

# UNIT-1

## Design of a Common Emitter Transistor

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### **1.1 Aim**

1. Design a common emitter amplifier.
2. Measure its voltage gain, input resistance and output resistance.

## 1.2 Apparatus

Protoboard, Function Generator, Digital Multimeter, Power Supply, Resistors, Capacitors, Transistors: 1 x 2N3904

## 1.3 Diagram

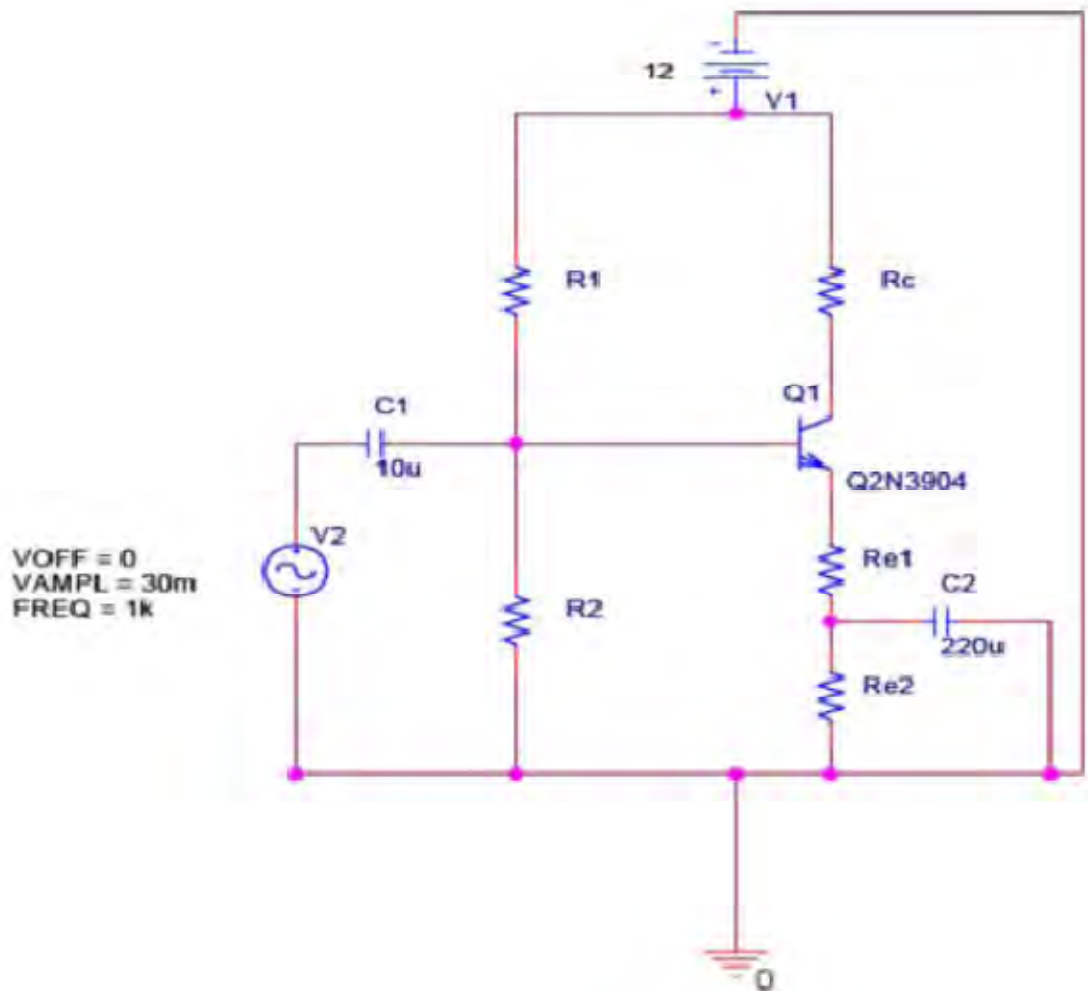
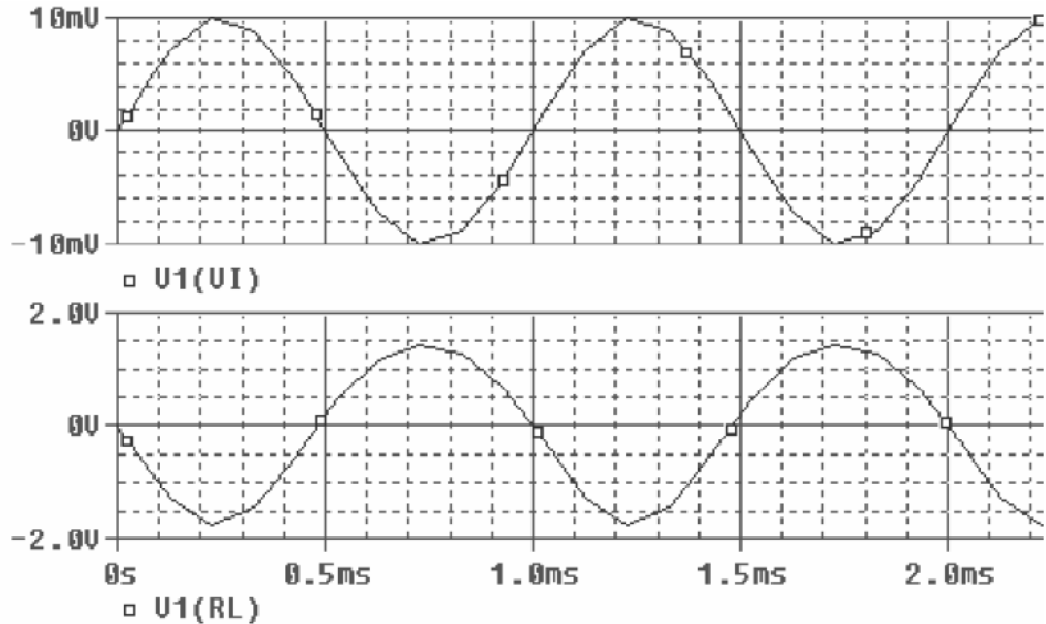


Figure 1: Circuit to be designed

## 1.4 Model Graph



## 1.5 Theory and Description

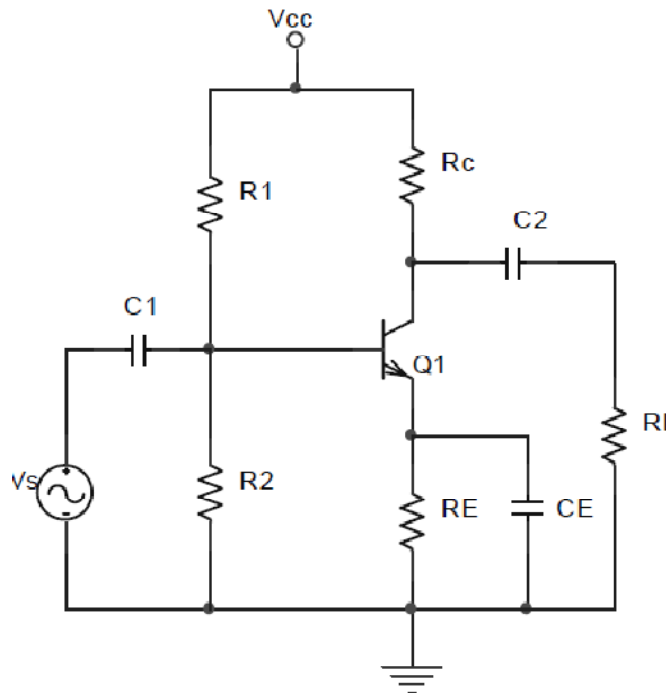
**Amplifier** is an electronic circuit that is used to raise the strength of a weak signal. The process of raising the strength of a weak signal is known as **amplification**. One important requirement during amplification is that only the magnitude of the signal should increase and there should be no change in signal shape. The transistor is used for amplification. When a transistor is used as an amplifier, the first step is to choose a proper configuration in which device is to be used.

### CE amplifier operation

Consider a CE amplifier circuit as shown in fig.2 When the capacitors are regarded as ac short circuits, it is seen that the circuit input terminals are the transistor base and emitter, and the output terminals are the collector and the emitter. So, the emitter terminal is common to both input and output, and the circuit configuration is termed *Common -Emitter (CE)*.

From the voltage waveforms for the CE circuit shown in figure it is seen that there is a 180 degree phase shift between the input and output waveforms. This can be understood by considering the effect of a positive going input signal. When  $V_s$

increases in a positive direction, it increases the transistor  $V_{BE}$ . The increase in  $V_{BE}$  raises the level of  $I_C$ , thereby increasing the drop across  $R_C$ , and thus reducing the level of the  $V_C$ . The changing level of  $V_C$  is capacitor-coupled to the circuit output to produce the ac output voltage,  $V_O$ . As  $V_S$  increases in a positive direction,  $V_O$  goes in a negative direction. Similarly, When  $V_S$  changes in a negative direction, the resultant decrease in  $V_{BE}$  reduces the  $I_C$  level, thereby reducing  $V_{RC}$ , and producing a positive going output.



**Figure 2**

The circuit in figure has input impedance ( $Z_i$ ) and output impedance ( $Z_o$ ). These can cause voltage division of the circuit input and output voltages. So, for most transistor circuits  $Z_i$  and  $Z_o$  are important parameters. The circuit voltage amplification ( $A_v$ ), or voltage gain, depends on the transistor parameters and on resistor  $R_C$  and  $R_L$ .

### **CE amplifier circuit elements and their functions**

- **Biasing circuit:** The resistances  $R_1$ ,  $R_2$  and  $R_E$  form the biasing and stabilization circuit. The biasing circuit must establish a proper operating point; otherwise a part of the negative half cycle of the signal may be cut-off in the output.

- **Input capacitor,  $C_1$ :** An electrolyte capacitor  $C_1$  is used to couple the signal to the base of the transistor. If it is not used, the signal source resistance,  $R_S$  will come across  $R_2$  and thus change the bias.  $C_1$  allows only ac signal to flow but isolates the signal source from  $R_2$
- **Emitter bypass capacitor,  $C_E$ :** An Emitter bypass capacitor,  $C_E$  is used parallel with  $R_E$  to provide low reactance path to the amplified ac signal. If it is not used, then ac amplified ac signal following through  $R_E$  will cause a voltage drop across it, thereby reducing the output voltage.
- **Coupling capacitor,  $C_2$ :** The coupling capacitor,  $C_2$  couples one stage of amplification to the next stage. If it is not used, the bias conditions of the next stage will be drastically changed due to the shunting effect of  $R_C$ . This is because  $R_C$  will come in parallel with the upper resistance  $R_1$  of the biasing network of the next stage, thereby altering the biasing conditions of the latter. In short, the coupling capacitor  $C_2$  isolates the dc of one stage from the next stage, but allows the passage of ac signal.

### CE amplifier circuit currents

#### (i) Base current

$$i_B = I_B + i_b$$

where  $I_B$  = dc base current when no signal is applied,  $i_b$  = ac base when ac signal is applied and  $i_B$  = total base current

#### (ii) Collector current

$$i_C = I_C + i_c$$

where  $I_C$  = zero signal collector current,  $i_c$  = ac collector current when ac signal is applied and  $i_C$  = total collector current

#### (iii) Emitter Current

$$i_E = I_E + i_e$$

where  $I_E$  = Zero signal emitter current,  $i_e$  = ac emitter current when ac signal is applied and  $i_E$  = total emitter current

It is useful to keep in mind that

$$I_E = I_B + I_C \text{ and}$$

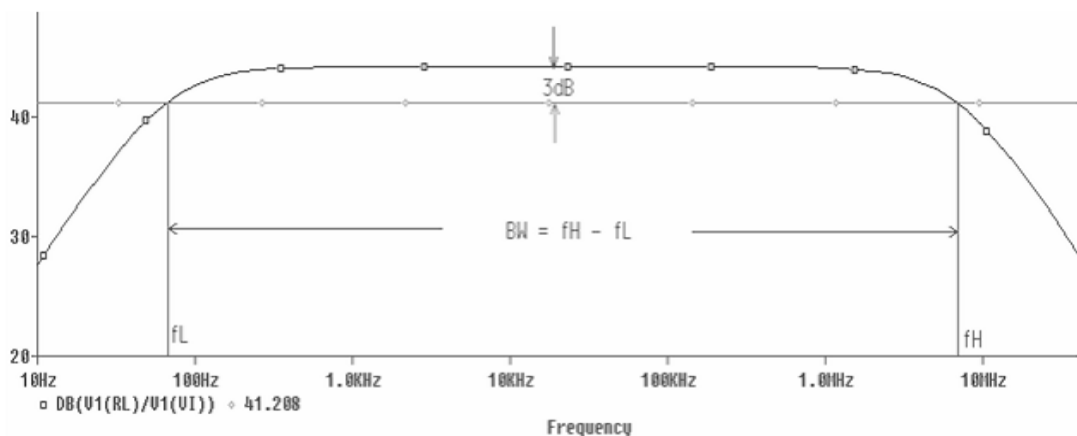
$$i_e = i_b + i_c$$

Also,  $I_E \approx I_C$  and  $i_e \approx i_c$

### CE amplifier frequency response

The voltage gain of an amplifier varies with signal frequency. It is because reactances of the capacitors in the circuit changes with signal frequency and hence affects the output voltage. The curve between voltage gain and signal frequency of an amplifier is known a frequency response. Figure shows the frequency response of a typical CE amplifier.

Then the transistor is biased to get the desired *Q-point*. The signal is applied to the amplifier input and gain is achieved.



It is clear that the voltage gain drops off at low ( $< f_L$ ) and high ( $> f_H$ ) frequencies whereas it is uniform over mid-frequency range ( $f_L$  to  $f_H$ ).

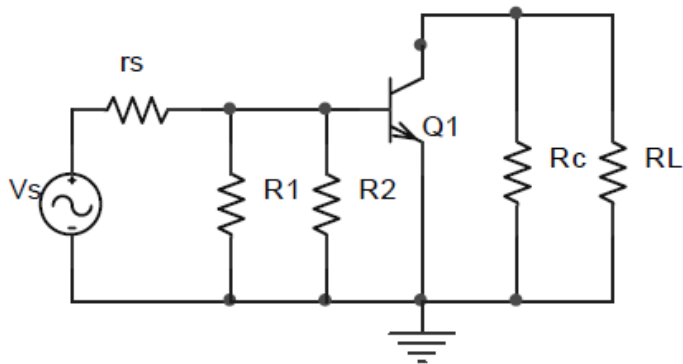
(i) At low frequencies ( $< f_L$ ), the reactance of coupling capacitor is quite high and hence very small part of signal will pass from amplifier stage to the load. Moreover,  $C_E$  cannot shunt the  $R_E$  effectively because of its large reactance at low frequencies. These two factors cause a falling of voltage gain at low frequencies.

(ii) At high frequencies ( $> f_H$ ), the reactance of  $C_2$  is very small and it behaves as a short circuit. This increases the loading effect of amplifier stage and serves to reduce the voltage gain. Moreover, at high frequency, capacitive reactance of base-emitters junction is low which increases the base current. These reduce the current amplification factor  $\beta$ . Due to these two reasons, the voltage gain drops off at high frequency.

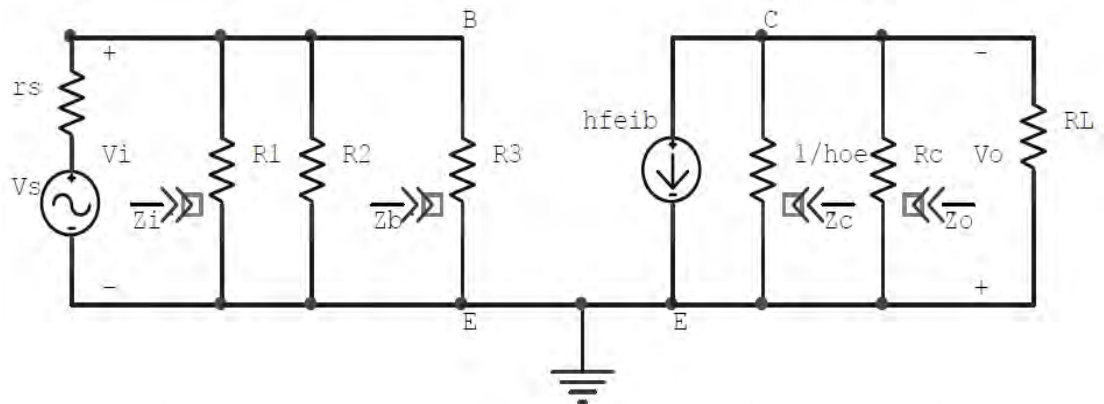
(iii) At mid frequencies ( $f_L$  to  $f_H$ ), the voltage gain of the amplifier is constant. The effect of coupling capacitor  $C_2$  in this frequency range is such as to maintain a uniform voltage gain. Thus, as the frequency increases in this range, reactance of  $C_C$  decreases which tend to increase the gain. However, at the same time, lower reactance means higher almost cancel each other, resulting in a uniform gain at mid-frequency.

### CE amplifier analysis

The first step in ac analysis of CE amplifier circuit is to draw ac equivalent circuit by reducing all dc sources to zero and shorting all the capacitors. Figure shows the ac equivalent circuit.



The next step in the ac analysis is to draw h-parameter circuit by replacing the transistor in the ac equivalent circuit with its h-parameter model. Figure shows the h-parameter equivalent circuit for CE circuit.



The typical CE circuit performance is summarized below:

Device input impedance,

$$Z_b = h_{ie}$$

Circuit input impedance

$$Z_i = R_1 \parallel R_2 \parallel Z_b$$

Device output impedance,

$$Z_c = 1 / h_{oe}$$

Circuit output impedance,

$$Z_o = R_c \parallel Z_c \approx R_c$$

Circuit voltage gain,

$$A_v = -\frac{h_{fe}}{h_{ie}}(R_c \parallel R_L)$$

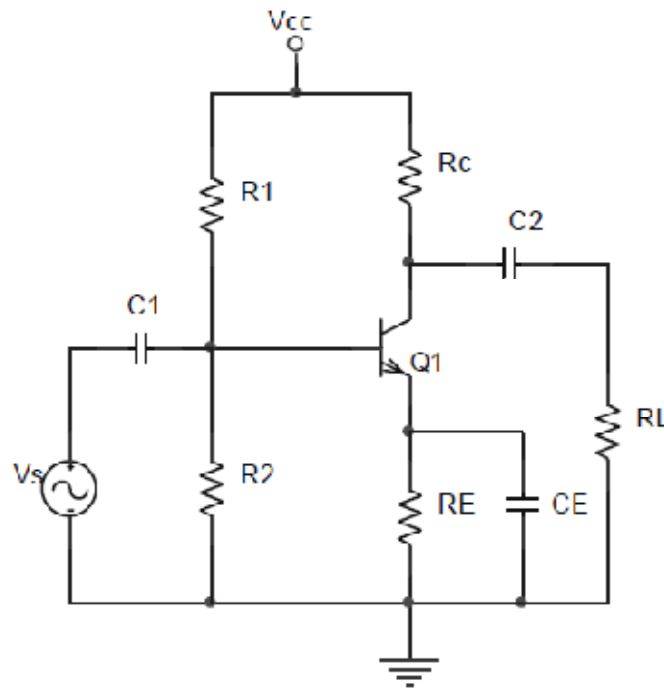
Circuit current gain,

$$A_i = -h_{fe} R_c R_B / (R_c + h_{ie})(R_c + R_L)$$

Circuit power gain,

$$A_p = A_v A_i$$

### CE Amplifier Circuit Design:





Design of CE circuit normally commences with a specification of supply voltage, minimum voltage gain, frequency response, source impedance, load impedance, stability factor and Q-point.

