Program : M.A./M.Sc. (Mathematics)

M.A./M.Sc. (Previous)

Paper Code:MT-04

Differential Geometry & Tensors

Section – B

(Short Answers Questions)

1. Show that the tangent at a point of the curve of the intersection of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=t$ is $\frac{x\ (x-x)}{a^2(b^2-c^2)(a^1-\lambda)}=\frac{y\ (Y-y)}{b^2(c^2-a^2)(b^2-\lambda)}=\frac{z(Z-z)}{c^2(a^2-b^2)(c^2-\lambda)}$

A M.T. -04, Pg. 08

2. Find the plane that has three point contact of origin with the curve
$$a=t^4-1, y=f^3-1\ , z=t^2-1$$

A M.T. -04, Pg. 10

3. Prove that the condition that four consecutive points of a curve should be coplanar is $\begin{vmatrix} x' y' z' \\ x'' y'' z'' \\ x''' y''' z''' \end{vmatrix}=0$

A M.T. -04, Pg. 13

4. Show that the tangent at any point of the curve whose equation are $x=3t, y=\ 3t^2, z=2t^3$ makes a constant angle with line $y=z-x=0$

A M.T. -04, Pg. 07

5. Find the osculating plane at the point ‘t’ on the helix $x=a\cos t, y=a\sin t$, $z=ct$

A M.T. -04, Pg. 19

6. The necessary and sufficient condition for the curve to be a straight line is that k = 0 at all points of the curve.

A M.T. -04, Pg. 34

7. If the tangent and binomial at al point of curve make angle v,ϕ respectively with a fixed directions then:

$$\frac{\sin v}{\sin \phi} \cdot \frac{dv}{d\phi} = \pm \frac{k}{\tau}$$

A M.T. -04, Pg. 35

8. Prove that the curve given by $x = a \sin \mu, y = 0, z = 0 \cos \mu$ lies on a sphere.

A M.T. -04, Pg. 52

9. Prove that :

$$x'''^{\,2} + y'''^{\,2} + z'''^{\,2} = \frac{1}{\delta^2 \sigma^2} + \frac{1 + \delta^{12}}{\delta^4}$$

Where dashes denote differentiation with respect to ‘s’.

A M.T. -04, Pg. 53

10. Prove that the distance between corresponding points of te two curve is constant.

A M.T. -04, Pg. 60

11. Prove that the tension of the twp Bertrand curves have the same sign and their product is constant.

A M.T. -04, Pg. 62

12. Find the evalute of a circular helix given by $x = a \cos \theta, y = a \sin \theta, z = a \tan \alpha$

A M.T. -04, Pg. 65

13. Find the equation to the gonoid generated by lines parallel to the plane XOY are drawn to intersect DX and the curve $x^2 + y^2 = r^2, \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}$

A M.T. -04, Pg. 71

14. Prove that the points of the surface $xyz = a\,(y = \; +zx + xy) = 0$ at which the indicates is a rectangular Hyperbola, lie on the cone $x^4(y + z) + y^4(z + x) + z^4(x + y) = 0$

A M.T. -04, Pg. 77

15. Find the equation of the developable surface which contains the two curves $y^2 = 4ax, z = 0 \; and \; (y - b)^2 = 4cz, x = 0$ and also show that its edge regression lies on the surface:

$$(ax + by + cz)^2 = 3abx\,(b + y)$$

A M.T. -04, Pg. 8965

16. Find the equations to the edge of regression of the devlopable $y = xt - t^3, z = \; t^3 - t^6$

A M.T. -04, Pg. 98

17. Prove that generators of a developable surface are tangents to curve.

A M.T. -04, Pg. 101

18. Prove that on a given surface a family of curves and their orthogonal tranhection can always be chosen as parametric curves.

A M.T. -04, Pg. 129

19. On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the setion by the planes z = constant.

A M.T. -04, Pg. 131

20. Let $v^2 du^2 + u^2 dv^2$ be the metric of a given surface. Then find the family of curves orthogonal to the curves UV = constant.

