Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous) Question Bank-2015
Paper Code:MT-04
Section-A

1) Define space curves. MT-04 (p.2)
2) Give the definition of binormal. MT-04 (p.25)
3) Define singular points. MT-04 (p.74)
4) Define envelope. MT-04 (p.81)
5) What is the condition for parametric transformation u^*=u (u, v), v^*=v(u, v) to be proper? MT-04 (p.107)
6) Define plane section of a surface? MT-04 (p.134)
7) Define principal sections of a surface? MT-04(p.148)
8) Define conjugate directions at a given point (u,v) on a surface $\vec{r} = \vec{r}(u, v)$. MT-04 (p.178)
9) Write unit tangent vector of a curve. (p.3)
10) Define normal plane. (p.27)
11) Define conoids. (p.71)
12) Define edge of regression. (p.81)
13) Define metric of a surface. (p.106)
14) Define direction coefficients. (p.122)
15) Define principal directions. (p.148)
16) Define kronecker delta. (p.257)
17) What do you understand by covariant derivative of contravariant vector? (P.298)
18) Define metric tensor and Riemannian space. (p.277)
19) What is invariant? (p.261)
20) Write Gauss's characteristic equations. (p.242)
21) State clairut's theorem. (p.213)
22) Define asymptotic lines. (p.183)
23) Define principal radius of curvature. (p.148)

What are orthogonal trajectories? (p.122)

SECTION-B

24) Find the lines that have four point contact at (0,0,1) with the surface
$x^4+3xyz+x^2-y^2-z^2+2yz-3xy-2y+2z=1$ MT- 04 (p.11)

25) Prove that the tangent to the locus of the centre of curvature lies in the normal plane of the original curve and is inclined to $\hat{n}$ at an angle $\tan^{-1}(\frac{\rho\tau}{\rho_1})$.
(p.46)

26) Find the equation to the right conoid generated by lines which meet 0Z, are parallel to the plane X0Y and intersect the circle $x=a$, $y^2+z^2=r^2$. (p.72)

27) Show that a ruled surface generated by $x=az+\alpha,\ y = bz + \beta$ is developable or skew if $\alpha'b' - \beta'a' = 0 \ or \neq 0$ respectively. (p.97)

28) Prove that for the curve $x=r\cos\theta, y = r\sin\theta,\ z = 0, ds^2 = dr^2 + r^2 d\theta^2$.
(p.111)

29) If a sphere is described with ρ_n as diameter then all centres of curvature lie on this sphere, provided unit tangent vector $\hat{t}$ is the same.

30) If $\emptyset = a_{ij}A^iA^j$, then prove that we can always write $\emptyset = b_{ij}A^iA^j$ where b_{ij} is symmetric. (p.268)

31) Prove the following :
   i. R_{rijk} is symmetric in two pairs (first and last) of indices i.e. $R_{rijk} = R_{jkri}$
   ii. R_{rijk} has cyclic property in last three indices i.e.
   $R_{rijk} + R_{rjki} + R_{rkij} = 0$

32) Find the equation to the tangent at the point θ on the circular helix
$x=a\cos\theta, y = a\sin\theta, z = c\,\theta$. (p.7)

33) Principal normal to c is normal to c_1 at the points where curvature is stationary.
` (p.48)

34) Find and classify the singular points of the surface : $xyz-a^2(x+y+z)+2a^3=0$
(p.74)

35) Prove that the generators of a developable surface are tangents to the curve.
(p.101)

36) Prove that the equation $Edu^2 - Gdv^2 = 0$ denote the curves bisecting the angles between the parametric curves u=constant, v=constant on a surface $\vec{r} = \vec{r}(u, v)$.
(p.126)

37) Prove that in general three lines of curvature pass through an umbilic.
(p.170)

38) If a_{ij} is a symmetric covariant tensor and b_i a covariant vector which satisfy the relation $a_{ij}b_k + a_{jk}b_i + a_{ki}b_j = 0$, prove that either $a_{ij} = 0$ or $b_i = 0$. (p.268)