### Selection of $I_C$ , $R_C$ and $R_E$

$$A_V = -h_{fe} / h_{ie} (R_C \parallel R_L)$$

- For satisfactory transistor operation,  $I_C$  should not be less than  $500\mu\text{A}$ . A good minimum  $I_C$  to aim for is  $1\text{mA}$ .
- The  $V_{CE}$  should typically be around  $3\text{V}$  to ensure that the transistor operates linearly and to allow a collector voltage swing of  $\pm 1\text{V}$  which is usually adequate for small-signal amplifier  
**Note:**  $R_C$  should normally be very much larger than  $R_L$ , so that  $R_L$  has little effect on voltage gain.
- Select  $V_E = 5\text{V}$  for good bias stability in most circumstances. **Note:** When  $V_E \gg V_{BE}$ ,  $V_E$  will be only slightly affected by any variation in  $V_{BE}$  (due to temperature change or other effects)

- Once  $V_E$ ,  $V_{CE}$  and  $I_C$  are selected,  $V_{RC}$  is determined as

$$V_{RC} = V_{CC} - V_{CE} - V_E$$

Then,  $R_C$  and  $R_E$  are calculated as

$$R_C = V_{RC} / I_C \text{ and } R_E = V_E / I_C$$

### Selection of bias resistors

Voltage divider current ( $I_2$ ) as  $I_C/10$  gives good bias stability and reasonably high input resistance. The bias resistors are calculated as

$$R_2 = V_B / I_2$$

$$R_1 = \frac{V_{cc} - V_B}{I_2}$$

Selecting  $R_2 = 10R_E$  gives  $I_2 = I_C/10$  the precise level of  $I_2$  can be calculated as

$I_2 = V_B/R_2$  and this can be used in the equation for  $R_1$ .

### Selection of bypass capacitor, CE

Basically the capacitor values are calculated at the lowest signal frequency that the circuit is required to amplify. This frequency is the lower cut-off frequency,  $f_L$ .

Choose  $X_{CE} = \frac{h_{ie}}{1 + h_{fe}}$

at  $f_L$  for  $C_E$  calculation to give the smallest value for the bypass capacitor.

### **Selection of coupling capacitors, $C_1$ and $C_2$**

The coupling capacitors  $C_1$  and  $C_2$  should have a negligible effect on the frequency response of the circuit. To minimize the effects of  $C_1$  and  $C_2$ , the reactance of each coupling capacitor is selected to be approximately equal to one-tenth of the impedance in series with it at the lowest operating frequency of the circuit ( $f_L$ ).

$$X_{C1} = \frac{Z_i + r_s}{10}$$

$$X_{C1} = \frac{Z_o + R_L}{10}$$

Usually,  $R_L \gg Z_o$  and often  $Z_i \gg r_s$ , so that  $Z_o$  and  $r_s$  can be omitted in the above equations.

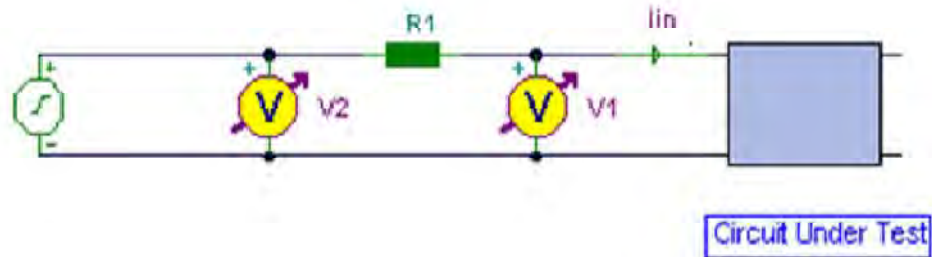
## **1.6 Self Learning Exercise-I**

- Q.1** What is amplifier?
- Q.2** What is amplification?
- Q.3** What is the difference between amplifier and oscillator
- Q.4** How are amplifiers classified according to the transistor configuration?
- Q.5** Define current amplification factor of a transistor ?

## **1.7 Procedure**

1. Wire the circuit.
2. Apply the 12V supply voltage to the bread board .With a VOM or DMM, individually measure the transistor DC base, emitter, and collector voltages with respect to ground, recording your results in Table 1. Also measure the emitter and the collector current.
3. Connect channel 1 of your Oscilloscope to point 1( $v_{in}$ ) and channel 2 to point O ( $v_{out}$ ). Then connect the signal generator to the circuit as shown in figure, and adjust the sine wave out-put level of the generator at 30mV *peak-to-peaks* at a frequency of 10 kHz (Use a voltage divider if required).You should observe that

the output signal level ( $v_{out}$ ) is greater than the input level ( $v_{in}$ ) in addition,  $v_{out}$  is inverted or  $180^\circ$  out-of-phase, with respect to the input. These points are two major characteristics of a common-emitter.



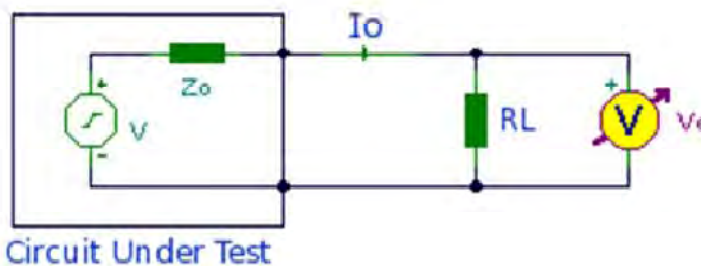
4. Use a fix resistor for instance of  $10k\Omega$  in series with the signal generator as shown in fig. Measure the AC voltage at points  $V_1$  and  $V_2$ , then the input current,  $I_{IN}$  becomes:

$$I_{in} = \frac{V_2 - V_1}{R_1}$$

The input impedance of the circuit under test is then found by

$$Z = \frac{V_1}{I_{in}}$$

5. Output impedance may also be determined using a similar technique. A fixed load resistor is used and the output voltage is measured first with full load, then without the load.



In the diagram above,  $Z_o$  is the internal output impedance of the network to be measured. First, the load resistor  $R_L$  is removed and output voltage ( $V$ ) measured and recorded. Then  $R_L$  is placed back in circuit and the output voltage under load ( $V_L$ ). The output impedance,  $Z_o$  is now found by Ohm's Law for AC circuits. As the load is purely resistive  $Z=V/I$ , where "V" is voltage drop across the output impedance:  $(V - V_L)$ , and "I" the output current,  $V_L/R_L$ .

## 1.8 Observation

**Table I**

Parameter	Measured value	Calculated value	Percentage error
$V_B$			
$V_E$			
$V_C$			
$I_C$			
$I_E$			

**Table II**

Parameter	Measured value	Calculated value	Percentage error
$Z_{in}$			
$Z_{Out}$			

## 1.9 Graph

Plot the required Graph

## 1.10 Calculations

Show all the steps by which the dc operating points have been determined.

.....  
.....

.....  
.....  
.....

### **1.11 Result**

The gain of the amplifier as measured is:.....

The input impedance as measured is:.....

The output impedance as measured is:.....

### **1.12 Discussion**

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.....  
.....  
.....  
.....

### **1.13 Precautions and Sources of Error**

1. Make the connections properly.
2. While doing the experiment do not exceed the ratings of the diode. This may lead to damage of the diode.
3. Connect voltmeter and Ammeter in correct polarities as shown in the circuit diagram.
4. Do not switch ON the power supply unless you have checked the circuit connections as per the circuit diagram.
5. Over heating of the transistor should be avoided.

### **1.14 Self Learning Exercise -II**

- Q.1** What is the biasing rule of transistor ?
- Q.2** For amplification CE is preferred, why?
- Q.3** To operate a transistor as amplifier, emitter junction is forward biased and collector junction is reverse biased, why?

**Q.4** Which transistor configuration provides a phase reversal between the input and output signals?

## 1.15 Glossary

**Amplifier:** A device that uses an active component to increase the voltage or power of a signal without distorting its waveshape.

**Base:** (1) The control portion of an NPN or PNP junction transistor.

(2) The P-type diffusion done using boron that forms the base of NPN transistors, the emitter and collector of lateral PNP transistors, and resistors.

**Bipolar transistor :** A transistor consisting of an emitter, base and collector, whose action depends on the injection of minority carriers from the base by the collector. Sometimes called NPN or PNP transistor to emphasize its layered structure.

**Capacitor :** A discrete device which stores electrical charge on two conductors separated by a dielectric.

**Collector :** Along with the emitter and base, one of the three regions of the bipolar type of transistor.

**Device :** A single-function component such as a transistor, resistor, capacitor or an integrated circuit.

**Emitter :** (1) The region of a transistor that serves as the source or input end for carriers. (2) The N-type diffusion usually done using phosphorus, which forms the emitter of NPN transistors, the base contact of PNP transistors, the N<sup>+</sup> contact of NPN transistors, and low-value resistors.

**Intrinsic Semiconductor:** An element or compound that has four electrons in its outer ring (i.e. elements from Group IV of the periodic table or compounds of Group III and V).

**Semiconductor:** An element such as silicon or germanium, intermediate in electrical conductivity between the conductors and the insulators, in which conduction takes place by means of holes and electrons. Common single-element semiconductors are Si (silicon) and Ge (germanium); a common compound semiconductor is GaAs (gallium arsenide). silicon (Si): The Group IV element used for fabricating diodes, transistors and integrated circuits.

**Transistor:** A semiconductor device that uses a stream of charge carriers to produce active electronic effect. The name was coined from the electrical characteristic of "transfer resistance".

**Common Base:** A transistor circuit in which the base electrode is the common element to both input and output circuits. Normally the emitter is used as the input terminal and the collector is used as the output terminal. The common base configuration does not produce a phase shift between the input and output and is commonly used as a voltage amplifier.

**Common Collector:** A transistor circuit configuration in which the collector is the element common to both the input and the output circuits. The common collector configuration does not produce a phase shift between the input and output.

**Common Emitter :**A circuit configuration in which the emitter of a transistor is the element common to both the input and the output circuits. The common emitter configuration produces a phase shift of 180 degrees between the input and output.

## 1.16 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** Amplifier is a device which is used to amplification purpose.

**Ans.2:** A low strength signal is converted into strengthen signal ie) boost up process .

**Ans.3:** Amplifier is working in the negative feedback while oscillator working in the positive feedback.

**Ans.4:** (i)Common emitter amplifier  
(ii)Common base amplifier  
(iii)Common collector amplifier

**Ans.5:** It is defined as ratio of change in collector current to change in base current.

### *Answers to Self Learning Exercise-II*

**Ans.1:** Emitter base junction should be forward biased and collector base junction should be reversed biased.

**Ans.2:** Because amplification factor beta is usually ranges from 20-500 hence this configuration gives appreciable current gain as well as voltage gain at its output on the other hand in the Common Collector configuration has very high input resistance( $\sim 750K\Omega$ ) & very low output resistance( $\sim 25\Omega$ ) so the voltage gain is always less than one & its most important application is for impedance matching for driving from low impedance load to high impedance source

**Ans.3:** Voltage is directly proportional to Resistance. Forward bias resistance is very less compared to reverse bias. In amplifier input forward biased and output reverse biased so voltage at output increases with reverse bias resistance.

**Ans.4:** Common emitter configuration.

### 1.17 Viva Questions

- Q.1 What is an Oscillator?
- Q.2 What is Quality factor?
- Q.3 What is the different analysis available to analyze a transistor?
- Q.4 What is the necessary of the coupling capacitor?
- Q.5 What is transistor? what are the types of transistors?
- Q.6 Define n-type and p-type semiconductors.
- Q.7 What is doping?
- Q.8 What are three basic transistor connection modes?  
Which mode is most preferred? Why?
- Q.9 What is semiconductor?
- Q.10 Which are the commonly used semiconductors?
- Q.11 Can we replace transistor by two back to back connected diodes?

### 1.18 Answers to Viva Questions

**Ans.1:** An Oscillator is a Circuit, which generates an alternating voltage of any desired frequency. It can generate an ac output signal without requiring any externally applied input signal.



**Ans.2:** The ratio of inductive reactance of the coil at resonance to its resistance is known as quality factor.  $Q = X_L / R$ .

**Ans.3:** 1. AC analysis 2. DC analysis

**Ans.4:** It is used to block the c signal to the transistor amplifier. It allows a c & blocks the d c.

**Ans.5:** Transistor is a three terminal semiconductor device. There are two types of semiconductor i.e. n-p-n and p-n-p.

**Ans.6:** If an intrinsic semiconductor is doped with pentavalent impurity. Then it is called n-type semiconductors. If it is doped with trivalent impurity then it is called p-type semiconductor.

**Ans.7:** Adding trivalent or pentavalent impurity to an intrinsic semiconductor is called doping.

**Ans.8:** Common-Emitter, Common-Base & Common-Collector connections. CE mode is most preferred because CE modes gives Highest voltage gain.

**Ans.9:** Semiconductor is a substance which has the resistivity in between that of conductor and insulator. semiconductor has negative temperature coefficient of resistance.

**Ans.10:** Germanium & Silicon

**Ans.11:** No, because the doping levels of emitter (heavily doped), base (lightly doped) and collector (doping level greater than base and less than emitter) terminals are different from p and n terminals in diode.

## References and Suggested Readings

1. G.E. Transistor Manual 1964.
2. Handbook of Semiconductor Electronics by Lloyd P. Hunter, 1970.
3. Fundamentals of Electronics by E. Norman Lurch, 1981.
4. Electronic and Radio Engineering by Frederick Terman, 1955.
5. Laboratory Manual by Jiann S. Yuan, 2011.

# UNIT-2

## RC Oscillators

### Structure of the Unit

- 2.1 Aim
- 2.2 Apparatus
- 2.3 Diagram
- 2.4 Formula
- 2.5 Theory and Description
  - 2.5.1 RC Phase Shift Oscillator
    - (i) RC Phase Shift Network
    - (ii) OP-AMP RC Phase Shift Oscillator
    - (iii) Illustrative Example
  - 2.5.2 Wein Bridge Oscillator
    - (i) OP-AMP Wein Bridge Oscillator
    - (ii) Feedback Process
    - (iii) Illustrative Examples
- 2.6 Self Learning Exercise
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- 2.15 Answers to Self Learning Exercise

2.16 Viva Question

2.17 Answers to Exercise

References and Suggested Readings

## 2.1 Aim

To study the characteristics of RC oscillators, namely,

(i) Phase Shift Oscillators, and

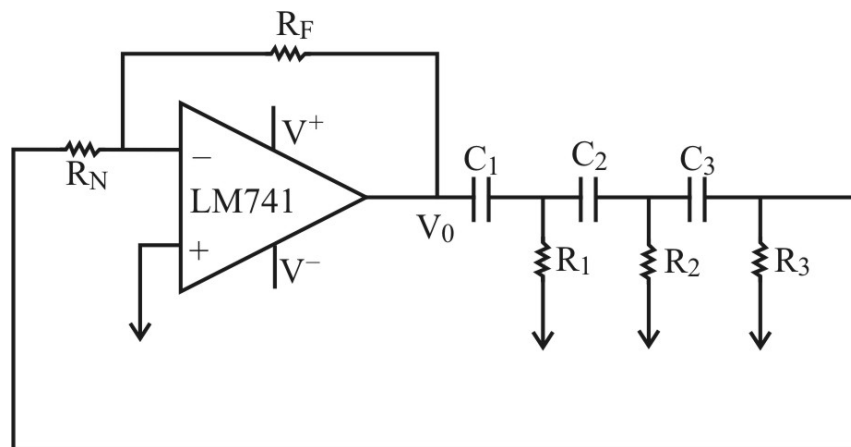
(ii) Wein Bridge Oscillators using IC-741 operational amplifier.

## 2.2 Apparatus

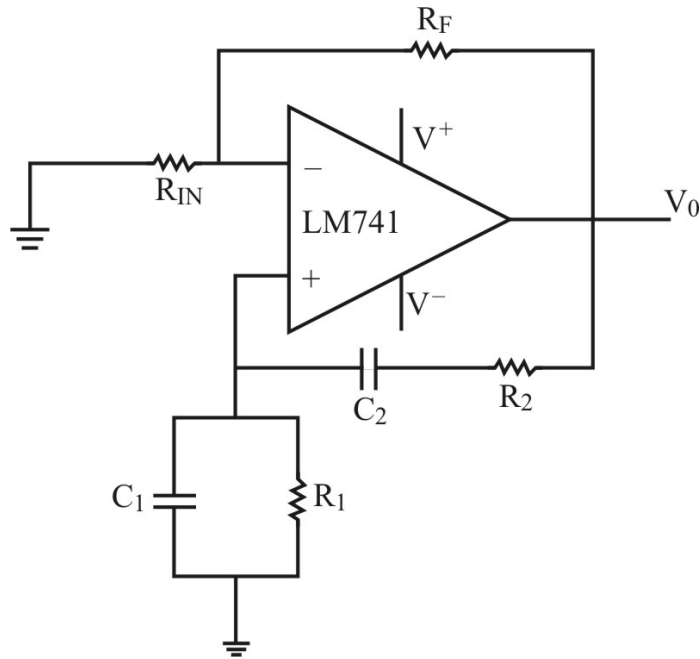
Dual Power Supply, CRO, Operational Amplifier (OP-AMP) 741, two variable capacitors ( $C_1$ ,  $C_2$ ), three variable non-inductive resistors ( $R_1$ ,  $R_2$ , and  $R_3$ ) and one fixed resistor, and connecting terminals.

## 2.3 Diagram

The circuit diagrams of RC Phase Shift Oscillator and of Wein Bridge oscillator are shown below as:



**Fig. 1:** OP-AMP RC Phase Shift Oscillator



**Fig. 2:** OP-AMP Wein Bridge Oscillator

## 2.4 Formula

Frequency of oscillation for RC phase shift oscillator (Fig. 1)

$$f_{osc} = \frac{1}{2\pi(\sqrt{6})RC} = \frac{0.065}{RC} \text{ Hz,}$$

where  $R_1 = R_2 = R_3 = R$  Resistances and  $C_1 = C_2 = C_3 = C$  Capacitances (F)

For Wein Bridge oscillator (Fig. 2), it is given by

$$f_{osc} = \frac{1}{2\pi RC} = \frac{0.159}{RC} \text{ Hz,}$$

where  $R_1 = R_2 = R$  Resistances and  $C_1 = C_2 = C$  Capacitances (F).

## 2.5 Theory and Description

### 2.5.1 RC Phase Shift Oscillator

It is known that single frequency RC oscillators produces a sine wave output at audio frequency range. Audio oscillators are widely used to produce warning sounds like beep in anything from automobiles to airliners. But oscillators

may also produce a single sine wave of fixed frequency carefully by using controlling elements, such as RC filters. Audio oscillators produce waves having frequencies from about 20Hz to 20kHz (the audio spectrum) and up to about 100kHz for ultrasonic purposes. Variable frequency oscillators are used in audio signal generators that are essential for testing amplifiers and fault tracing in many electronics systems. The RC oscillators used for these applications can be capable of producing signals up to around 1MHz or more.