A M.T. -04, Pg. 132

21. Show that on a right helicoids te family of curves orthogonal to the curves n cos v = constant is the family $(u^2 + a^2)\ \Delta m^2 v = constant$

A M.T. -04, Pg. 134

22. State and prove meunienis theorem.

A M.T. -04, Pg. 143

23. Show that the curvature k at any point p of the curve of intersection of two surfaces in given by $K^2 sin^2\alpha = {k_1}^2 + {k_2}^2 - 2k_1 k_2 \cos\alpha$ where $k_1 and\ k_2 are$ the normal of curvatures of the surface in the direction of the curve p and α is angle between their normals at that point.

A M.T. -04, Pg. 146

24. Show that the surface $e^z \cos x = \cos y\ \ is\ minimum\ surface.$

A M.T. -04, Pg. 163

25. Find the value of (i) first curvature (ii) Ganssion curvature at any point of right helicoids $x = u\cos v, y = \sin v, z = cQ$

A M.T. -04, Pg. 164

26. Find the principal radip at the origin of the surface $2z = 5x^2 + 4xy + 2y^2$. Find the radius of curvature of the section x=y.

A M.T. -04, Pg. 167

27. To show that the directions given by $Pdu^2 + 2l\ du\ dv +\ Rdv^2 = 0$ are conjugate if $LR = 2\ MR + NP = 0$

A M.T. -04, Pg. 180

28. Prove that on the surface $z = f(x, y)$ (Mange's form) the equation of asymptotic lines are :

$$rdx^2 + 2\ s\ dx\ dy + tdy^2 = 0$$

A M.T. -04, Pg. 186

29. Prove hat for the surface $x = 3u(1+v^2) - u^3, y = 3v\,(1+u^2) - v^3, z = 3u^2 - 3v^2$ the asymptotic lines are $u \neq v = constant.$

A M.T. -04, Pg. 188

30. Prove that osculating plane at any point of a curved asymptotic lines is the tangent plane to the surface.

A M.T. -04, Pg. 190

31. Prove that on the surface $z = f(x, y)$torsian the asymptotic lines are :

$$\pm\frac{\sqrt{s^2 - rt}}{(1 + p^2 + q^2)}$$

A M.T. -04, Pg. 194

32. Show that the curvature of an asymptotic line may be expressed as :

$$\frac{(\vec{r}_1, \vec{r}')\,\vec{r}_2.\vec{r}''\,)\,(\vec{r}_2.\vec{r}')(\vec{r}_1.\vec{r}''\,)}{H}$$

A M.T. -04, Pg. 195

33. Prove that the curves $u + v = constant$ are geodes pcs on a surface with metric $(1 + u^2)\,du^2 - 2uv$ and $v + (1 + v^2)\,dv^2$

A M.T. -04, Pg. 208

34. Prove that the curves of the family $\frac{v^3}{u^3} = \ constant$ and geodesies on a surface with metric $v^2du^2 = 2uv\,dv + 2u^2dv^2$; $(u > 0, v > 0)$

A M.T. -04, Pg. 209

35. Prove that a curve on a geodesies if and only if it is a great circle.

A M.T. -04, Pg. 215

36. Find the Geodesic curvature of the curve u = constant on the surface $x = u\cos\theta\,, y = u\sin\theta\,, z = \frac{1}{2}au^2$

A M.T. -04, Pg. 223

37. Gedoseics are drown on a catenoid so as to cross the meridpond at an angle whose sine is c/u where u is the distance of the point of crossing from the axis. Prove that the polar equation to their projection on the xy-plane is $\frac{u-c}{u+c} = e^{2(\theta+\alpha)}$ where α is an arbitrary constant.