39) Prove that

(i) $div\ grad\ I = \nabla^2 I = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^r} \{\sqrt{g} g^{rk} I_{,k}\}$

(ii). $div\ grad\ I = \nabla^2 I = g^{jk} \left[\frac{\partial^2 I}{\partial x^j \partial x^k} - \frac{\partial I}{\partial x^r} \left\{ {r \atop jk} \right\} \right]$

Section-C

40) Show that when the curve is analytic, there exists a definite osculating plane at a point of inflexion, provided the curve is not a straight line. (p.16)

41) Find the inflexional tangent at (x_1, y_1, z_1) on the surface $y^2 z = 4ax$. (p.73)

42) Derive a formula for metric of a surface. (p.110)

43) Determine the radius of curvature of a given section through any point of a surface, z=f(x, y). (p.165)

44) For the curve $x=3t$, $y=3t^2$, $z=2t^3$, show that any plane meets it in three points and deduce the equation to the osculating plane at t=ti. (p.18)

45) Prove that the product of the torsion of c_1 at corresponding points is equal to the product of curvatures at these points. (p.51)

46) Prove that the z-axis is a nodal line with unodes at the points (0,0,-2) and (0,0,2) for the surface $2xy+x^3-3x^2y-3xy^2+y^3+z(x^2-xy+y^2)=0$

(P.75)

47) Examine whether the surface z=y sin x is developable. (p.105)

48) Examine whether the parametric curves x=b sin u cos v , y=b sin u sin v , z=b cos u on a sphere of radius b constitute an orthogonal system.

49) Find the asymptotic lines on the surface z=y sin x. (p.187)

50) If $u_{ij} \neq 0$ are the components of a tensor of the type (0,2) and if the equation $f\, u_{ij} + g\, u_{ji} = 0$ holds, then prove that either f=g and u_{ij} is skew symmetric or f=-g and u_{ij} is symmetric. (p.269)

51) Show that the covariant differentiation of invariants is commutative i.e. $(I_{,i})_{,j} = (I_{,j})_{,i}$

52) Show that the necessary and sufficient condition that a given curve be a plane curve is that $\tau = 0$ at all the points of the curve or in other words $[\vec{r'}\,\vec{r''}\,\vec{r'''}] = 0.$ (p.35)

53) Find the envelope of the family of planes $F(x, y, z, \theta, \emptyset) = \frac{x}{a} cos\theta\ sin\emptyset + \frac{y}{b} sin\theta\ sin\emptyset + \frac{z}{c} cos\emptyset - 1 = 0.$ (p.86)

54) Show that the curves $du^2-(u^2+c^2)\ dv^2=0$ form an orthogonal system on the right helicoids $\vec{r}$=(u cos v, u sin v ,c v). (p.133)

55) "The geodesic curvature vector of any curve is orthogonal to the curve." prove it. (p.218)

56) Show that the curvature and torsion of either associate Bertrand curves are connected by a linear relation. (p.61)

57) Prove that the indicatrix at a point of the surface z=f(x ,y) is a rectangular hyperbola if $(1+p^2)t+(1+q^2)r-2pqs=0$ (p.76)

58) Determine fundamental magnitude of Monge's form surface. (p.121)

59) Prove the necessary and sufficient condition for the parametric curves through a point to have conjugate directions is M=0.

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(Section-C)

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62) Determine fundamental magnitude of Monge's form surface. (p.121)

63) Prove the necessary and sufficient condition for the parametric curves through a point to have conjugate directions is M=0. (p.181)

64) Show that the necessary and sufficient condition that a given curve be a plane curve is that $\tau = 0$ at all the points of the curve or in other words $[\overrightarrow{r'}\overrightarrow{r''}\overrightarrow{r'''}] = 0.$ (p.35)

65) Find the envelope of the family of planes F(x ,y ,z , $\theta, \emptyset) = \frac{x}{a} cos\theta\ sin\emptyset + \frac{y}{b}\ sin\theta\ sin\emptyset + \frac{z}{c}\ cos\emptyset - 1 = 0.$ (p.86)

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