**(i) RC Phase Shift Network**

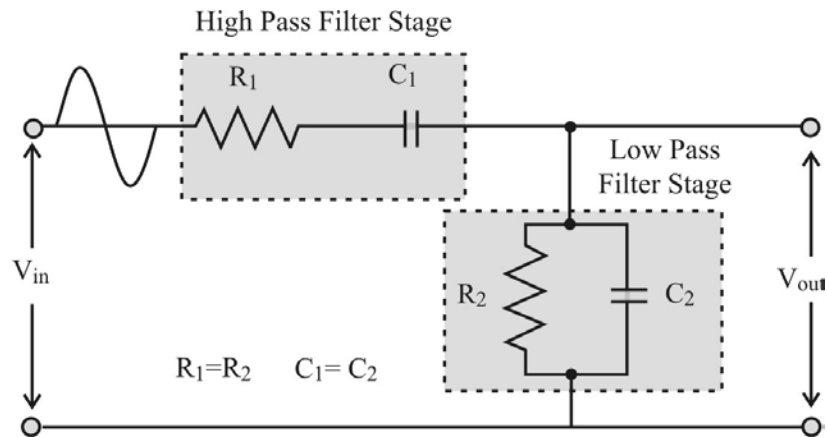
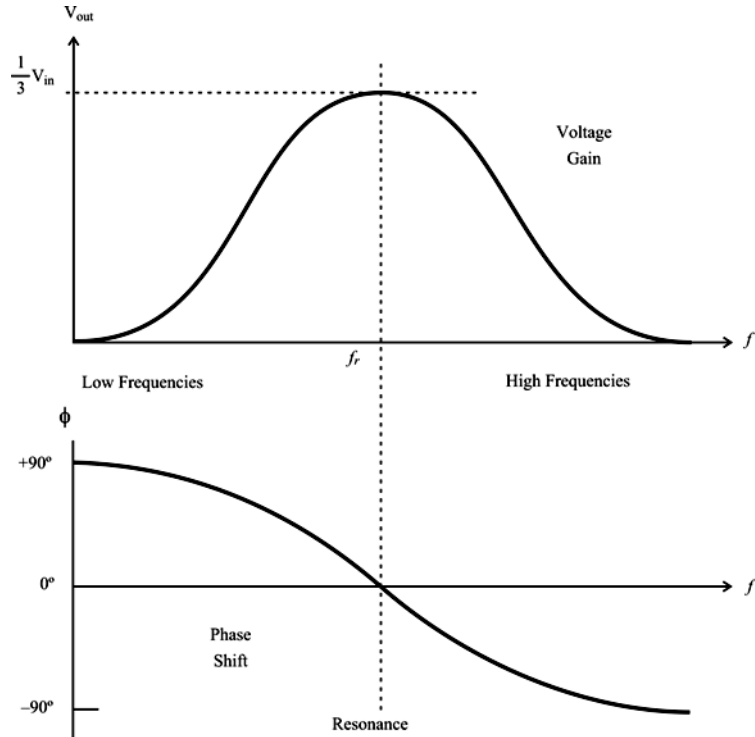


Fig. 3

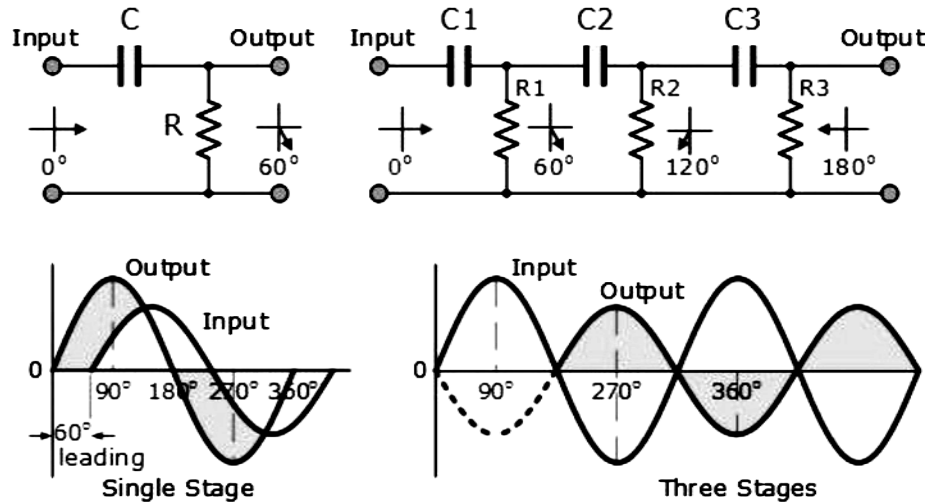
Now let us understand about the Phase shift due to RC network briefly. For low frequency applications (up to 100kHz), the passive circuits (filters) are constructed using simple RC (Resistor-Capacitor) networks, while for higher frequency (above 100kHz) filters are made from RLC (Resistor-Inductor-Capacitor) components, generally. The above RC network consists of basically a High Pass Filter (series RC circuit) connected to a Low Pass Filter (parallel RC circuit) producing a particular second-order frequency dependant Band Pass Filter with a high Q factor at the selected frequency,  $f_r$ .

Since, at low frequencies the reactance of capacitor  $C_1$  is very high and thus behaves like an open circuit and does not allow any input signal at  $V_{in}$  and therefore there is no output signal,  $V_{out}$ , whereas, at high frequencies, the reactance of capacitor  $C_2$  is very low then therefore this parallel connected capacitor acts like a short circuit at the output and thus again there is no output signal. However, between these two extremes the output voltage reaches a maximum value with the frequency at which this happens being called the *Resonant Frequency*, ( $f_r$ ). At this

resonant frequency (when  $X_C = R$ ), the phase shift is zero degree between the input and output. The magnitude of the output voltage is therefore at its maximum and is equal to one third (1/3) of the input voltage as shown below.



**Fig. 4**



**Fig. 5: Shifting of phase**

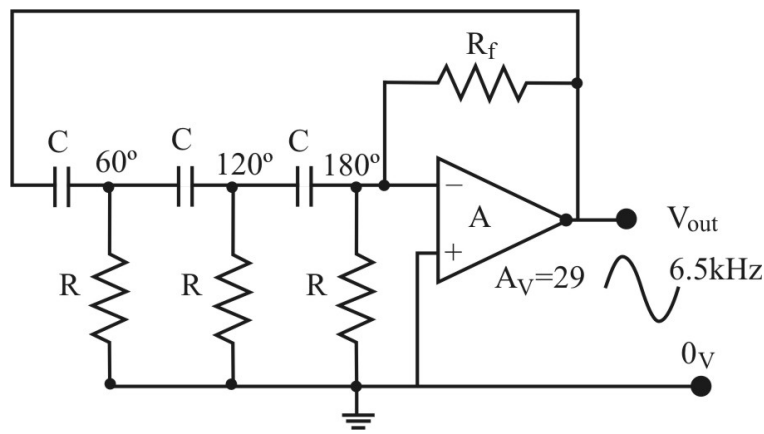
On looking into left side of the circuit in Fig. 4, a single resistor-capacitor network, the output voltage "leads" the input voltage by some angle less than  $90^\circ$ . In ideal

situation, single RC circuit would produce a phase shift of exactly  $90^\circ$ , and because  $180^\circ$  of phase shift is required for oscillation, at least two single RC networks must be used in an *RC oscillator* design. However in reality it is difficult to obtain exactly  $90^\circ$  of phase shift so more stages are used. If a three-stage RC phase-shift network is connected between this input and output of the amplifier, the total phase shift necessary for regenerative feedback will become  $3 \times 60^\circ + 180^\circ = 360^\circ$ . The amount of actual phase shift in the circuit depends upon the values of the resistor and the capacitor, and the chosen frequency of oscillations with the phase angle ( $\phi$ ) being given by

$$\phi = \tan^{-1} \frac{X_C}{R}, \text{ where } X_C = \frac{1}{2\pi fC} \text{ and resistance is } R.$$

### (ii) OP-AMP RC Phase Shift Oscillator

The oscillator circuit consists of a negative-gain operational amplifier and a three section RC network. The phase shift network is connected from the op-amps output back to its “inverting” input as shown below:



**Fig. 6**

As the feedback is connected to the inverting input terminal of operational amplifier which produces the required  $180^\circ$  phase shift while the RC network produces the other  $180^\circ$  phase shift at the required frequency ( $180^\circ + 180^\circ$ ). Unique feature of an RC oscillator is its frequency stability which is its ability to provide a constant frequency sine wave output under varying load conditions. By cascading more RC stages together, the stability of the oscillator can be greatly improved.

Frequency of RC Oscillators is proportional to  $1/RC$  and thus, a broad range of

frequency is possible when using a variable capacitor.

**Advantages:-**

- (i) This circuit is very simple and cheap as it comprises resistors and capacitors.
- (ii) It provides good frequency stability.
- (iii) The output of this circuit is sinusoidal and almost distortion free.
- (iv) The phase shift oscillator circuit is simpler because it does not need negative feedback and the stabilization arrangements.
- (v) They have a wide range of frequency (from a few Hz to several hundred of kHz).
- (vi) They are particularly suitable for low frequencies; say of the order of 1 Hz, so these frequencies can be easily gained by using R and C of large values.

**Disadvantages:-**

- (i) The output is small because of its smaller feedback.
- (ii) The frequency stability is not as good as that of the Wien bridge oscillator.
- (iii) It is difficult for the circuit to start oscillations as the feedback is usually small.
- (iv) It needs high voltage power supply (say 12V) so as to develop a sufficiently large feedback voltage.

**(iii) Illustrative Example**

**Example 1:** A 3-stage RC Phase Shift Oscillator is required to produce an oscillation frequency of 6.5 kHz. If 1nF capacitors are used in the feedback circuit, calculate the value of the frequency determining resistors and the value of the feedback resistor required to sustain oscillations.

**Sol.** The standard equation given for the phase shift RC oscillator is given by:

$$f_r = \frac{1}{2\pi RC\sqrt{2N}}$$

Using the value of  $f_r = 6.5\text{kHz}$ ,  $C = 1\text{nF}$  and  $N = 3$ , we get

$$R \cong 10\text{k}\Omega$$

The operational amplifiers gain must be equal to 29 in order to sustain oscillations.

Then using  $A_v = \frac{R_f}{R}$ , we get  $R_f \cong 290\text{k}\Omega$ .



In the next we will discuss another type of RC Oscillator called a Wein Bridge Oscillators which uses resistors and capacitors as its tank circuit to produce a low frequency sinusoidal waveform.

## 2.5.2 Wein Bridge Oscillator

### (i) OP-AMP Wein Bridge Oscillator

One of the simplest sine wave oscillators, which has a RC network in place of LC tuned tank circuit to produce a sinusoidal output waveform, is called a Wein Bridge Oscillator as shown in fig. 2. The circuit of this oscillator is based on a frequency-selective form of the Wheatstone bridge circuit. Actually, this type of oscillator is simple in design, compact in size, and it is a two-stage RC coupled amplifier circuit provide remarkable stability at its resonant frequency, low distortion and is very easy to tune. This is widely used in audio and sub-audio frequency ranges (20 – 20 kHz). However, the maximum frequency output of a typical Wein bridge oscillator is only about 1 MHz. This, in fact, behaves like a phase-shift oscillator.

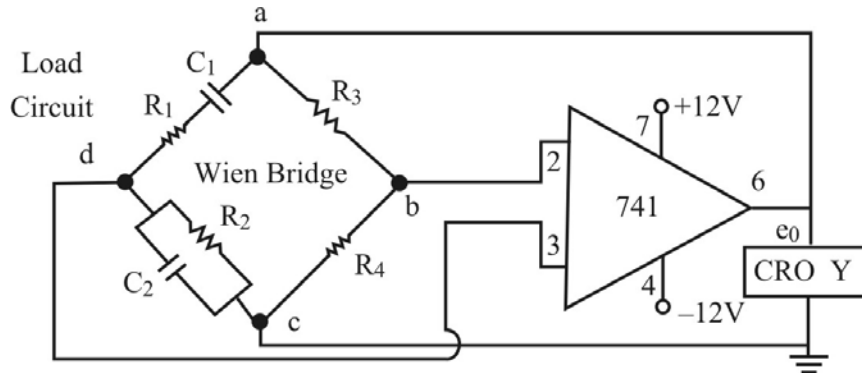
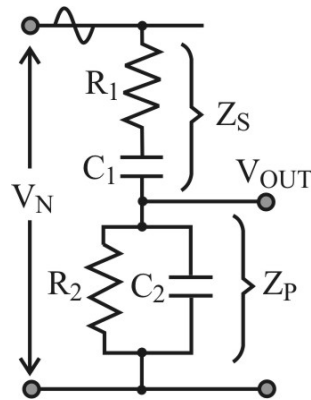


Fig. 7

This oscillator circuit is also known as **lead-lag circuit**. At high frequencies, the reactance of capacitor  $C_1$  and  $C_2$  approaches to zero. This causes  $C_1$  and  $C_2$  appears short. Here, capacitor  $C_2$  shorts the resistor  $R_2$ . Hence, the output voltage  $V_0$  will be zero since output is taken across  $R_2$  and  $C_2$  combination. So, at high frequencies, circuit acts as a "**lag circuit**", i.e. the phase angle between input and output signal becomes "negative" (Phase Delay) and at low frequencies, both capacitors act as open because capacitor offers very high reactance. Again output voltage will be zero because the input signal is dropped across the  $R_1$  and  $C_1$

combination. Here, the circuit acts like a “**lead circuit**”, i.e. the phase angle between the input and output signal becomes “positive” (Phase advanced). But at one particular frequency between the two extremes, the output voltage reaches to the maximum value. At this frequency only, resistance value becomes equal to capacitive reactance and gives maximum output. Hence, this particular frequency is known as resonant frequency or oscillating frequency ( $f_r$ ).

As it is said that the output voltage ( $V_0$ ) from the RC network is at its maximum value and equal to one third ( $1/3$ ) of the input voltage ( $V_{in}$ ) to allow for oscillations to occur. But the question arises that why one third and not some other value? Thus, in order to resolve this quest, we have to consider the complex impedance ( $Z = R \pm iX_c$ ) of the two connected RC circuits. Consider the following RC network:



**Fig. 8**

Let us define the total impedance of the series combination ( $R_1C_1$ ), and of the parallel combination ( $R_2C_2$ ) are  $Z_S$  and  $Z_P$  respectively. As  $Z_S$  and  $Z_P$  are effectively connected together in series across the input ( $V_{in}$ ), they form a voltage divider network with the output taken from across  $Z_P$  as shown.

For numerical testing, assuming that the component values of  $R_1$  and  $R_2$  are the same at:  $12k\Omega$ , capacitors  $C_1$  and  $C_2$  are the same at  $3.9nF$  and the supply frequency,  $f$  is  $3.4$  kHz.

**Series Circuit:** The total impedance of the series combination with resistor,  $R_1$  and capacitor,  $C_1$  is simply:

$$X_c = \frac{1}{2\pi fC} = 12k\Omega \text{ and } R = 12k\Omega$$

$$Z_s = \sqrt{R^2 + X_c^2} \cong 17k\Omega$$

**Parallel Circuit:** For the lower parallel impedance  $Z_p$ , as the two components are in parallel, we have to treat this differently because the impedance of the parallel circuit is influenced by this parallel combination. The total impedance of the lower parallel combination with resistor,  $R_2$  and capacitor,  $C_2$  is given as:

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_c} \Rightarrow Z = 6k\Omega$$

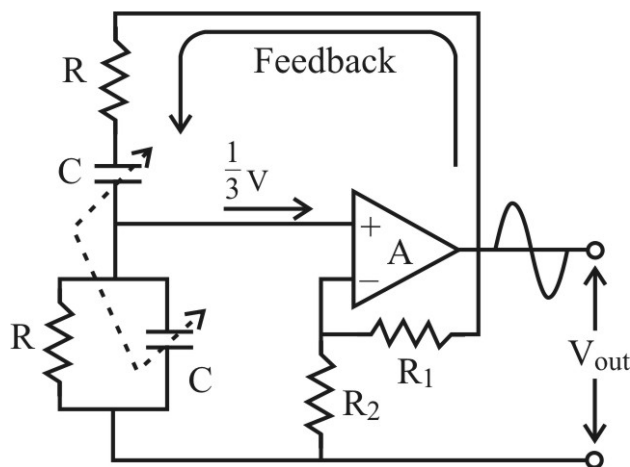
Therefore, the combined resistance and reactance of the RC parallel circuit becomes  $6k\Omega$  ( $R \parallel X_c$ ) and their parallel impedance will be:

$$Z_p = \sqrt{R^2 + X_c^2} = 8.5k\Omega$$

So we now have the value for the series impedance  $Z_s = 17k\Omega$  and for the parallel impedance  $Z_p = 8.5k\Omega$ . Therefore the output impedance,  $Z_{out}$  of the voltage divider network at the given frequency is:

$$Z_{out} = \frac{Z_p}{Z_p + Z_s} \cong \frac{1}{3}$$

Then at the oscillation frequency, the magnitude of the output voltage,  $V_o$  will be equal to  $Z_{out}V_{in}$  which as shown is equal to one third ( $1/3$ ) of the input voltage,  $V_{in}$  and it is this frequency selective RC network which forms the basis of the Wein Bridge Oscillator circuit.



**Fig. 9**

### **(ii) Feed Back Process**

The process of feedback is done by circuit consisting of a RC circuit in series connected with a parallel combination of RC having same component values, which produces a phase delay or phase advance circuit depending upon the frequency. At the resonant frequency  $f_r$  the phase shift is  $0^\circ$ . The output of the operational amplifier is fed back to both the inputs of the amplifier as shown. (i) *Negative Feedback* - One part of the feedback signal is connected to the inverting input terminal through the resistor divider network of  $R_1$  and  $R_2$  which allows the amplifiers voltage gain to be adjusted within narrow limits. (ii) *Positive Feedback* - The other part is fed back to the non-inverting input terminal through the RC Wein Bridge network. Therefore, the RC network is connected in the positive feedback path of the amplifier produces zero phase shift a just one frequency. Then at the selected resonant frequency, ( $f_r$ ) the voltages applied to the inverting and non-inverting inputs will be equal and "in-phase" so the positive feedback will cancel out the negative feedback signal causing the circuit to oscillate. To start the oscillation with the constant amplitude, positive feedback is not the only sufficient condition but also oscillator circuit must satisfy the conditions known as *Barkhausen conditions*. The feedback signal does not produce any phase shift. This is the "basic principle of a Wein bridge oscillator".

Hence, the voltage gain of the amplifier circuit *must* be equal to or greater than three for oscillations to start as it is understood that the input is 1/3 of the output. This value, ( $A_v \geq 3$ ) is set by the feedback resistor network,  $R_1$  and  $R_2$  and for a non-inverting amplifier this is given as the ratio  $1 + R_1 / R_2$ .

The advantages and disadvantages of Wein bridge oscillators are given below:

#### ***Advantages:-***

1. Provides a stable low distortion sinusoidal output over a wide range of frequency.
2. The frequency range can be selected simply by using decade resistance boxes.
3. The frequency of oscillation can be easily varied by varying capacitances  $C_1$  and  $C_2$  simultaneously.

#### ***Disadvantages:-***

1. Large number of components required to design 2 stage amplifiers.
2. The maximum frequency output is limited because of amplitude and the phase-shift characteristics of amplifier.

**(iii) Illustrative Examples**

**Example 2:** Determine the maximum and minimum frequency of oscillations of a Wein Bridge Oscillator circuit having a resistor of  $10k\Omega$  and a variable capacitor of  $1nF$  to  $1000nF$ .

**Sol.** The frequency of oscillations for a Wein Bridge Oscillator is given by

$$f_r = \frac{1}{2\pi RC}$$

(i) **Lowest Frequency:**  $f_{\min} = \frac{1}{2\pi(10k\Omega) \times (1000 \times 10^{-9})} = 15.9 \text{ Hz}$

(ii) **Highest Frequency:**  $f_{\max} = \frac{1}{2\pi(10k\Omega) \times (1 \times 10^{-9})} = 15915 \text{ Hz}$

**Example 3.** A Wein Bridge Oscillator circuit is required to generate a sinusoidal waveform of  $5.2 \text{ kHz}$ . (a) Calculate the values of the frequency determining resistors  $R_1$  and  $R_2$  and the two capacitors  $C_1$  and  $C_2$  to produce the required frequency. (b) If the oscillator circuit is based around a non-inverting operational amplifier configuration, determine the minimum values for the gain resistors to produce the required oscillations.

**Sol.** Given  $f_r = \frac{1}{2\pi RC} = 5.2 \text{ kHz}$

Assuming the value for the feedback capacitors is of  $3.0nF$ , the corresponding value of the feedback resistors is  $R = \frac{1}{2\pi f_r C} = 10200\Omega$ . As it is known that the

voltage gain of the Wein Bridge circuit must be equal to or greater than 3 for starting the sinusoidal oscillations. For a non-inverting op-amp configuration, this value is set by the feedback resistor network of  $R_3$  and  $R_4$  and is given as:

$$A_v = \frac{V_{out}}{V_{in}} = 1 + \frac{R_3}{R_4} \geq 3$$

Choosing the resistor  $R_3$  of  $100k\Omega$  (say), then  $R_4 = \frac{R_3}{2}$ . If  $R_3 = 100k\Omega$  then  $R_4 = 50k\Omega$ , while  $A_v = 3$  is the minimum value required to ensure oscillations. Thus, practically value on little higher than that is generally required, i.e. 3.1, then  $R_4$  would be of  $47k\Omega$ 's.

## 2.6 Self Learning Exercise

- Q.1** What is amplifier?  
**Q.2** What do you mean by sustained Oscillation?  
**Q.3** How is phase angle determined in RC phase shift oscillator?  
**Q.4** State the two conditions of oscillations.

## 2.7 Procedure

### (A) RC Phase shift Oscillator

1. Connect the circuit as shown in figure1.
2. Connect Oscilloscope (CRO) at output terminals  $V_0$  observe the output sine wave.
3. Keep the resistance and capacitor values  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$  and switch on the power
4. Record the output waveform and measure the experimental frequency of the output waveform as mentioned below:
  - a. Adjust the voltage sensitivity switch and time – base switch such that at least two or more complete sine waves are observed on the screen of CRO.
  - b. Also adjust the resistance  $R_3$  value till the wave formed on the CRO screen is stationary. Note R and C values in the table and measure the peak to peak horizontal length ( $l$ ) of one sine wave.
  - c. Multiply this value with the corresponding time-base ( $t$ ) value. This product ( $T = l \times t$ ) gives the time period ( $T$ ) of the generated sine wave. The reciprocal of time period gives the experimental frequency of the sine wave.
5. From the given values of R & C calculate theoretical frequency using the

formula  $f_{th} = \frac{1}{2\pi\sqrt{6}RC}$ .