A M.T. -04, Pg. 230

38. Show that for a geodesic :

$$z^2 = (K - K_a)(K_b - K)or\ \frac{1}{\sigma^2} = \left(\frac{1}{\delta} - \frac{1}{\delta_a}\right)\left(\frac{1}{\delta_b} - \frac{1}{\delta}\right)$$

A M.T. -04, Pg. 234

39. Find the Gaussian curvature at the point (u,v) of the ancher ring :

$$\vec{r} = (g(u)\cos v\,, g(u)\sin v, f(u))$$

A M.T. -04, Pg. 236

40. For any surface prove that :

$$\frac{\partial}{\partial u}(\log H) = 1 + u \quad , \quad \frac{\partial}{\partial v}(\log H) = m + v$$

Where u and v are parameters and symbols having their usual meaning.

A M.T. -04, Pg. 246

41. From the Gauss characteristic equation deduce that, when the parametric curves are orthogonal :

$$k = \frac{1}{\sqrt{EG}}\left[\frac{\partial}{\partial u}\left(\frac{1}{E}\frac{\sqrt{G}}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v}\right)\right]$$

A M.T. -04, Pg. 247

42. State and prove Bonner's theorem on parallel surfaces.

A M.T. -04, Pg. 252

43. If a vector has components $\dot{x}, \dot{y}$ $\left(\dot{x} = \frac{dx}{dt}, \dot{y} = \frac{dy}{dt}\right)$ is rectangular Cartesian coordinates then $\dot{r}\,, \dot{v}$ are its components in polar coordinates.

A M.T. -04, Pg. 259

44. A Wuartant tensor of first order has components $xy, 2y - z^2, xz$ in rectangular coordinates. Determine its covariant components in spherical coordinate.

A M.T. -04, Pg. 261

45. If A_{ij} be covariant tensor of second order and B^i, C^i are contravariant vectors, prove that $A_{ij}\ B^i C^i$ is an invariant

A M.T. -04, Pg. 263

46. If $A^i\ and\ B^i$ are arbitrary contravariant vectors and $C_{ij}\,, A^i B^i$ is an invariant show that C_{ij} is a covariant tensor of second order.

A M.T. -04, Pg. 271

47. If $A_{ij} = 0$ for i + j show that the conjugate tensor $B^{ij} = 0$ for i + j and $B^{''} = \frac{1}{A^{ii}}$ (no summation)

A M.T. -04, Pg. 274

48. Show that :

(i) $\left(g_{nj}\ g_{ik} - g_{nk}\ g_{ij}\right)g^{hi} = (N-1)g_{ik}$

(ii) $\frac{\partial k}{\partial x^j}\ (g_{nk}\ g_{il} - g_{nl}\ g_{ik})\ g^{hj} = \frac{\partial k}{\partial x^k}\ g_{il} - \frac{\partial k}{\partial x^l}\ g_{ik}$

A M.T. -04, Pg. 283

49. Calculate the christoffed symbols corresponding tometric $ds^2 = (dx^{'})^2 + G\ (x^1, x^2)(dx^2)^2$ where G is a function of $x^1\ and\ x^2$.

A M.T. -04, Pg. 289

50. Surface of sphere is a two dimensional Riemannian space. Compute the christoffel symbols.

A M.T. -04, Pg. 290

51. Prove that :

$$A_{ij} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} (A_j^i \sqrt{g}) - A_k^j \begin{Bmatrix} k \\ ij \end{Bmatrix}$$

Show that if associate tensor A^{ij} is symmetric then:

$$A_{i,j}^j = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(A_i^j \sqrt{g}\right) - \frac{1}{2} A^{jk} \frac{\partial g_{jk}}{\partial x^i}$$

A M.T. -04, Pg. 301

52. Prove that :

$$A_j^{i,j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(\sqrt{g}\ A^{ij}\right) + A^{ik} \begin{Bmatrix} i \\ jk \end{Bmatrix}$$

A M.T. -04, Pg. 302

53. To prove that :

$$div\ A_i = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^r} \left\{\sqrt{g} g^{rk} A_k\right\} = div\ A^i$$

Where $A^i\ and\ A_i$ ar the contravariant and covariant components of the same vector A.

