in the direction of flow and q is the speed.
(Pg. 324)
55. A mass of fluid is in motion such that the lines of motion lie on the surface of coaxial cylinders, show that the equation of continuity is $\frac{\partial P}{\partial t} + \frac{1}{r}\frac{\partial (Pu)}{\partial \theta} + \frac{\partial (Pv)}{\partial z} = 0$. where u, v are the velocity perpendicular and parallel to z.
(Pg. 327)
56. Show that the ellipsoid $\frac{x^2}{a^2k^2t^{2n}} + \ kt^n \left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$ is a possible form of the boundary surface of a liquid.
(Pg. 333)
57. Show that :
$\frac{x^2}{a^2}\ f(t) + \frac{y^2}{b^2}\ \phi(t) + \frac{z^2}{c^2}\ \Psi(t), where\ f(t).\phi(t).\Psi(t) = 1$ is a possible form of the boundary surface.
(Pg. 334)
58. State and prove the "Bernoulli's theorem". (Pg. 344)
59. State and Prove the principle of "Permanence of irrotational motion".
(Pg. 348)
60. Air, abeying Boyle's law is in motion in a uniform tube of small section, prove that if P be the density and v be the velocity at a distance x from a fixed point at time t $\frac{\partial^2 P}{\partial t^2} = \frac{\partial^2}{\partial x^2}\{(v^2 + k)P\}$.
(Pg. 349)
61. Steam is rushing from a boiler through a conical pipe the diameter of the ends of which are D and d. If V and v be the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone. Prove that $\frac{v}{V} = \frac{D^2}{d^2}\ e^{(v^2 - V^2)/2k}$
Where k is the pressure divided by the density and supposed constant.
(Pg. 353)
62. A stream in a horizontal pipe after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area if B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth $\frac{S^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$

Below the pipe, S being the delivery per second.
(Pg. 366)

63. Deduce the Lagrange's equation of motion.
(Pg. 370)
64. Find the equation of motion under Impulsive forces in vector form.
(Pg. 376)
65. A mass of liquid surrounds a solid sphere of radius a and its outer surface, which is a concentric spheres of radius b, is subject to a given constant pressure p, no other force being in action on the liquid. The solid sphere suddenly shrinks into a concentric sphere. It is required to determine the subsequent motion.
(Pg. 382)
66. Explain mathematically the "Lagrange's stream function".
(Pg. 391)
67. Explain mathematically the "Irrotational motion in two-dimensions".
(Pg. 391)
68. Find the Cauchy-Riemann equations in polar coordinates.
(Pg. 394)
69. Explain the "Strength of a source".
(Pg. 394)
70. Obtain an Image of a source with respect to a straight line.
(Pg. 403)
71. What arrangement of sources and sinks will give rise to the function $w = log\left(z - \frac{a^2}{z}\right)$?
Draw a rough sketch of a stream line. Prove that two of the stream lines sub divide into the circle r = a and the axis of y.
(Pg. 410)
72. A source S and a sink T of equal strength m are situated within the space bounded by a circle, centre if O. If S and T are at equal distances from O an opposite side of it and on the same diameter. AOA'. Show that the velocity of the liquid at any point O is $2m.\frac{OS^2+OA^2}{OS}.\frac{PA.PA'}{PS.PS'.PT.PT'}$
Where S' and T' are the inverse points of S and T with respect to the circle.
(Pg. 418)