6. Compare the theoretical and Experimental frequencies at output signal.
7. Repeat the experiment for various R and C values.

### **(B) Wein-Bridge Oscillator**

1. Connect the circuit as shown in the Fig-2.
2. Connect the Oscilloscope (CRO) at output terminals to observe sine wave.
3. Keep the resistance and capacitor values  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$  and switch on the power.
4. For the measurement of experimental frequency, follow the procedure no. (iv) as for the case of RC phase shift oscillator.
5. On substitution of R and C values in  $f_{th} = \frac{1}{2\pi RC}$ , it gives theoretical frequency.
6. Compare the theoretical and experimental frequencies at output signal.
7. The experiment is repeated by changing the value of R or C.

## **2.8 Observation**

### **(i) RC Phase Shift Oscillator**

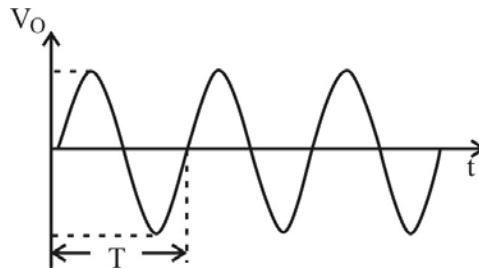
S. No.	Theoretical Frequency			Experimental Measurement of frequency at output			
	$R_1 = R_2 = R$ ( $\Omega$ )	$C_1 = C_2 = C$ ( $\mu F$ )	$f_{th} = \frac{1}{2\pi\sqrt{6}RC}$	peak to peak horizontal length (l) (Div.)	Time Base (t) (Sec./Div.)	Time Period $T = l \times t$ (Sec.)	$f_{exp} = 1/T$ (Hz)

**(ii) Wein Bridge Oscillator**

S. No.	Theoretical Frequency			Experimental Measurement of frequency at output			
		$R_1 = R_2 = R$ ( $\Omega$ )	$C_1 = C_2 = C$ ( $\mu F$ )	$f_{th} = \frac{1}{2\pi RC}$	peak to peak horizontal length ( $l$ ) (Div.)	Time Base ( $t$ ) (Sec./Div.)	Time Period $T = l \times t$ (Sec.)

**2.9 Graph**

Expected wave form should be



**2.10 Calculations**

.....

.....

.....

.....

**2.11 Result**

Operation of RC Phase shift Oscillator and Wein –Bridge Oscillator using IC 741 Op-Amp is studied and found that

Experimental frequency: .....

Theoretical frequency: .....



## 2.12 Discussion

### ***Phase Shift Oscillator:-***

- (i) In RC phase shift oscillator, the feedback circuit consists of three RC cascade sections.
- (ii) Feedback circuit introduces a phase shift of  $180^{\circ}$  at frequency of oscillations.
- (iii) Additional, phase shift of  $180^{\circ}$  is introduced by amplifier.
- (iv) Using OP-AMP, the value of  $\beta$  in phase shift oscillator must be at least  $-1/29$ .
- (v) For sustained oscillations, minimum value of gain ( $A_v$ ) should be 29.

### ***Wein Bridge Oscillator:-***

- (i) Without input signal a Wein bridge oscillator produces continuous output oscillations.
- (ii) The Wein bridge oscillator can produce a large range of frequencies.
- (iii) The Voltage gain of the amplifier must be greater than 3.
- (iv) The RC network can be used with a non-inverting amplifier.
- (v) The input resistance of the amplifier must be high compared to R so that the RC network is not overloaded and alter the required conditions.
- (vi) The output resistance of the amplifier must be low so that the effect of external loading is minimized.
- (vii) If the voltage gain of the amplifier is too small then the desired oscillation will decay and stop gradually.

## 2.13 Precautions and Sources of Error

1. Check the continuity of the connecting terminals before connecting them.
2. Keep the band switches of the C.R.O. and adjust the value of R3 such that steady wave forms are observed on the screen.
3. Observe the output sine wave on the screen of CRO and measure the horizontal length accurately.

## 2.14 Glossary

**The Low Pass Filter:** the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency,  $f_c$  point to pass while blocking those any higher.

**The High Pass Filter:** the high pass filter only allows high frequency signals from its cut-off frequency,  $f_c$  point and higher to infinity to pass through while blocking those any lower.

**Band Pass Filter:** the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

## 2.15 Answers to Self Learning Exercise

**Ans.1:** Amplifier is a device which is used to amplification purpose.

**Ans.2:** The electrical oscillations in which amplitude does not change with time are called as sustained oscillations. It is also called as undamped Oscillation.

**Ans.3:** Since the impedance of an RC network is capacitive, the current flowing through it leads the applied voltage by a specific phase angle. The phase angle is determined by the amount of resistance and capacitance of the RC section. If the capacitance is a fixed value, a change in the resistance value will change the phase angle. If the resistance could be changed to zero, we could get a maximum phase angle of 90 degrees. But since a voltage cannot be developed across zero resistance, thus 90-degree phase shift is not possible.

## 2.16 Viva Questions

- Q.1 What is amplification?
- Q.2 What is the difference between amplifier and oscillator?
- Q.3 What is an Oscillator?
- Q.4 What is meant by resonant Circuit Oscillators?
- Q.5 What is a phase shift oscillator?
- Q.6 Why RC oscillators cannot generate high frequency oscillations?
- Q.7 What are the applications of RC phase shift oscillators?

- Q.8** What amount of phase shift does RC phase shift oscillator produce?
- Q.9** Why do we need a phase shift between input and output signal?
- Q.10** How can we get a maximum phase angle of 90 degrees in RC phase shift oscillator?
- Q.11** What are Barkhauesan conditions?
- Q.12** What are the applications of oscillators?
- Q.13** Draw the basic structure of feedback oscillator.
- Q.14** What is the frequency range of RC phase shift oscillator?
- Q.15** Define Wein bridge oscillator.
- Q.16** For sustained oscillations, what is the gain of RC phase shift oscillator?
- Q.17** Design a phase shift oscillator to oscillate at 100 Hz.
- Q.18** Draw the comparison between RC phase shift oscillator and Wein bridge oscillator.

## 2.17 Answers to Exercise

- Ans.1:** A low strength signal is converted into strengthen signal i.e. boost up process
- Ans.2:** Amplifier is working in the negative feedback while oscillator working in the positive feedback.
- Ans.3:** An Oscillator is a Circuit, which generates an alternating voltage of any desired frequency. It can generate an AC output signal without requiring any externally applied input signal.
- Ans.4:** LC Oscillators are known as resonant circuit oscillator because the frequency of operation of LC oscillator is nothing but a resonant frequency of tank circuit or LC tank circuit produces sustained Oscillation at the resonant circuit oscillator.
- Ans.5:** The oscillator, which uses an RC network in the feedback loop to generate the required phase shift at a particular frequency to sustain oscillations.
- Ans.6:** Because at high frequencies, resistors behaves like inductors or capacitors so the equations that govern oscillation in RC oscillators does not hold.

**Ans.7:** RC phase shift oscillators are mostly used at audio frequencies. Other than this, it is used in electronic musical instruments like pianos. It is also used in equipment that emits beeps.

**Ans.8:** RC phase shift oscillator produces a phase shift of 180 degrees between output and input signal. In the phase-shift oscillator, at least three RC sections are needed to give the required 180-degree phase shift for regenerative feedback. The values of resistance and capacitance are generally chosen so that each section provides about a 60-degree phase shift.

**Ans.9:** To obtain the regenerative feedback in the phase-shift oscillator, we need a phase shift of 180 degrees between the output and the input signal. This purpose is resolved by using an RC network consists of three RC sections which provides the proper feedback and phase inversion to provide this regenerative feedback.

**Ans.10:** By changing the resistance to zero, we can get a maximum phase angle of 90 degrees. But since, we cannot develop voltage across zero resistance, so a 90 degree phase shift is not possible.

**Ans.11:** The first condition is that the magnitude of the loop gain ( $A\beta = 1$ ), and the second condition is that the phase shift around the loop must be  $360^\circ$  or  $0^\circ$ .

## References and Suggested Readings

1. [www.learnabout-electronics.org/Oscillators/](http://www.learnabout-electronics.org/Oscillators/)
2. [www.nptel.ac.in/courses/](http://www.nptel.ac.in/courses/)
3. <https://www.classle.net/book/rc-phase-shift-oscillators-0>
4. *Other useful and trusted internet resources*

# UNIT-3

## Zener Diode

### Structure of the Unit

#### Part A : Characteristics of a Zener Diode

- 3.1A Aim
- 3.2A Apparatus
- 3.3A Diagram
- 3.4A Model Graph
- 3.5A Formula
- 3.6A Theory and Description
- 3.7A Procedure
- 3.8A Observations
- 3.9A Graph
- 3.10A Calculations
- 3.11A Result
- 3.12A Discussion
- 3.13A Precautions and Sources of Error
- 3.14A Self Learning Exercise-I

#### Part B : Line and Load Regulations with Zener Diode

- 3.1B Aim
- 3.2B Apparatus
- 3.3B Diagram
- 3.4B Model Graph
- 3.5B Formula
- 3.6B Theory and Description
- 3.7B Procedure
- 3.8B Observation

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  - 3.13B Precautions and Sources of Error
  - 3.14B Self Learning Exercise-II
  - 3.15 Glossary
  - 3.16 Answers to Self Learning Exercises
  - 3.17 Viva Questions
  - 3.18 Answers to Viva Questions
- References and Suggested Readings

## Part A : Characteristics of a Zener Diode

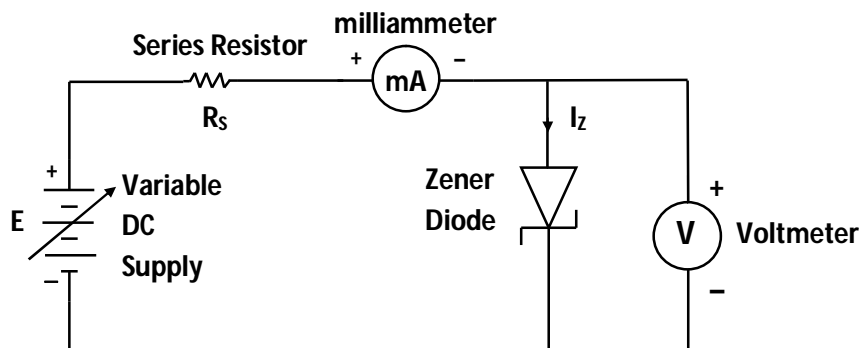
### 3.1A Aim

To study the characteristics of the Zener diode.

### 3.2A Apparatus

Zener diode, a variable dc power supply (0-15 Volts), dc voltmeters (0-1.5 Volts range and 0-10 Volts range ) milliammeter (0-30 mA) or multimeter, current limiting resistor (about  $1K\Omega$ ), bread board, connecting wires [or a trainer board having all these components].

### 3.3A Diagram



**Figure 3.1A Forward Bias**

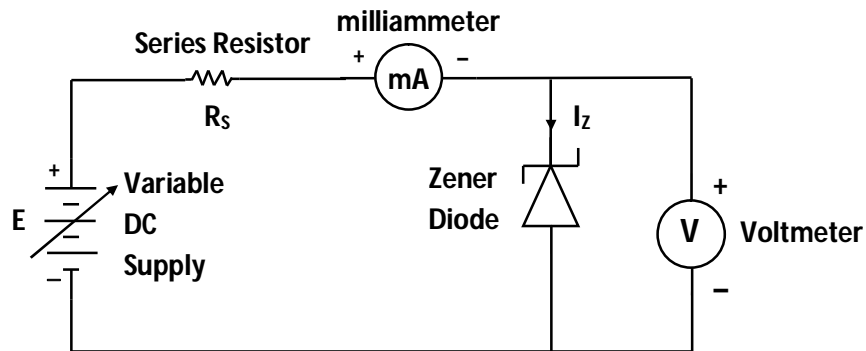


Figure 3.2A Reverse Bias

### 3.4A Model Graph

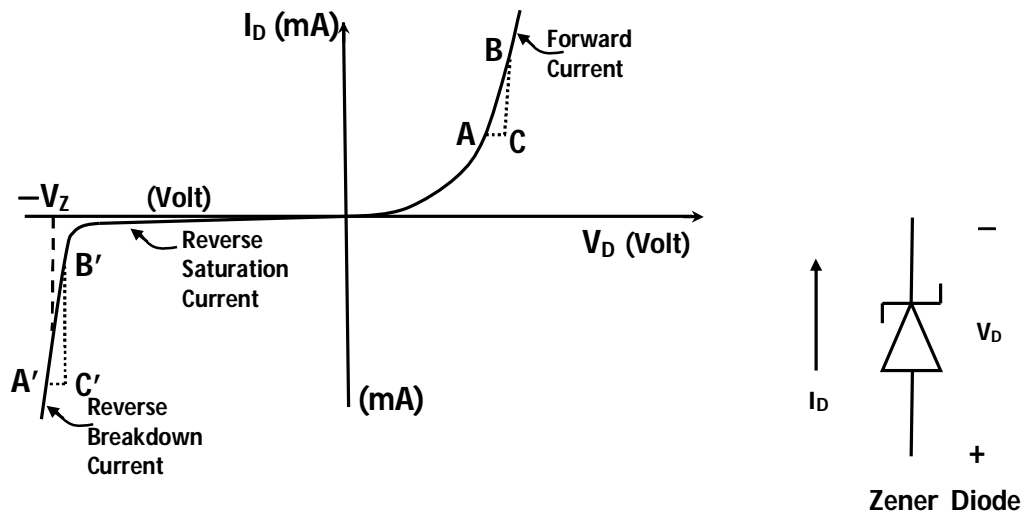


Figure 3.3 A Typical I-V characteristics of a Zener diode

$I_D$  = Diode current

$V_D$  = Voltage across the diode

$V_z$  = Breakdown Voltage

### 3.5A Formula

Zener diode conducts like an ordinary PN junction diode in forward bias and when the diode is reverse biased, then reverse saturation current (of the order of  $\mu A$ ) flows until the breakdown voltage is reached. At the breakdown voltage, reverse current increases abruptly. Inverse of the slope of I-V graph (I on y axis and V on x axis) represents the dynamic resistance.

$$\text{Diode dynamic resistance } r_{\text{diode}} = \frac{\Delta V_{\text{diode}}}{\Delta I_{\text{diode}}}$$

From model graph

$$\text{Forward bias dynamic resistance } r_F = \frac{AC}{BC} \text{ (in } \Omega \text{)}$$

$$\text{Zener dynamic resistance } r_Z = \frac{A'C'}{B'C'} \text{ (in } \Omega \text{)}$$

### 3.6A Theory and Description

Zener diode acts like an ordinary PN junction diode in forward bias and it exhibits approximately voltage independent saturation current in reverse bias until critical reverse voltage is reached. At this critical breakdown voltage  $V_Z$ , the reverse current through the diode increases drastically with voltage. This breakdown voltage is also known as Zener voltage. Current limiting series resistor is used to limit the current to a safe level. Zener diodes can be made with reverse breakdown voltages from about 2 Volts to over 1000 Volts by varying the doping level. Value of  $V_Z$  depends on amount of doping level. There are two mechanisms by which reverse breakdown occurs.

#### (i) Zener Breakdown :

When a heavily doped PN Junction is reverse biased, then electric field at junction becomes high of the order of  $10^7$  V/m. Due to this high electric field, breaking of covalent bonds occurs i.e. this tunneling of free electrons constitutes a large reverse current from N to P. This phenomena is known as Zener breakdown. Zener diodes have a steep current- voltage characteristic at breakdown voltage.

#### (ii) Avalanche Breakdown:

Electrons gain kinetic energy in the electric field of depletion region and electron hole pairs are created by impact ionization by these energetic carriers. These generated electrons further create new electron hole pairs by collisions. Due to this carrier multiplication i.e. avalanche process, a large reverse current flows through the diode.

The process may be avalanche or Zener, the name Zener diode is commonly used for all breakdown diodes. Zener breakdown has low reverse breakdown voltages





### 3.9A Graph

Choose the appropriate scales for  $I_{\text{diode}}$  and  $V_{\text{diode}}$  and plot the graph of  $I_{\text{diode}}$  (on y axis) versus  $V_{\text{diode}}$  (on x axis).

### 3.10A Calculations

Slope of I -V graph =  $\frac{1}{\text{resistance}}$

Forward bias resistance (dynamic)  $r_F = \frac{AC}{BC} = \dots$

Zener resistance (dynamic)  $r_z = \frac{A'C'}{B'C'} = \dots$

### 3.11A Result

1. Characteristics of Zener diode has been obtained and drawn.
2. Zener breakdown voltage  $V_z$  (approximated) = .... Volts
3. The forward bias dynamic resistance  $r_F = \dots \Omega$
4. The Zener resistance  $r_z = \dots \Omega$

### 3.12A Discussion

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### 3.13A Precautions and Sources of Error

1. Zener diode ,voltmeter and ammeter must be connected with proper polarities.
2. Applied voltages and currents must not exceed the ratings of the Zener diode.
3. Current limiting series resistor should be used to prevent the Zener diode from damage.
4. All connections should be tight. Avoid using long wires.
5. Power supply should be turned off while connecting or disconnecting the circuit components, wires etc.
6. Least counts and ranges of voltmeter and ammeter should be appropriate.

7. If there is zero error in the voltmeter or milliammeter, then it should be removed.

### 3.14A Self Learning Exercise-I

- Q.1** "Critical reverse (breakdown) voltage decreases as the doping of PN Junction increases." Is this statement true ?
- Q.2** Zener diode can be operated in forward, leakage and breakdown regions. What do you mean by leakage region?
- Q.3** Zener diode has specification such as power rating 500 mW and Zener voltage 10 Volts. What is the maximum value (rated) current?
- Q.4** What are the values of cut in voltages in forward bias for Si and Ge diodes?

## Part B : Line and Load Regulations with Zener Diode

### 3.1B Aim

To study the line and load regulations with Zener diode.

### 3.2B Apparatus

Zener diode, a variable dc power supply (0-15 Volts), dc voltmeters (0-15 Volts range) or multimeter, resistors, bread board, connecting wires [or a trainer board having all these components].