A M.T. -04, Pg. 305

54. If two unit vectors $A^i\ and\ B^i$ are defines along a curve C such that intrinsic derivatives along are zero show that the angle between them is constant.

A M.T. -04, Pg. 312

55. If the intrinsic Derivative of a vector A^i along a curve C vanish at every point of the curve, then show that the magnitude of the vector A^i is constant along the curve.

A M.T. -04, Pg. 312

56. The necessary and sufficient condition that a system of coordinates be geodesic with the pole P_0 are that their second covariant derivative, with respect to the metric of the space, al vanish at P_0.

A M.T. -04, Pg. 35

57. Show that the coordinate system x^{-i} $defined\ by\ x^{-i} = x^i + \frac{1}{2}\left\{ {i \atop mn} \right\} x^m . x^n$ is a geodesic cootdinate system with the pole at origin.

A M.T. -04, Pg. 326

58. Show that the vector B^i of variable magnitude suffers a parallel displacement along a curve C if and only if :

$$\left(B^l B_j{}^i - B^i B_j{}^l\right) \frac{dx^i}{ds} = 0$$

A M.T. -04, Pg. 331

59. Prove that :

(a) $R^{\alpha}_{.ijk}$ has cyclic property in its subscripts i.e.

$R^{\alpha}_{.ijk} + R^{\alpha}_{.jkp} + R^{\alpha}_{.kij} = 0$

(b) $R^{\alpha}_{.ijk}$ vanish an contradiction in $\alpha\ and\ i$ i.e.

$R^{\alpha}_{.ijk} = 0$

A M.T. -04, Pg. 335

60. State and prove Banachs uf Identty.

A M.T. -04, Pg. 337

61. Prove that $R_{1212} = -G\frac{\partial^2 G}{\partial u^2}$ for the V_2 whose line element is $ds^2 = du^2 + G^2 dv^2$, where G is a function of u and v.

A M.T. -04, Pg. 338

62. Prove that the divergence of Einstein tensor vanishes i.e. $G^i_{j,i} = 0$

A M.T. -04, Pg. 340

63. Prove that an Einstein space $V_N (N > 2)$ has constant curvature.

A M.T. -04, Pg. 341

64. The metric of V_2 formed by the surface of sphere of radius a is : $ds^2 = a^2 dv^2 + a^2 sin^2\theta\ d\theta^2$ in a spherical polar coordinates. Show that the curvature of the surface is $\frac{1}{a^2}$, which is constant.

A M.T. -04, Pg. 342

65. State and prove schur's theorem.

A M.T. -04, Pg. 344

66. If the metric of a two dimensional flat space it $f(r)[(dx^1)^2(dx^2)^2]$ where $(r)^2 = (x^1)^2 + (x^2)^2$ show that $f(r) = C\,(r)^k$ where c and k aree constants.

A M.T. -04, Pg. 345

67. In a V_2 Prove that :

(i) $R\left(g_{ij}\ g_{rj} - g_{ij}\ g_{rk}\right) = -2\,R_{.rijk}$

And hence that : $gR_{ik} = -2R_{1212}$

A M.T. -04, Pg. 346

68. Show that for the right holicoed:

$$\vec{r} = (u\cos v, u\sin v, cv)\ ,\qquad l = 0, m = 0,$$
$$n = -u\ , \lambda = 0, \mu = \frac{u}{(n^2 + c^2)}, v = 0$$

A M.T. -04, Pg. 246

69. State and prove gauss characteristic equations :

A M.T. -04, Pg. 242

70. Show that the curves $du^1 - (u^2 + c^2)\ dv^2 = 0$ from an orthogonal system on the right helicoids :

$$\vec{r} = (u\cos v, u\sin v, cv)$$

A M.T. -04, Pg. 133

71. Determine the function f(0) so that $x = \cos\theta, y = a\sin v, z = f(v)$ shall be a plane curve.

A M.T. -04, Pg. 42

72. Prove that the principals normed at cinseculwe points of a curve do not intersect unless z = 0

A M.T. -04, Pg. 36