### 3.3B Diagram

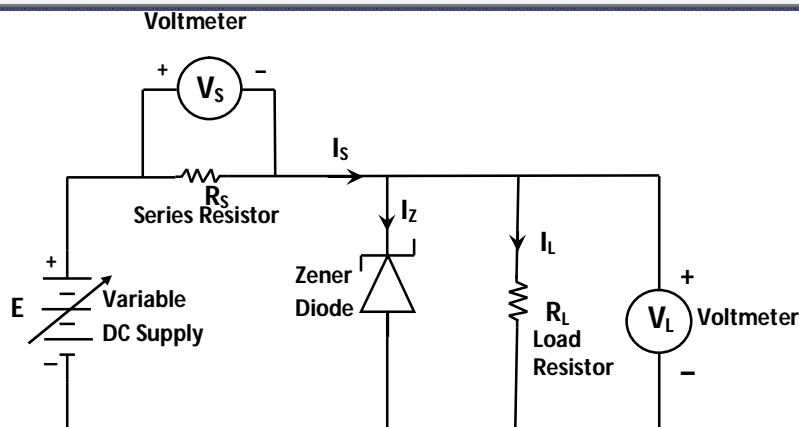


Figure 3.1B : Line regulation (Variable input supply voltage E and constant load resistance R<sub>L</sub>)

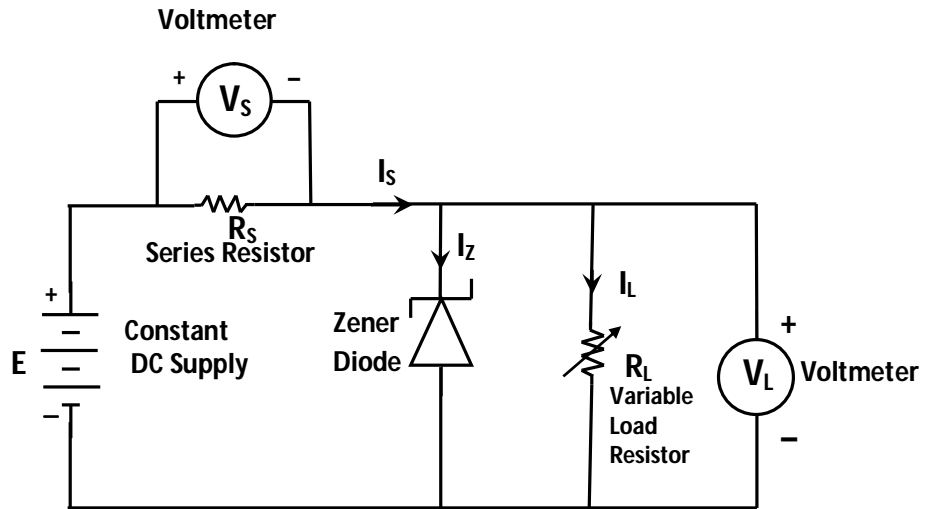


Figure 3.2B : Load regulation (Variable load resistance  $R_L$  and fixed supply voltage  $E$ )

### 3.4B Model Graph

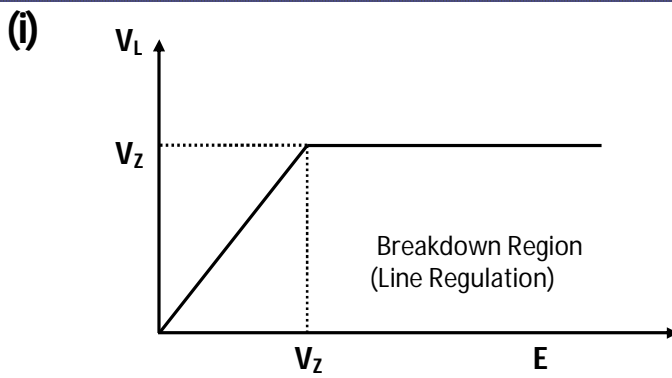


Figure 3.3B : Line Regulation

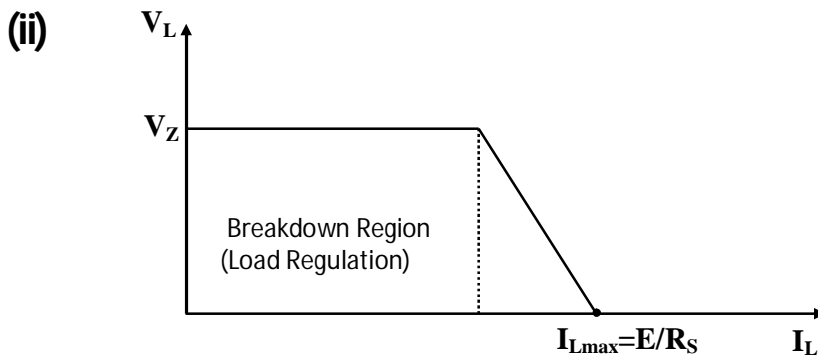


Figure 3.4B : Load Regulation

### 3.5B Formula

Line (source) regulation =  $\frac{\text{change in output voltage}}{\text{change in source voltage}}$

$$= \frac{\Delta V_{\text{out}}}{\Delta V_{\text{source}}} = \frac{\Delta V_L}{\Delta E} \quad (\text{in breakdown region})$$

Line regulation is usually expressed in  $\frac{mV}{V}$

Load regulation =  $\frac{\text{change in output voltage}}{\text{change in load current}}$

$$= \frac{\Delta V_{\text{out}}}{\Delta I_{\text{Load}}} = \frac{\Delta V_L}{\Delta I_L} \quad (\text{in breakdown region})$$

Load regulation is usually expressed in  $\frac{mV}{mA}$

### 3.6B Theory and Description

Basic Zener regulator circuit is shown in Figure 3.5B

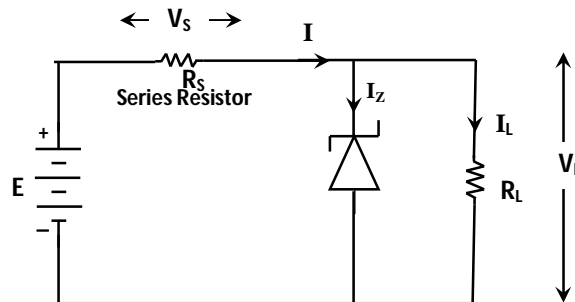


Figure 3.5B

If voltage across Zener diode is less than  $V_Z$  (breakdown voltage) ,then current through the diode is extremely small (of the order of  $\mu A$ ) and diode is considered in off state. Equivalent open circuit (off state) is shown in figure 3.6B

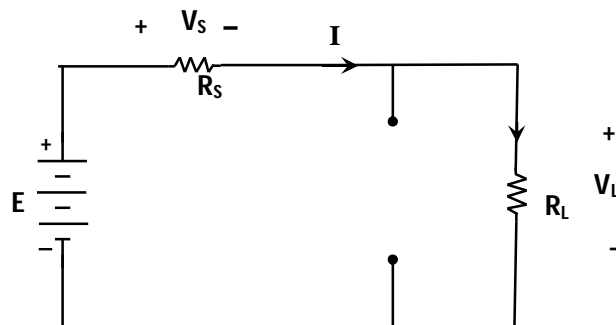


Figure 3.6B

Voltage across the load resistor is

$$V_L = IR_L = \left( \frac{E}{R_s + R_L} \right) R_L$$

In this case voltage across the load can be changed by varying  $R_L$  and  $E$ .

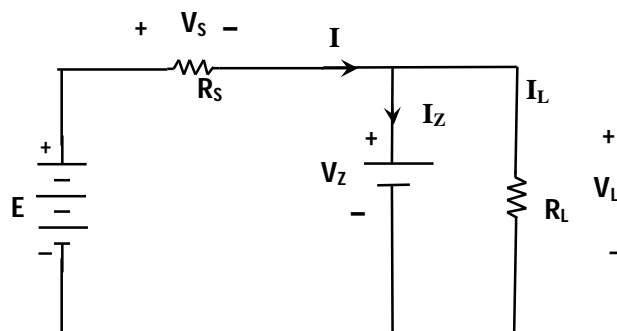
Voltage across the Zener diode is given by

$$V_L = \frac{E}{\left( 1 + \frac{R_s}{R_L} \right)}$$

For given value of  $E$ , if load resistance  $R_L$  increases, then  $V_L$  also increases and viceversa. The minimum load resistance at which voltage across the Zener diode becomes  $V_Z$  is  $R_{Lmin}$ , then we have

$$\begin{aligned} V_Z &= \frac{E}{1 + \frac{R_s}{R_{Lmin}}} \\ \Rightarrow 1 + \frac{R_s}{R_{Lmin}} &= \frac{E}{V_Z} \\ \Rightarrow R_{Lmin} &= R_s \left( \frac{V_Z}{E - V_Z} \right) \end{aligned}$$

For implementing the voltage regulator circuit, the Zener diode must be operated in the breakdown region. In that case voltage across the diode remains almost constant. Since load resistance  $R_L$  and the Zener diode are in parallel, so voltages across both the elements must be same i.e.  $V_L = V_Z$  and Zener diode is called in 'on state'. Equivalent 'on state' circuit is shown in figure 3.7B



**Figure 3.7B**

By Kirchoff's current law

$$I = I_Z + I_L \Rightarrow I_Z = I - I_L$$

$$I_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} \text{ (Here current } I_L \text{ is constant for given load resistance)}$$

If supply voltage E increases, then current through the Zener diode increases whereas load current  $I_L$  remains constant because voltage across the load resistor is constant. This circuit works as a voltage regulator or as reference voltage.

For given Zener diode, series resistor and load resistor, there are restrictions on the minimum and maximum values of supply voltage E for proper functioning of circuit as a voltage regulator.

If Zener current  $I_Z$  exceeds  $I_{Zmax}$  (rated Zener current), then Zener diode burns out.

**Line (Source) Regulation:**

It is the change in load voltage with variation in line (input source) voltage. Zener diode is operated in breakdown region for constant load resistance  $R_L$ .

We can summarize the result (in the breakdown region) for diagram 3.1B in the following table.

Here  $R_S$  and  $R_L$  have fixed values.

E(supply voltage) (Greater than $V_Z$ )	Voltage across series resistor $R_S$ $V_S = E - V_Z$	Current $I_S$ $I_S = \frac{V_S}{R_S}$	Load voltage $V_L$ (or breakdown voltage $V_Z$ )	Load current $I_L = \frac{V_Z}{R_L}$	Zener current $I_Z = I_S - I_L$
Increase	Increases	Increases	Constant	Constant	Increases
Decreases	Decreases	Decreases	Constant	Constant	Decreases

Hence we may conclude that even though Zener current  $I_Z$  may change, output voltage  $V_L$  remains constant.

If supply voltage E is decreased continuously, then  $I_Z$  decreases continuously, so circuit may come out of the breakdown region.

**Load Regulation:**

In this there is change in load voltage with variation in load resistance. Zener diode is operated in breakdown region for constant supply voltage E. We can summarize the result (in breakdown region) for diagram 3.2B in the following table.

Here supply voltage  $E$  and series resistance  $R_s$  are fixed.

Load Resistance	Voltage across load $V_L$ (i.e. $V_Z$ )	Load current $I_L = \frac{V_Z}{R_L}$	Voltage across series resistor $V_s = E - V_Z$	Current $I_s = \frac{V_s}{R_s}$	Zener current $I_Z = I_s - I_L$
Increases	Constant	Decreases	Constant	Constant	Increases
Decreases	Constant	Increases	Constant	Constant	Decreases

We may conclude that output voltage remains constant with variation in load resistance.

If load resistance  $R_L$  is continuously increased, then  $R_L$  becomes infinity (open circuit) and load voltage still remains  $V_Z$  (breakdown voltage) and Zener current increases (it should be ensured that Zener current  $I_Z$  must be less than rated current  $I_{ZM}$ ).

If load resistance is continuously decreased, then load current  $I_L$  increases and Zener current  $I_Z$  decreases. Due to this, Zener diode may go into reverse saturation region (come out of the breakdown region) and there will be failure of voltage regulation. If output voltage does not change much with varying of load resistor, then load regulation will be good.

Power supply has some internal resistance.  $R_s$  includes the internal resistance of power supply also. For an ideal power supply, internal resistance of it should be zero. If load resistance  $R_L$  and source voltage  $E$  are fixed and series resistance  $R_s$  is increased then series current  $I_s$  will decrease. Hence Zener current  $I_Z$  will decrease and circuit may come out of the breakdown region. There is a limit on maximum value of  $R_s$ .

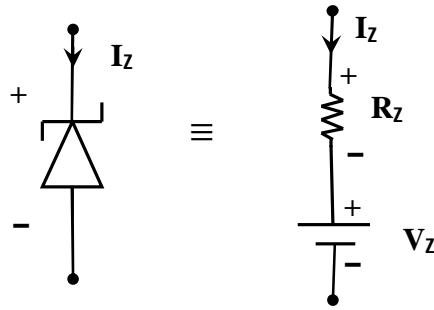
For an ideal approximation of a Zener diode, we have ignored the Zener resistance. Since Zener diode has small but finite resistance  $R_Z$  (Zener impedance), so output voltage slightly varies with Zener current. Total voltage across Zener diode is

$$V_{out} = V_Z + I_Z R_Z$$

This Zener resistance is responsible for slight increase in reverse voltage with reverse current in breakdown region.



Equivalent Zener diode is shown below.



**Figure 3.8B**

### 3.7B Procedure

1. Make the connections as per circuit diagram 3.1B for line regulation.
2. Keep the fix value of load resistance  $R_L$ .
3. Connect the circuit to an appropriate power source and turn it on.
4. By varying the applied dc voltage  $E$ , record the reading for load voltage  $V_L$ .
  - *Load Regulation:*
5. Make the connections as per circuit diagram 3.2B for load regulation.
6. For different values of load resistance  $R_L$ , measure the corresponding load voltages  $V_L$ .

### 3.8B Observation

**Line Regulation:**

Load resistance = .....  $K\Omega$

S.No.	Input Voltage $E$ (Volts)	Load Voltage $V_L$ (Volts)


**Load Regulation:**

Input supply voltage E= ....Volts

S.No.	Load Resistance $R_L$ (K $\Omega$ )	Load Voltage $V_L$ (in Volts)	Load Current (Calculated) $I_L = \frac{V_L}{R_L}$ (mA)

**3.9B Graph**

(i) Line regulation:

Plot the graph of load voltage  $V_L$  (on y axis) and input supply voltage E (on x axis)

(ii) Load Regulation:

Plot the graph of load voltage  $V_L$  (on y axis) and load current  $I_L$  (on x axis).

### 3.10B Calculations

Model Calculation:

$$\begin{aligned} \text{(i) Line Regulation} &= \frac{\text{change in output voltage}}{\text{change in source voltage}} \\ &= \frac{\Delta V_{\text{out}}}{\Delta V_{\text{source}}} = \frac{\Delta V_L}{\Delta E} \\ &= \text{Slope of } V_L \text{ versus } E \text{ curve in breakdown region} \\ &= \frac{(78)\text{mV}}{(4)\text{V}} = 19.5 \frac{\text{mV}}{\text{V}} \end{aligned}$$

$$\begin{aligned} \text{(ii) Load Regulation} &= \frac{\text{change in output voltage}}{\text{change in load current}} = \frac{\Delta V_{\text{out}}}{\Delta I_{\text{Load}}} = \frac{\Delta V_L}{\Delta I_L} \\ &= \text{Slope of } V_L \text{ versus } I_L \text{ curve in breakdown region} \\ &= \frac{72(\text{mV})}{-4.2(\text{mA})} = -17.1 \frac{\text{mV}}{\text{mA}} \end{aligned}$$

### 3.11B Result

1. Graph of load voltage  $V_L$  versus input supply voltage  $E$  has been plotted at fixed value of load resistance.

$$\text{Line regulation} = \dots\dots \frac{\text{mV}}{\text{V}}$$

2. Graph of load voltage  $V_L$  versus load current  $I_L$  has been plotted at fixed value of supply voltage.

$$\text{Load Regulation} = \dots\dots \frac{\text{mV}}{\text{mA}}$$

### 3.12B Discussion

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### 3.13B Precautions and Sources of Error

See section 3.13A

### 3.14B Self Learning Exercise-II

- Q.1** Suppose a Germanium diode has power rating of 300 mW for a lead temperature 50° C and derating factor is 3.5 mW/°C . What do you mean by this derating factor?
- Q.2** Write the any two uses of Zener diode?
- Q.3** What is the major drawback of Zener voltage regulator circuit?
- Q.4** What is the function of a Zener voltage regulator?

### 3.15 Glossary

**Zener breakdown voltage:** The voltage at which reverse current through zener diode increases abruptly with slight increase in reverse voltage.

**Dynamic resistance:** Slope of V-I (voltage -ampere) curve represents the a.c. (dynamic) resistance. Here V on y axis and I on x axis are plotted.

### 3.16 Answers to Self Learning Exercises

#### *Answers to Self Learning Exercise-I*

**Ans.1:** True (absolute value of the breakdown voltage decreases).

**Ans.2:** In reverse bias, leakage region lies between zero and breakdown voltage, in which small reverse saturation current (of the order of  $\mu A$ ) flows.

**Ans.3:**  $P_{\max} = V_Z I_{ZM}$   
 $500 \text{ mW} = 10 \text{ V } I_{ZM}$   
 $50 \text{ mA} = I_{ZM}$

**Ans.4:** For Si  $V_C \cong 0.7 \text{ V}$   
For Ge  $V_C \cong 0.3 \text{ V}$

#### *Answers to Self Learning Exercise-II*

**Ans.1:** If lead temperature of the diode is above 50°C, then we have to reduce the power rating by 3.5 mW for each degree above 50°C.

**Ans.2:** It is used in DC regulated power supply (voltage stabilizers and voltage references), clipping and clamping circuits.

**Ans.3:** There is loss as heat in series element. Zener voltage regulator is suitable for low power applications.

**Ans.4:** The function of the Zener voltage regulator is to provide constant output voltage i.e. output voltage is constant and it is independent of the input supply voltage and also independent of the load current.

### 3.17 Viva Questions

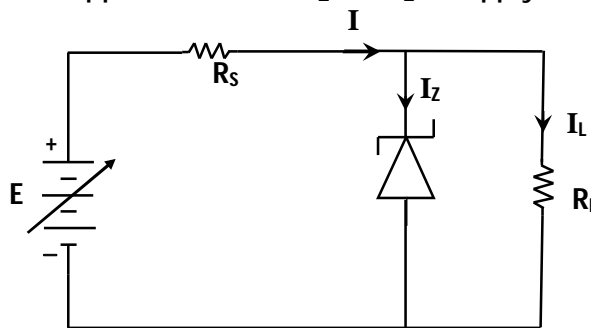
**Q.1** Can PN-diode be used as a switch ?

**Q.2** What type of bonding exists in silicon crystal?

**Q.3** Zener diode has been specially made for ..... bias.

**Q.4** Assume that Zener diode is ideal and it is working in breakdown region.

What will happen to currents  $I_Z$  and  $I_L$  if supply voltage  $E$  increases.



**Q.5** In previous question  $R_s = 1K\Omega$ ,  $R_L = 2K\Omega$ , breakdown voltage  $V_Z = 5.5$  Volts and  $E = 12$  Volts. Find Zener current  $I_Z$  and load current  $I_L$ .

**Q.6** Which of the following diode is operated commonly in reverse bias mode?

(A) Zener diode

(B) Schottky diode

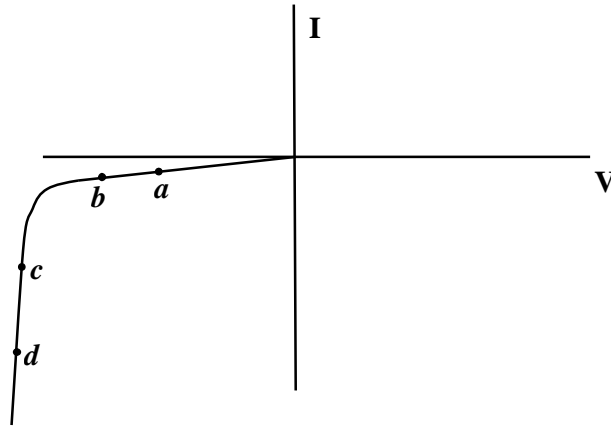
(C) PN Junction diode

(D) Tunnel diode

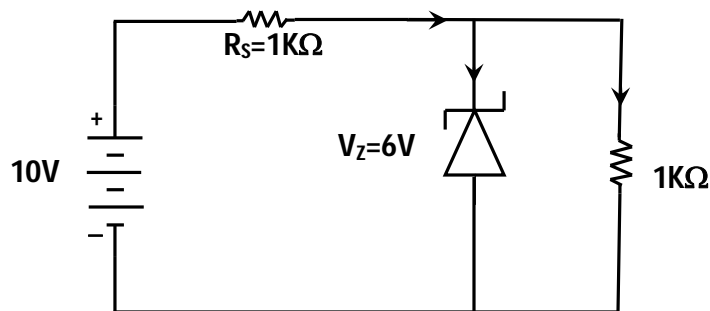
**Q.7** Write the typical values of Zener voltage, maximum Zener impedance, maximum dc Zener current and maximum reverse leakage current for Zener diode 1N751A at  $25^{\circ}C$ .

**Q.8** Write the any three name of type number of Zener diode.

**Q.9** Consider two regions  $ab$  (leakage region) and  $cd$  (breakdown region) in the given figure. In which region resistance is minimum?

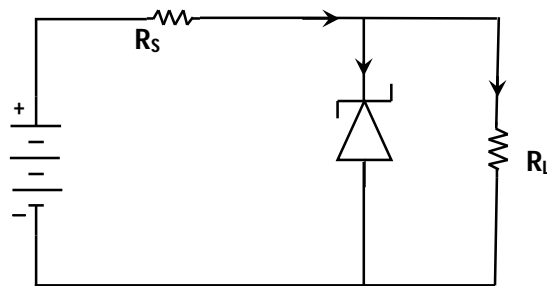


**Q.10** Determine the value of current flowing through load resistor  $R_L=1K\Omega$  ?



**Q.11** Consider the load resistance  $R_L = 2 K \Omega$  instead of  $1 K \Omega$  in the previous question. Find the current flowing through the Zener diode.

**Q.12** Consider the following Zener regulator circuit.



Zener current flowing through diode is 10 mA and Zener resistance of diode is  $8\Omega$ . If the breakdown voltage of diode is 6 Volts, then calculate the load voltage taking into effect of Zener resistance.

**Q.13** What is sign of temperature coefficient of avalanche breakdown mechanism?

**Q.14** Why silicon is preferred to germanium for manufacturing of PN Junction

diode?

**Q.15** Does the dynamic resistance of a Zener diode vary?

**Q.16** Why is series resistor used in Zener diode circuit?

**Q.17** Who discovered the quantum mechanical tunneling effect of Zener diode?

**Q.18** What do you mean by power rating of Zener diode?

**Q.19** What is the Zener impedance of an ideal Zener diode?

**Q.20** What do you mean by voltage regulation?

**Q.21** Which type of breakdown occurs in a lightly doped diode?

**Q.22** A ring is marked on some diodes. What does it give information ?

### 3.18 Answers to Viva Questions

**Ans.1:** Yes

**Ans.2:** Covalent bonding.

**Ans.3:** Reverse.

**Ans.4:**  $I_L$  remains constant &  $I_Z$  increases .

**Ans.5:**  $I_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} = \frac{5.5}{2 \times 1000} = 2.75 \text{mA}$

$$I = \frac{12 - 5.5}{1000} = \frac{6.5}{1000} = 6.5 \text{ mA}$$

$$I_Z = I - I_L = 6.5 - 2.75 = 3.75 \text{ mA}$$

**Ans.6:** (A) Zener diode

**Ans.7:** Zener voltage  $\approx$  5 Volts

Maximum Zener impedance  $\approx$  15  $\Omega$

Maximum DC Zener current  $\approx$  65 mA

Maximum reverse leakage current  $\approx$  1  $\mu\text{A}$

**Ans.8:** 1N4370A, 1N755A, 1N975B by Motorola.

**Ans.9:** *cd* (breakdown region)

**Ans.10** : Firstly we have to check that whether Zener diode is in breakdown region or in leakage region. If we assume that Zener diode is in off state

(leakage region), then load voltage  $V_L = \frac{E R_L}{R_S + R_L} = 10 \frac{1}{1+1} = 5 \text{ Volts}$ . Since  $V_L < V_Z$ , thus our assumption is true that Zener diode is in off state. Load current  $I_L = \frac{V_L}{R_L} = \frac{5}{10^3} = 5 \text{ mA}$

**Ans.11:** The minimum value of load resistance that will turn the Zener diode on (in the breakdown region ) is given by

$$R_{L\min} = \frac{R_S V_Z}{E - V_Z} = \frac{(1)(6)}{10 - 6} = 1.5 \text{ K}\Omega$$

Thus  $R_L > R_{L\min}$

Hence circuit behaves as voltage regulator and voltage across the diode is  $V_L = 6\text{V}$

$$I = \frac{10 - V_Z}{R_S} = \frac{10 - 6}{1 \times 10^3} = 4 \text{ mA}$$

$$I_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} = \frac{6}{2 \times 10^3} = 3 \text{ mA}$$

By Kirchhoff's current law

$$I_Z = I - I_L = 4 - 3 = 1 \text{ mA}$$

**Ans.12:**  $V_L = V_Z + I_Z R_Z$   
 $= 6 + 10 \times 10^{-3} \times 8 = 6.08 \text{ Volts}$

**Ans.13:** It is positive for avalanche process i.e. breakdown voltage increases with temperature, but for Zener effect (high field emission) temperature coefficient is negative.

**Ans.14:** Reverse saturation current is less in silicon diodes.

**Ans.15:** Yes, it depends on the slope of V-I (Volt-Ampere) graph for diode.

**Ans.16:** Series resistor is used in Zener circuit so that excessive current through diode must not exceed the rated current of the diode.

**Ans.17:** Clarence Zener

**Ans.18:** Power rating  $P_{ZM}$  gives the maximum Zener current  $I_{ZM}$  that can be flowed through the Zener diode.

$$P_{ZM} = I_{ZM} V_Z$$



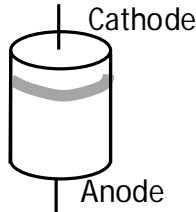
Current rating and power rating will be simultaneously satisfied.

**Ans.19:** An ideal Zener diode has zero Zener impedance. Impedance of real Zener diode is

$$Z_{\text{Zener}} = \frac{dV_{\text{diode}}}{dI_{\text{diode}}}$$

**Ans.21:** Avalanche breakdown.

**Ans.22:** Generally a band is marked at its cathode for its identification.



## References and Suggested Readings

1. Albert Paul Malvino, Electronic Principle (Sixth Edition, 1999), Tata McGraw Hill.
2. Robert L. Boylestad & Louis Nashelsky, Electronic Devices and Circuit Theory (Tenth Edition 2009), Prentice-Hall India.
3. Streetman & Banerjee, Solid State Electronic Devices (Fifth Edition 2002), Prentice-Hall India.
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5. Jacob Millman & Christos C. Halkias, Integrated Electronics (Second Edition 2011), Tata McGraw Hill.
6. Gupta & Kumar, (Handbook of Electronics (Twenty Seventh Edition 2000), Pragati Prakashn.

# **UNIT-4**

## **Monostable and Bistable Multivibrators**

### **Structure of the Unit**

- 4.1 Aim
  - 4.2 Apparatus
  - 4.3 Diagram
  - 4.4 Formula
  - 4.5 Model Graph
  - 4.6 Theory and Description
  - 4.7 Self Learning Exercise-I
  - 4.8 Procedure
  - 4.9 Observation
  - 4.10 Graph
  - 4.11 Calculations
  - 4.12 Result
  - 4.13 Discussion
  - 4.14 Precautions And Sources Of Error
  - 4.15 Self Learning Exercise -II
  - 4.16 Glossary
  - 4.17 Answers To Self Learning Exercises
  - 4.18 Viva Questions
  - 4.19 Answer To Viva Questions
- References and Suggested Readings

### **4.1 Aim**

1. To design and test performance of a monostable multivibrator to generate clock pulse for a given frequency and obtain the waveforms.

2. To design a fixed bias Bistable Multivibrator and to measure the stable state voltages before and after triggering.

## 4.2 Apparatus

Components Required:

1. Resistors
2. Capacitors
3. Transistors 2N2369 – 2

Apparatus Required:

1. CRO
2. Power supply 0-30V
3. Bread board
4. Connecting wires

## 4.3 Diagram

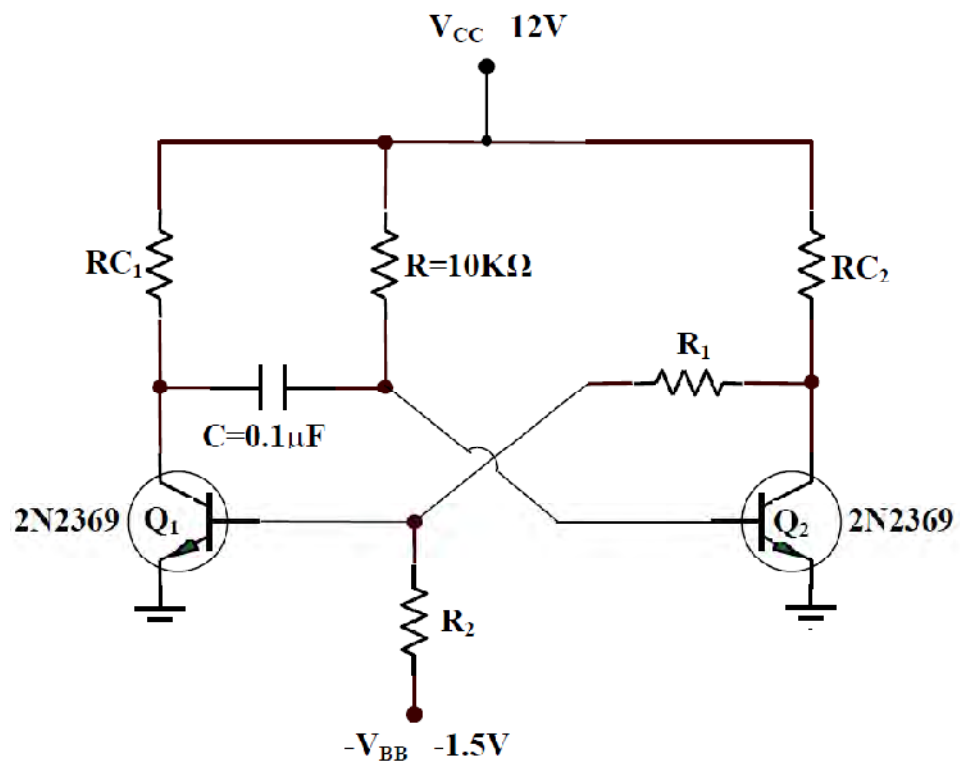
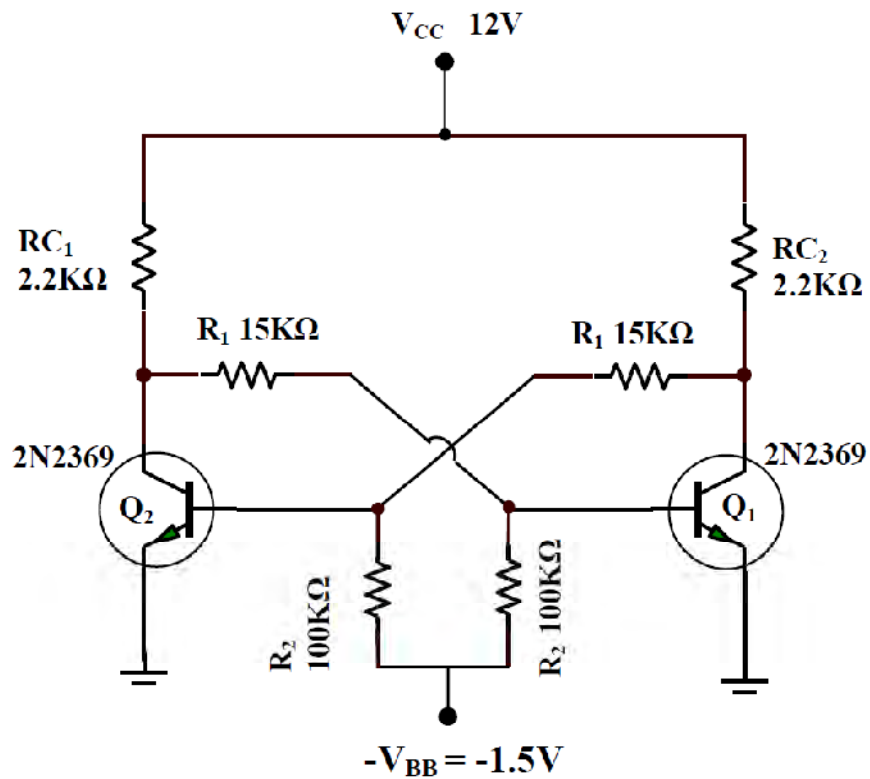


Figure 2: Monostable Multivibrator



**Figure 3: Bistable Multivibrator**

### 4.4 Formula

Time period

$$T = 0.693 RC$$

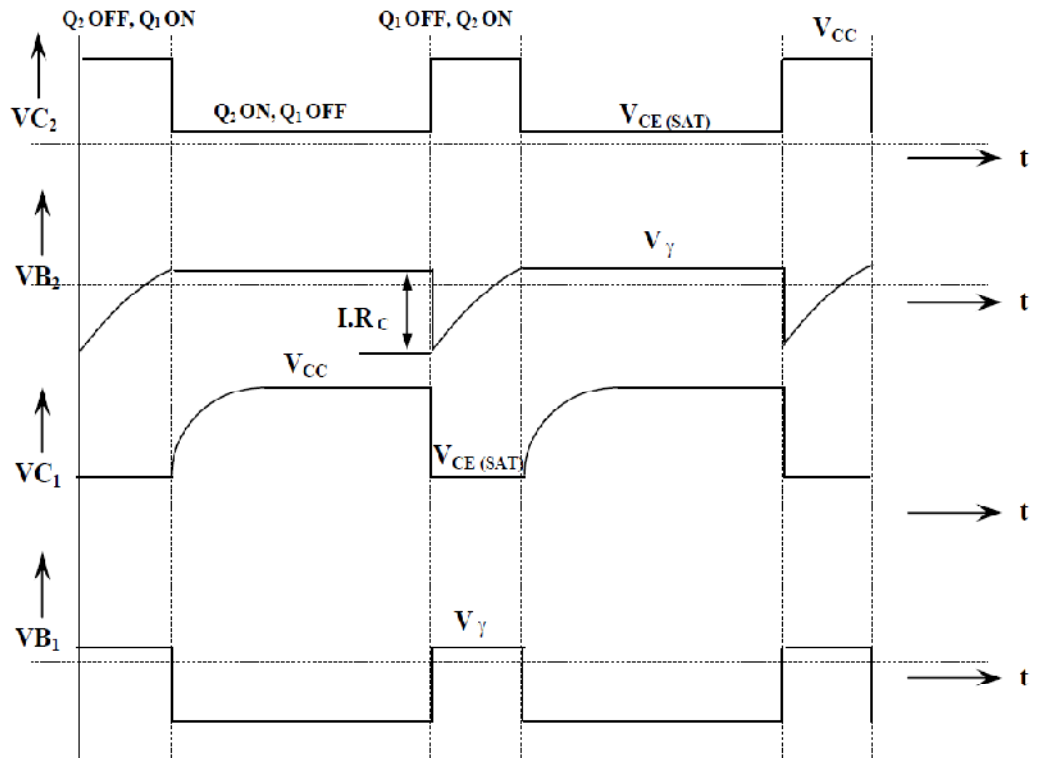
Frequency

$$F = 1/T$$

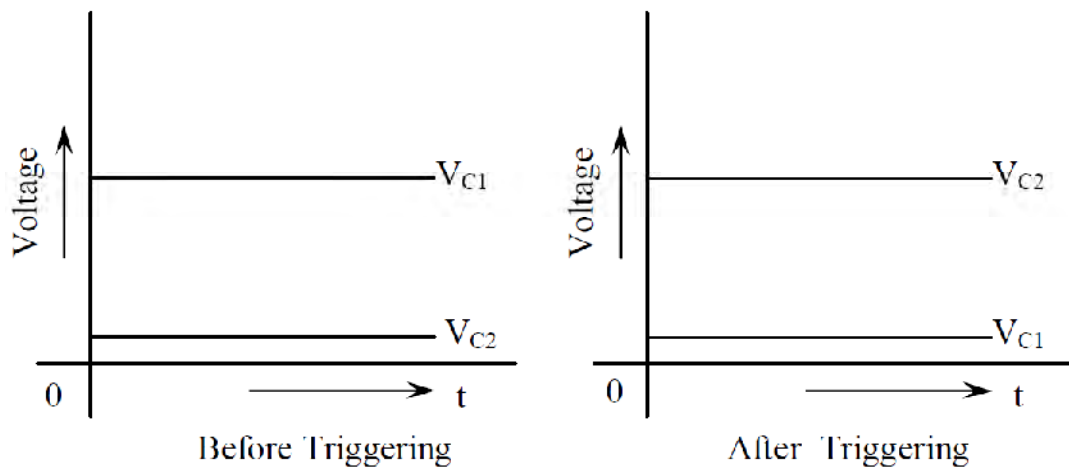
### 4.5 Model Graph

For monostable multivibrator waveforms are shown in figure 3.

For Bistable multivibrator waveforms are shown in figure 4.



**Figure 4: Monostable Multivibrator**



**Figure 5: Bistable multivibrator**

## 4.6 Theory And Description

### Monostable Multivibrator

'A monostable multivibrator has only one stable state, the other state being

quasistable. Normally the multivibrator is in the stable state, and when an external triggering pulse is applied, it switches from the stable to the quasi-stable state. It remains in the quasi-stable state for a short duration, but automatically reverts i.e. switches back to its original stable state, without any triggering pulse'.

### **Principle of operation**

A collector-coupled Monostable multivibrator of the two transistors  $Q_1$  and  $Q_2$  as shown in figure 1,  $Q_1$  is normally OFF and  $Q_2$  is Normally ON. Resistor  $R_1$  and  $R_2$  are connected to the normally OFF transistor, and the capacitor  $C$  is connected to the normally ON transistor. It is seen from the circuit of the monostable multivibrator that, under normal conditions, the supply voltage  $V_{CC}$  provides enough base drive to the transistor  $Q_2$  through resistor  $R$ , with the result that  $Q_2$  goes into saturation. With  $Q_2$  ON,  $Q_1$  goes OFF, as already studied in the context of binary operation. With  $Q_2$  ON and  $Q_1$  OFF, the capacitor finds a charging path. The voltage across the capacitor is  $V_{CC}$  with polarity. It is obvious that in the stable state of the multivibrator,  $Q_2$  is ON and  $Q_1$  is OFF. If the negative triggering pulse is applied to the collector of  $Q_1$ , it is transmitted to the base of  $Q_2$  through the capacitor, and hence makes the base of  $Q_2$  negative. Immediately  $Q_2$  goes OFF and  $Q_1$  becomes ON. However, this is only a quasi-stable state as is obvious from the following observation.

With  $Q_1$  ON and  $Q_2$  OFF, the capacitor  $C$  finds a discharging path. As the capacitor discharges, it is seen that the potential at the base of the transistor  $Q_2$  becomes less and less negative, and after a time, we have  $V_B = V_\gamma$ , the cut-in-voltage of  $Q_2$ . As soon as  $V_B$  crosses the level of  $V_\gamma$ ,  $Q_2$  starts conducting and gets saturated. When  $Q_2$  becomes ON,  $Q_1$  becomes OFF. Thus the original stable state of the multivibrator is restored.

### **[In quasi-stable state: Q1 is ON and Q2 is OFF]**

The interval during which the quasi-stable state of the multivibrator persists i.e.,  $Q_2$  remains OFF is dependent upon the rate at which the capacitor  $C$  discharges. This duration of the quasi-stable state is termed as delay time or pulse width or gate time. It is denoted as  $T$ . The wave forms of the voltage at base of the transistor  $Q_2$  and  $C$  (Collector of  $Q_1$ )

## Use of the Monostable Multivibrator

- Used to generate a clean pulse of the correct height and duration for a digital system
- Used to turn circuits or external components on or off for a specific length of time.
- Used to generate delays.
- Can be cascaded to create a variety of sequential timing pulses. These pulses can allow you to time and sequence a number of related operations.

## Bistable Multivibrator

A bistable multivibrator has two stable output states. It can remain indefinitely in any one of the two stable states, and it can be induced to make an abrupt transition to the other stable state by means of suitable external excitation. It would remain indefinitely in this stable state, until it is again induced to switch into the original stable state by external triggering.

Bistable multivibrators are also termed as 'Binary's or Flip-flops'. A binary is sometimes referred to as '*Eccles-Jordan Circuit*'.

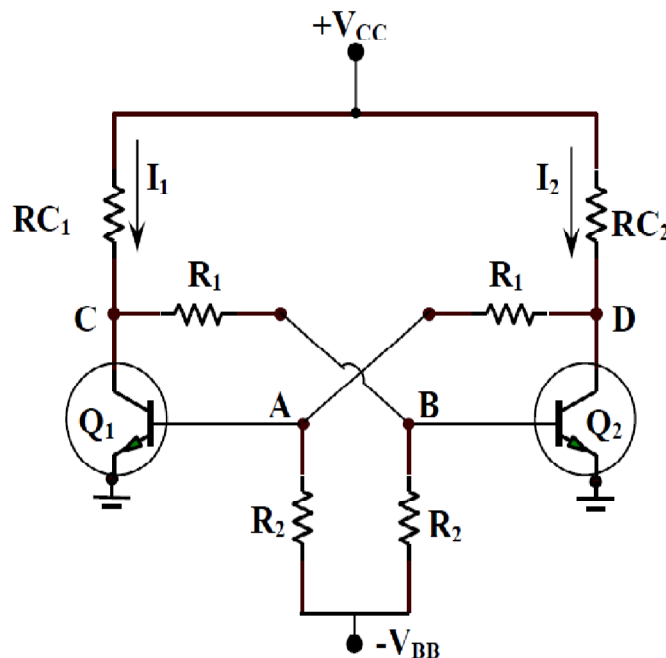


Figure 6 Eccles-Jordan Circuit

## Principle of Operation

Consider the circuit as shown in the figure. The transistor  $Q_1$  and  $Q_2$  are n-p-n transistors. They are coupled to each other as shown in figure. It is evident that the output of each transistor is coupled to the input of the other transistor. Since the transistors are identical, their quiescent currents would be the same, unless the loop gain is greater than unity. When  $I_1$  increases slightly, the voltage drop across the collector resistance  $R_{C1}$  increases. Since  $V_{CC}$  is fixed, the voltage of point C decreases. This has the effect of decreasing the base current of  $Q_2$ . This, in turn, decreases the collector current of  $Q_2$  viz.  $I_2$  decreases, the voltage drop  $I_2 R_{C2}$  decreases. Hence the voltage of point D increases.

Due to increase of  $V_D$ , the base current of  $Q_1$  increases. This increases the collector current of  $Q_1$  viz  $I_1$ . Thus  $I_1$  further increases.  $I_1 R_{C1}$  drop further increases,  $V_C$  further decreases, the base current of  $Q_2$  further decreases, with the result that  $I_2$  further decreases. Thus it can easily be seen that if the collector current  $I_1$  increases even marginally,  $I_2$  would go on progressively decreasing and as a result,  $I_1$  would progressively increase. Eventually  $I_2$  would become practically zero, cutting off the transistor  $Q_2$ , at the same time transistor  $Q_1$  would conduct heavily with the result that it would be driven into saturation. Thus  $Q_2$  becomes OFF and  $Q_1$  becomes ON. It can similarly be shown that if  $I_2$  increases even marginally similar sequence of operation would result and ultimately  $Q_2$  would be ON and  $Q_1$  OFF. Thus when  $Q_1$  is ON,  $Q_2$  is OFF and when  $Q_1$  is OFF  $Q_2$  is ON.

### 4.7 Self Learning Exercise-I

- Q.1 What is Multivibrator?
- Q.2 What are the other names of Monostable Multivibrator?
- Q.3 How many stable and semi stable states present in the Monostable Multivibrator?
- Q.4 What are the other names of Bistable Multivibrator?
- Q.5 How many stable and semi stable states present in the Bistable Multivibrator?
- Q.6 Explain the operation of Monostable Multivibrator?
- Q.7 Explain the operation of Bistable Multivibrator?



## 4.8 Procedure

### Monostable Multivibrator

1. Connect the circuit as shown in figure 1.
2. With the help of a triggering circuit and using the condition  $T(\text{trig}) > T(\text{Quasi})$  a pulse waveform is generated.
3. The output of the triggering circuit is connected to the base of the off transistor.
4. The Off transistor goes into ON state.
5. Observe the waveforms at  $V_{BE1}$ ,  $V_{BE2}$ ,  $V_{CE1}$ ,  $V_{CE2}$
6. Keep the DC- AC control of the Oscilloscope in DC mode.

### Bistable Multivibrator

1. Connect the circuit as shown in figure 2.
2. Observe the waveforms at  $V_{BE1}$ ,  $V_{BE2}$ ,  $V_{CE1}$ ,  $V_{CE2}$
3. Observe which transistor is in ON state and which transistor is in OFF state.
4. Apply -ve triggering at the base of the ON transistor and observe the voltages  $V_{C1}$ ,  $V_{C2}$ ,  $V_{B1}$ , and  $V_{B2}$ .
5. Apply + ve triggering at the base of the OFF transistor and observe the Voltages  $V_{C1}$ ,  $V_{C2}$ ,  $V_{B1}$ ,  $V_{B2}$ .

## 4.9 Observations

Sample Readings

### Before Triggering

$Q_1$ (OFF)	$Q_1$ (ON)
$V_{BE1} = \dots V$	$V_{BE2} = \dots V$
$V_{CE1} = \dots V$	$V_{CE2} = \dots V$

## After Triggering

$Q_1$ (ON)	$Q_1$ (OFF)
$V_{BE1} = \dots V$	$V_{BE2} = \dots V$
$V_{CE1} = \dots V$	$V_{CE2} = \dots V$

### 4.10 Graph

Take the required waveforms on CRO for monostable multivibrator and for bistable multivibrator.

### 4.11 Calculations

#### Model Calculations

Time period

$$\begin{aligned} T &= 0.693 RC \\ &= 0.693 \ 10k\Omega \ 0.1\mu f \\ &= 0.693 \ 10 \times 10^3 \times 0.1 \times 10^{-6} \text{ sec} \\ &= 0.693 \ 1 \times 10^{-3} \text{ sec} \\ &= 0.693 \text{ ms} \end{aligned}$$

Frequency

$$\begin{aligned} f &= 1/T = 1/0.693 \text{ ms} \\ &= 1443 \text{ Hz} \end{aligned}$$

### 4.12 Result

1. Monostable multivibrator is designed and studied.
2. The stable state voltages of a bistable multivibrator are observed.

	$V_{BE1}$	$V_{BE2}$	$V_{CE1}$	$V_{CE2}$
Stable state voltages				

### 4.13 Discussion

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### 4.14 Precautions And Sources Of Error

1. Connections should be tight.
2. Should take care when applying proper supply.
3. Note down the parameters without parallax error.
4. The supply voltage levels should not exceed the maximum rating of the transistor.
5. Verify the circuit before giving supply.

### 4.15 Self Learning Exercise -II

- Q.1** Define Stable state.
- Q.2** Explain Bidirectional sampling gate.
- Q.3** Define Astable multivibrator.
- Q.4** Draw the waveforms of  $V_{C1}$  and  $V_{C2}$  of a Bistable Multivibrator?
- Q.5** Draw the waveforms of  $V_{C1}$  and  $V_{C2}$  of a Monostable Multivibrator?
- Q.6** What is the formula for the theoretical value of T in?

## Q.7 What is a linear waveform-shaping circuit?

### 4.16 Glossary

**Astable multivibrator:** The resulting circuit of the multivibrator that employs capacitive coupling in both the directions is known as an astable multivibrator. Astable multivibrator can also be treated as a relaxation oscillator. The astable multivibrator has two quasi-stable states.

**Basic waveshape:** The sinusoidal waveform can be viewed as a kind of a basic waveshape as all periodic waveforms of any waveshape can be expressed in terms of the sinusoidal waveshapes of different frequencies.

**Bidirectional sampling gate:** A bidirectional sampling gate can transmit an input signal that assumes both positive and negative polarities under the influence of an appropriate control input. This circuit is obtained by simply combining two unidirectional diode sampling gates. These two unidirectional gates take care of the two polarities of the input pulse

**Bistable Multivibrator:** If the multivibrator employs resistive or direct coupling in both the directions, the resulting circuit is known as a bistable multivibrator. The bistable multivibrator has two stable states.

**Clock:** Clock is a periodic waveform that is formed by a train of pulses assuming high and low values alternately. It plays a very important role in the functioning of a sequential logic circuit.

**Free running mode:** If the voltage sweep waveform is generated in free running mode, the waveform is initiated immediately, and the circuit does not wait for any external trigger input to initiate the next cycle of the sweep waveform. Free-running mode operation is also described by terms such as astable mode and recurrent mode.

**Multivibrator:** A multivibrator is a kind of relaxation oscillator consisting of a two-stage cross-coupled amplifier with the output of each stage coupled regenerative to the other.

**Relaxation oscillator:** The term relaxation oscillator is employed to any circuit in which timing intervals are determined by the exponential charging (tensing) or discharging (relaxing) of a capacitor. A charged capacitor functions like a stiff

spring that has stored potential energy in it. A discharging capacitor is analogous to a relaxing spring in which stored potential energy is being converted to kinetic energy.

**Schmitt trigger circuit:** The transistor Schmitt trigger circuit is a variation of the conventional bistable multivibrator. Transitions in stable states of an Schmitt trigger circuit can be induced by a waveform of an arbitrary shape and the only condition is that the maximum amplitude of the waveform has to exceed VUTP of the circuit. The Schmitt trigger circuit cannot function as a 1-bit memory element. Also see upper-tripping point and lower tripping point.

**Stable state:** If the multivibrator can remain indefinitely in a state until the circuit is triggered by an external signal, that state is known as a stable state.

**Triggered mode:** In triggered-mode operation, the sweep waveform generator does not generate a periodic output waveform and the output waveform is present at irregular intervals. The output of a triggered-mode circuit remains at a quiescent value and the circuit waits for the arrival of an external trigger pulse. The triggered-mode operation is also known as monostable operation.

**Unsymmetrical triggering:** In unsymmetrical triggering the trigger pulse is effective in inducing a transition in only one direction. In order to induce a reverse transition, the second trigger pulse from a second source must be applied at a different point.

**Waveform:** The visual form of a signal that can be observed on the CRO screen is known as a waveform.

## 4.17 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-I*

**Ans.1:** A multivibrator is a kind of relaxation oscillator consisting of a two-stage cross-coupled amplifier with the output of each stage coupled regeneratively to the other.

**Ans.2:** One Short Multivibrator, single shot, a single cycle, a single swing, a single step, a univibrator.

**Ans.3:** A monostable multivibrator has only one stable state, the other state being quasi stable.

**Ans.4:** Eccles-Jordan circuit, trigger circuit, scale of two toggle circuit, flip flop and binary

**Ans.5:** A bistable multivibrator has two stable states

### ***Answers To Self Learning Exercise-II***

**Ans.1:** If the multivibrator can remain indefinitely in a state until the circuit is triggered by an external signal, that state is known as a stable state.

**Ans.2:** A bidirectional sampling gate can transmit an input signal that assumes both positive and negative polarities under the influence of an appropriate control input. This circuit is obtained by simply combining two unidirectional diode sampling gates. These two unidirectional gates take care of the two polarities of the input pulse.

**Ans.3:** The resulting circuit of the multivibrator that employs capacitive coupling in both the directions is known as an astable multivibrator. Astable multivibrator can also be treated as a relaxation oscillator. The astable multivibrator has two quasi-stable states.

**Ans.4:** See Section Model graph

**Ans.5:** See Section Model graph

**Ans.6:** Time period  $T=0.693 RC$

**Ans.7:** The process by which the shape of a nonsinusoidal signal is changed by passing the signal through the network consisting of linear elements is called Linear Wave Shaping.

## **4.18 Viva Questions**

**Q.1** Define the Bistable multivibrator.

**Q.2** Define resolving time.

**Q.3** What is meant by linear wave shaping circuit?

**Q.4** What are different types of triggering of bistable multivibrator?

**Q.5** Define monostable multivibrator.

**Q.6** Define integrator.

**Q.7** What is the use of commutating capacitors?

- Q.8** Define transition time.
- Q.9** What is a linear waveform-shaping circuit?
- Q.10** What is meant by multivibrator?
- Q.11** Define Astable multivibrator.

### 4.19 Answer To Viva Questions

- Ans.1:** Bistable multivibrator signifies a circuit which can exist indefinitely in either of two stable states and which can be induced to make an abrupt transition from one state to other by applying an external triggering signal.
- Ans.2:** It is the minimum time interval between two consecutive trigger pulses and equals to transition time plus the settling time.
- Ans.3:** The action of a linear network in producing a waveform at its outputs different from its output is known as linear wave shaping circuit.
- Ans.4:** Asymmetrical triggering. Symmetrical triggering.
- Ans.5:** When a trigger pulse is applied to the input circuit, the circuit state is changed abruptly to unstable state for a predetermined time after which the circuit returned to its original stable state automatically.
- Ans.6:** Integrator is a circuit that passes low frequencies of the input and attenuates high frequencies. Integrator implies that the output voltage is an integral of the input voltage.
- Ans.7:** The Commutating capacitors can be used to reduce the transition time in a low to high level and vice versa.
- Ans.8:** The time interval during which the conduction transfer from one transistor to another transistor is defined as transition time.
- Ans.9:** The process by which the shape of a nonsinusoidal signal is changed by passing the signal through the network consisting of linear elements is called Linear Wave Shaping.
- Ans.10:** Multivibrators are two stage switching circuits in which the output of the first stage is fed to the input of the second state and vice-versa. The outputs of two stages are complementary.

**Ans.11:** Astable multivibrator is a multivibrator in which neither state is stable. There are two temporary states. The circuit changes state continuously from one quasi stable state to another at regular intervals without any triggering. This generates continuous square waveform without any external signal.

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# UNIT-5

## Combinational Logic

### Structure of the Unit

- 5.1 Aim
  - 5.2 Apparatus
  - 5.3 Diagram
  - 5.4 Formula
  - 5.5 Theory and Description
  - 5.6 Self Learning Exercise-I
  - 5.7 Procedure
  - 5.8 Observation
  - 5.9 Result
  - 5.10 Discussion
  - 5.11 Precautions and Sources of Error
  - 5.12 Self Learning Exercise -II
  - 5.13 Glossary
  - 5.14 Answers to Self Learning Exercises
  - 5.15 Viva Questions
  - 5.16 Answers to Viva Questions
- References and Suggested Readings

### 5.1 Aim

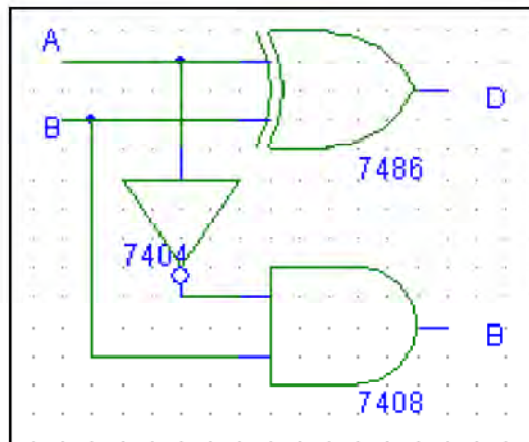
- (i) To simplify the given expression and to realize it using Basic gates and Universal gates.
- (ii) To realize Half Subtractor and Full Subtractor by using Basic gates and NAND gates

## 5.2 Apparatus

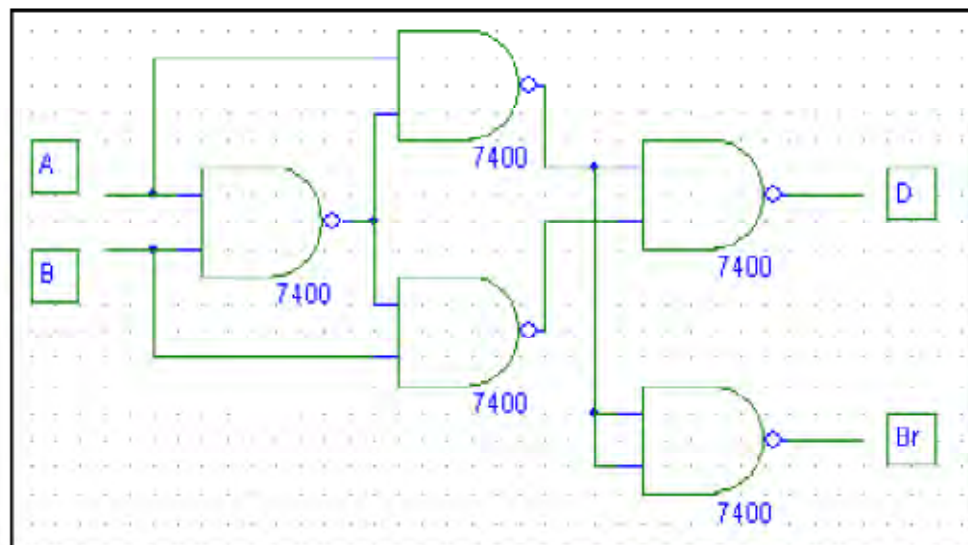
IC 7400, IC 7408, IC 7486, IC 7432, IC 7406, IC 7402, Patch Cords & IC Trainer Kit as required.

## 5.3 Diagram

- **Half Subtractor**  
Basic gate

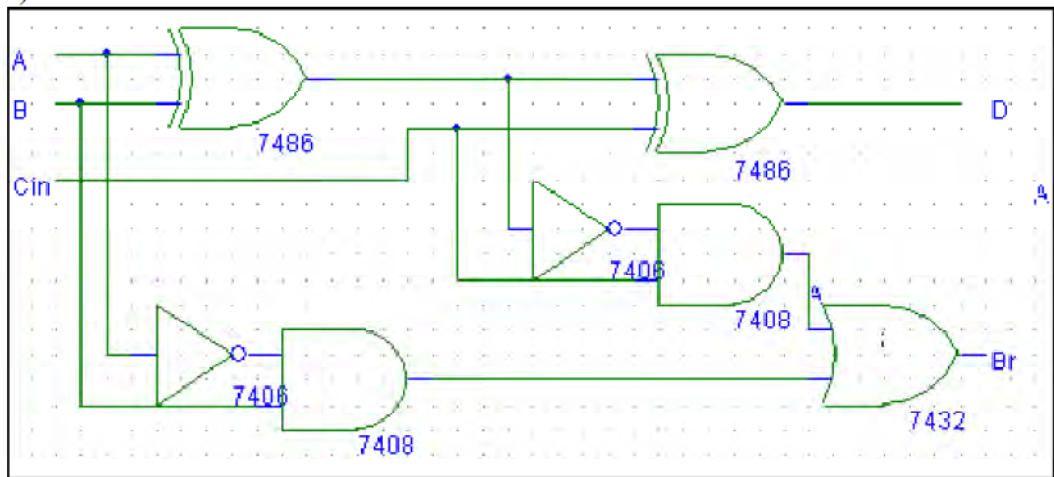


NAND gate

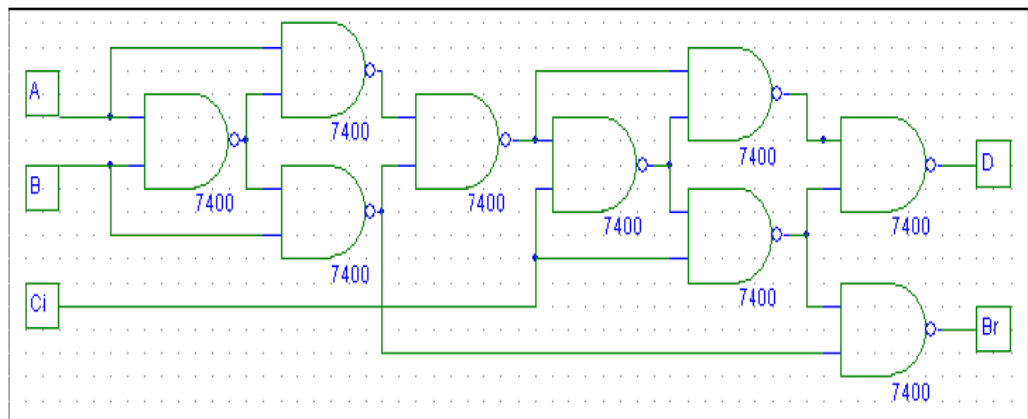


- **Full Subtractor**

Basic gate



NAND gate



## 5.4 Formula

### Karnaugh Maps

For a function of two variable say,  $f(x,y)$

	$X'$	$X$
$Y'$	$f(0,0)$	$f(1,0)$
$Y$	$f(0,1)$	$f(1,1)$

For a function of two variable say,  $f(x,y,z)$

	$X'Y'$	$X'Y$	$XY$	$XY'$
$Z'$	$f(0,0,0)$	$f(0,1,0)$	$f(1,1,0)$	$f(1,0,0)$
$Z$	$f(0,0,1)$	$f(0,1,1)$	$f(1,1,1)$	$f(1,0,1)$

For a function of two variable say,  $f(w,x,y,z)$

	$W'X'$	$W'X$	$WX$	$WX'$
$Y'Z'$	0	4	12	8
$Y'Z$	1	5	13	9
$YZ$	3	7	15	11
$YZ'$	2	6	14	10

**Half Subtractor:**

$$D = A \oplus B$$

$$Br = A' B$$

INPUTS		OUTPUTS	
A	B	D	Br
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

**Full Subtractor:**

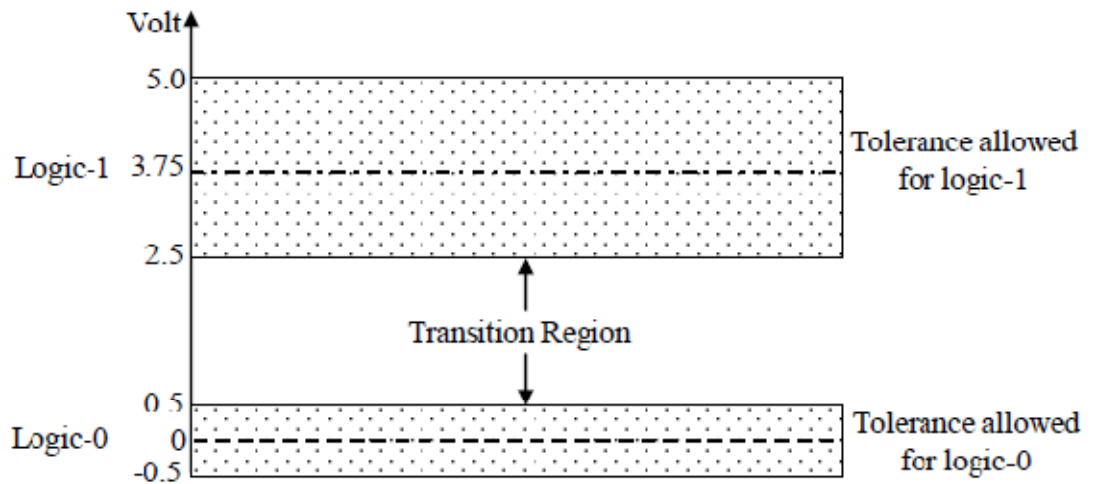
$$D = (A \oplus B) \oplus C$$

$$Br = A'B + B C_{in} + A'C_{in}$$

INPUTS			OUTPUTS	
A	B	Cin	D	Br
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

## 5.5 Theory and Description

In the Integrated Circuit's (IC) inputs and outputs occur as voltage levels. These voltage levels may be measured by some instruments and depending on the predefined voltage ranges they are accepted as logic-1 or logic-0. The two digits in the binary system, 1 and 0, are called bits, which is contraction of binary digit. During our experiments, any measured voltage between 2.5 and 5.0 V will be considered as logic-1 and the voltage between -0.5 and 0.5 V will be accepted as logic-0. Due to the type of the ICs used, these ranges may vary considering the supply voltage connected. Despite the difference in voltage levels, a basic operation will be the same. As shown in Figure, each voltage range has an acceptable deviation from each other by a transition region.



Electronic digital circuits are also called logic circuits. Logic gates are the basic building blocks for forming digital electronic circuitry. Logic circuits that perform the logic operations of AND, OR, NOT (Inverter), NAND and NOR are shown with their graphics symbols, algebraic functions and truth tables in Table. Digital circuits, switching circuits, logic circuits and logic gates are used in the same means. The operation principles of basic five logic gates can be explained as follows:

**The AND Gate:** The AND gate is one of the basic gates from which all functions are constructed. It is a multi-input circuit that may have two or more than two inputs in which the output is a logic-1 only if all inputs are logic-1. The output produces a logic-0 signal if any input is logic-0. The truth table, algebraic function (Boolean equation) and graphic symbol of 2-input AND Gate are shown in Table.

**The OR Gate:** The OR gate is another of the basic gates from which all logic functions are constructed. It is also a multi-input circuit that may have two or more than two inputs in which the output is a logic-1 when any of the inputs is a logic-1. Its output becomes logic-0 if all input signals are logic-0. The truth table, algebraic function (Boolean equation) and graphic symbol of 2-input OR Gate are shown in Table.

**The INVERTER (NOT) Gate:** The inverter (NOT circuit) performs the operation called inversion or complementation. The inverter changes one logic level to the opposite level. In terms of bits, the output is logic-0 when input is logic-1, and the output is logic-1 when the input is logic-0. The truth table, algebraic function

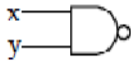

(Boolean equation) and graphic symbol of an INVERTER (NOT) Gate are shown in Table.

**The NAND Gate:** The NAND gate is a popular logic element because it can be used as a universal gate; that is, NAND gates can be used in combination to perform the AND, OR and inverter operations. The NAND gate is simply AND followed by INVERTER gate. Therefore, NAND gate produces a logic-0 output only all inputs are logic-1. When any of the inputs is logic-0, the output will be logic-1. The truth table, algebraic function (Boolean equation) and graphic symbol of 2-input NAND Gate are shown in Table.

**The NOR Gate:** The NOR gate, like the NAND gate, is a useful logic element because it can also be used as a universal gate, that is, NOR gates can be used in combination to perform the AND, OR and inverter operations. Then it is simply OR followed by INVERTER gate. The truth table, algebraic function (Boolean equation) and graphic symbol of 2-input NOR Gate are shown in Table.

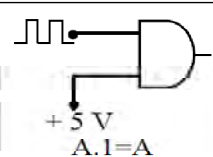
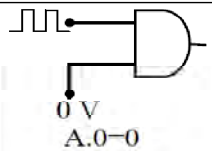
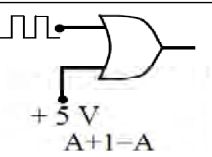
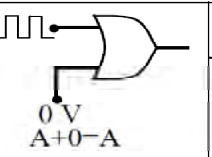
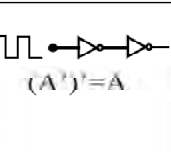
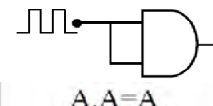
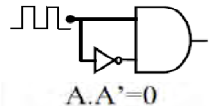
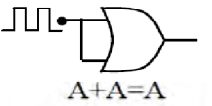
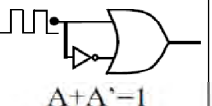
The NAND and NOR functions are the complements of AND and OR functions, respectively. The NAND and NOR gates are extensively used as standard logic gates.

Name	Graphic Symbol	Algebraic Function	Truth Table															
AND		$F=x.y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1
x	y	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F=x+y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	1
x	y	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
NOT		$F=x'$	<table border="1"> <thead> <tr> <th>x</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> </tr> </tbody> </table>	x	F	0	1	1	0									
x	F																	
0	1																	
1	0																	

NAND		$F=(x.y)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	x	y	F	0	0	1	0	1	1	1	0	1	1	1	0
x	y	F																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$F=(x+y)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	0
x	y	F																
0	0	1																
0	1	0																
1	0	0																
1	1	0																

Generally, we will find that the simple gate functions AND, OR, NAND, NOR and INVERTER are not enough by themselves to implement the complex requirements of digital systems. The basic gates will be used as the building blocks for the more complex logic that is implemented by using combinations of gates called *combinational logic*. Combinational logic employs the use of two or more of the basic logic gates to form a more useful, complex function. The systematic reduction of logic circuits is performed using *Boolean algebra*.

Boolean algebra is the mathematics of digital systems. A basic knowledge of Boolean algebra is indispensable to the study and analysis of logic circuits. This experiment reviews the material and provides additional definitions and information. Additionally, it demonstrates the relationship between a Boolean function and the corresponding logic diagram. The complexity of the digital logic gates that implement a Boolean function is directly related to the complexity of the algebraic expression from which the function is implemented. The exact implementation of many digital logic circuits will result in a circuit those functions as expected. Nevertheless, many such circuits can be reduced to a simpler form by the application of Boolean algebra and by Karnaugh mapping. The basic Boolean reducing equations are given below.

 + 5 V A.1=A	 0 V A.0=0	 + 5 V A+1=A	 0 V A+0=A	 $(A')'=A$
 A.A=A	 A.A'=0	 A+A=A	 A+A'=1	



Many times a complex logic circuit can reduce its simplest form or change its form to a more convenient one to implement the expression most efficiently by using these rules as a base. In simplification procedures, a variable and its complement are treated as separate variables and logic equations can be expanded and combined as in ordinary algebra using the priorities of ordinary algebra.

All Boolean expressions, regardless of their form, can be converted into either of two standard forms: the sum-of-products for (SOP) or the products-of-sum form (POS). Reductions can be made using the Boolean algebra simplification techniques or Karnaugh maps which provide a graphical/visual means of logic simplifications.

### **Half Subtractor:**

Subtracting a single-bit binary value B from another A (i.e. A - B ) produces a difference bit D and a borrow out bit B-out. This operation is called half subtraction and the circuit to realize it is called a half subtractor. The Boolean functions describing the half- Subtractor are:

$$S = A \oplus B$$

$$C = A' B$$

### **Full Subtractor:**

Subtracting two single-bit binary values, B, Cin from a single-bit value A produces a difference bit D and borrow out Br bit. This is called full subtraction. The Boolean functions describing the full-subtractor are:

$$D = (x \oplus y) \oplus \text{Cin}$$

$$\text{Br} = A' B + A' (\text{Cin}) + B(\text{Cin})$$

## **5.6 Self Learning Exercise-I**

- Q.1** What is digital gate?
- Q.2** What do you mean by universal gate?
- Q.3** What is truth table?
- Q.4** Make truth table of NAND gate?
- Q.5** What is different between Ex-OR & Ex-NOR gate?

## 5.7 Procedure

### Simplify the Boolean expression and to build the logic circuit

1. Simplifying given expression using K-Map method

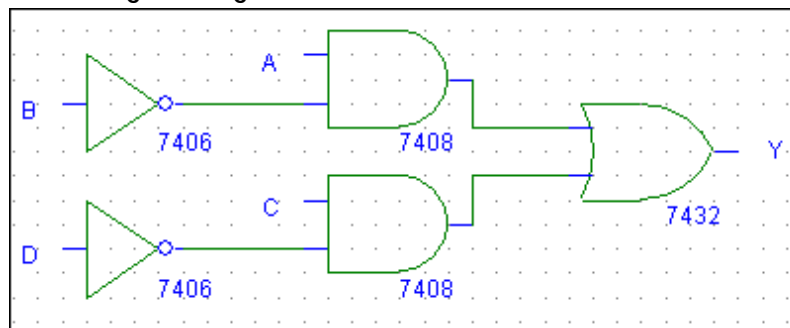
$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D}$$

	A'B'	A'B	AB	AB'
C'D'				1
C'D				1
CD				1
CD'	1	1	1	1

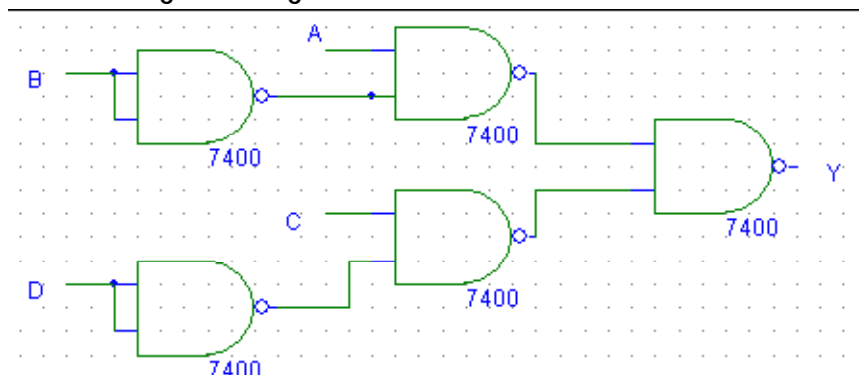
After simplifying we get

$$Y = A\bar{B} + C\bar{D}$$

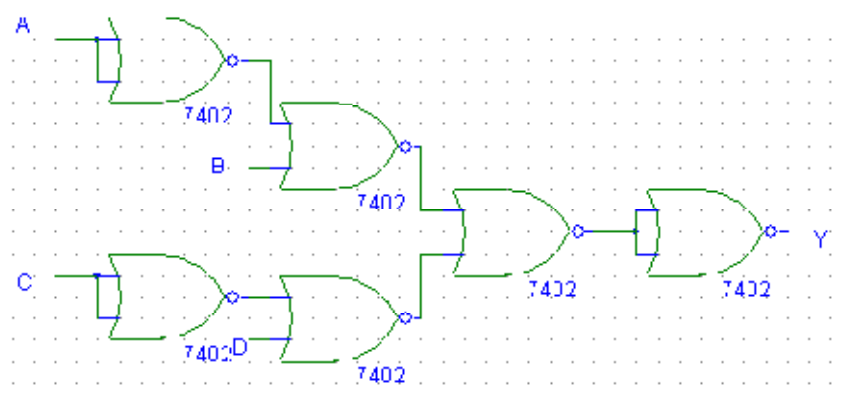
2. Insert the appropriate IC into the IC base.
3. Realization using Basic gates



4. Realization using NAND gates



## 5. Realization using NOR gates



6. Make connections as shown in the circuit diagram.
7. Provide the input data via the input switches and observe the output on output LEDs.
8. Verify the Truth Table

### Half subtractor, full subtractor

- Check the components for their working.
- Insert the appropriate IC into the IC base.
- Make connections as shown in the circuit diagram.
- Verify the Truth Table and observe the outputs.

### 5.8 Observation

INPUTS				OUTPUTS
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	

0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

### 5.9 Result

1. Simplified and verified the Boolean function using basic gates and universal gates
2. The truth table of the half subtractor, full subtractor circuits is verified.

### 5.10 Discussion

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## 5.11 Precautions and Sources of Error

1. All the connection should be tight.
2. It should be care that the values of the components of the circuit is does not exceed to their ratings (maximum value).
3. Before the circuit connection it should be check out working condition of all the component.
4. The continuity of the connecting terminals should be checked before going.
5. Always disconnect the power supply before connecting or charging to your circuit.
6. Digital lab kits and IC's should be handled with utmost care.

## 5.12 Self Learning Exercise -II

- Q.1 Draw the NAND gate?
- Q.2 Draw the Ex-OR gate?
- Q.3 Draw the EX-OR gate by using the NAND gate?
- Q.4 Draw the EX-NOR gate?
- Q.5 Draw the EX-NOR by using the NAND gate?
- Q.6 What is D'Morgans theorem.

## 5.13 Glossary

**Adder** : Circuitry that performs the operation of adding two numbers.

**AND Gate** : A logic circuit that produces a HIGH output only when all of the inputs are HIGH.

**And-Or-Invert Gate (AOI)** : An integrated circuit containing combinational logic consisting of several AND gates feeding into An OR gate and then an inverter. It is used to implement logic equations that are in the SOP format.

**Boolean Equation** : An algebraic expression that illustrates the functional

operation of a logic gate or combination of logic gates.

**Boolean Reduction** : An algebraic technique that follows specific rules to convert a Boolean equation into a simpler form.

**Combinational Logic** : Logic circuits formed by combining several of the basic logic gates to form a more complex function.

**Exclusive-NOR** : A gate that produces a HIGH output for both inputs HIGH or both inputs LOW.

**Exclusive-OR** : A gate that produces a HIGH output for one or the other input HIGH, but not both.

**Full-Adder** : An adder circuit having three inputs, used to add two binary digits plus a carry. It produces their sum and carry as outputs.

**Gate** : The basic building block of digital electronics. The basic logic gate has one or more inputs and one output and is used to perform one of the following logic functions: AND, OR, NOR, NAND, INVERTER, exclusive-OR, or exclusive-NOR.

**Half-Adder** : An adder circuit used in the LS position when adding two binary digits with no carry-in to consider. It produces their sum and carry as outputs.

**Karnaugh Map** : A two-dimensional table of Boolean output levels used as a tool to perform a systematic reduction of complex logic circuits into simplified equivalent circuits.

**Negative-AND** : A NOR gate equivalent operation in which there is a HIGH output when all inputs are LOW.

**Negative-OR** : A NAND gate equivalent operation in which there is a HIGH output when all inputs are LOW.

**NAND Gate** : A logic circuit in which a LOW output occurs if and only if all the inputs are HIGH.

**NOR Gate** : A logic circuit in which the output is LOW when any or all of the inputs are HIGH.

**NOT** : When reading a Boolean equation, the word used to signify an inversion bar. For example, the equation  $X=A'B'$  is read "X equals NOT AB".

**OR Gate :** A logic circuit that produces a HIGH output when any of the inputs is HIGH.

**Universal Gates :** The NOR and NAND logic gates are sometimes called universal gates because any of the other logic gates can be formed from them.

## 5.14 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** Digital gates are basically electronic components which are used for switching and manipulating binary data.

**Ans.2:** The universal gates are those gate from which we can make any gate by using them. The universal gates are- NAND & NOR

**Ans.3:** Truth table is a table from which we can get o/p of different gates.

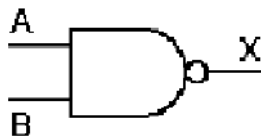
**Ans.4:**

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

**Ans.5:** The basic difference between this two gate is that Ex-OR gate gives output when both the input is different & Ex-NOR gate give output when both input same.

### *Answers to Self Learning Exercise-II*

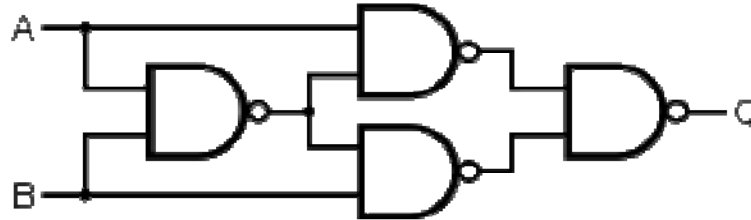
**Ans.1:**



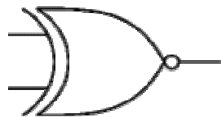
**Ans.2:**



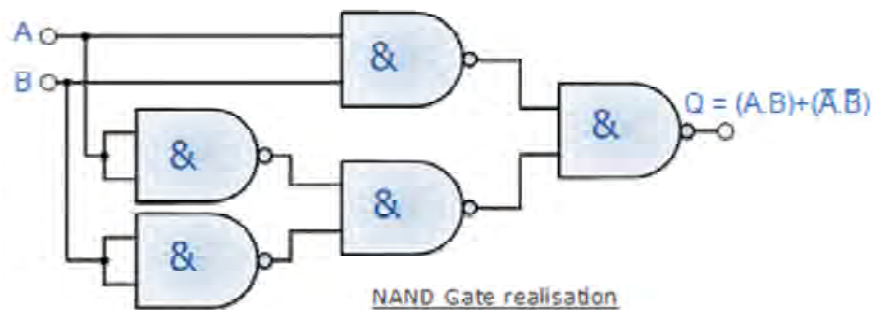
**Ans.3:**



**Ans.4:**



**Ans.5:**



**Ans.6:** D'Morgans theorem is theorems through which we can easily manipulate and reduce the given equation.

## 5.15 Viva Questions

- Q.1** Writes D'Morgan's theorem equations.
- Q.2** Solve following example by using D'Morgans theorem.
- Q.3** Solve following example by using D'Morgans theorem  
 $(A+B+C)'$
- Q.4** Solve following example by using D'Morgans theorem.  
 $(ABC)' (AB)'$
- Q.5** Who invent the D'Morgans theorem?
- Q.6** De Morgan theorem is used for what?
- Q.7** Draw the example of De Morgan theorem?



- Q.8** What are the applications of adders?
- Q.9** What is a half subtractor?
- Q.10** What is a full subtractor?
- Q.11** What are the applications of subtractors?
- Q.12** Obtain the minimal expression for above circuits.
- Q.13** Realize a full subtractors using two half subtractors
- Q.14** For what De Morgan theorem is used?
- Q.15** What is the Universal gates?
- Q.16** Types of universal gate?

## 5.16 Answers to Viva Questions

**Ans.1:**  $(A+B)'=A'B'$

$$A'B'=(A+B)'$$

**Ans.2:**  $(ABC)'=A'+B'+C'$

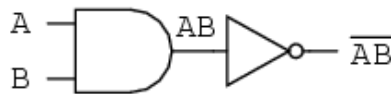
**Ans.3:**  $(A+B+C)'=A'B'C'$

**Ans.4:**  $(ABC)'(AB)'=(A'+B'+C')(A'+B')$

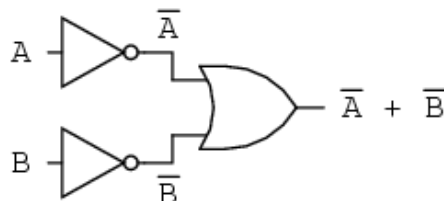
**Ans.5:** The law is named after Augustus De Morgan (1806–1871)

**Ans.6:** This theorem is useful for solving the different boolean expressions.

**Ans.7:**



*... is equivalent to ...*



$$\overline{AB} = \overline{A} + \overline{B}$$

**Ans.14:** De Morgan theorem may be thought of in terms of breaking a long bar symbol.

**Ans.15:** The universal gates are the gate by help of which we can make other all gates.

**Ans.16:** There are two universal gates

1. NAND
2. NOR

## References and Suggested Readings

1. Basic digital electronic experiments for physicists by A. Necmeddin Yazici, 2001.
2. William Kleitz, "*Digital Electronics*", (1996).
3. Thmos L.Floyd, "*Digital Fundamentals*", (1997).
4. M.Morris Mano, "*Digital Design*", (1984).
5. Morris E.Levine, "*Digital Theory and Experimentation Integrated Circuits*". (1982).
6. John F.Wakerly, "*Digital Design*", (2000).

# UNIT-6

## Flip Flops

### Structure of the Unit

- 6.1 Aim
  - 6.2 Apparatus
  - 6.3 Diagram
  - 6.4 Model Graph
  - 6.5 Theory and Description
  - 6.6 Self Learning Exercise-I
  - 6.7 Procedure
  - 6.8 Observation
  - 6.9 Graph
  - 6.10 Result
  - 6.11 Discussion
  - 6.12 Precautions and Sources of Error
  - 6.13 Self Learning Exercise-II
  - 6.14 Glossary
  - 6.15 Answers to Self Learning Exercises
  - 6.16 Viva Questions
  - 6.17 Answers to Viva Questions
- References and Suggested Readings

### 6.1 Aim

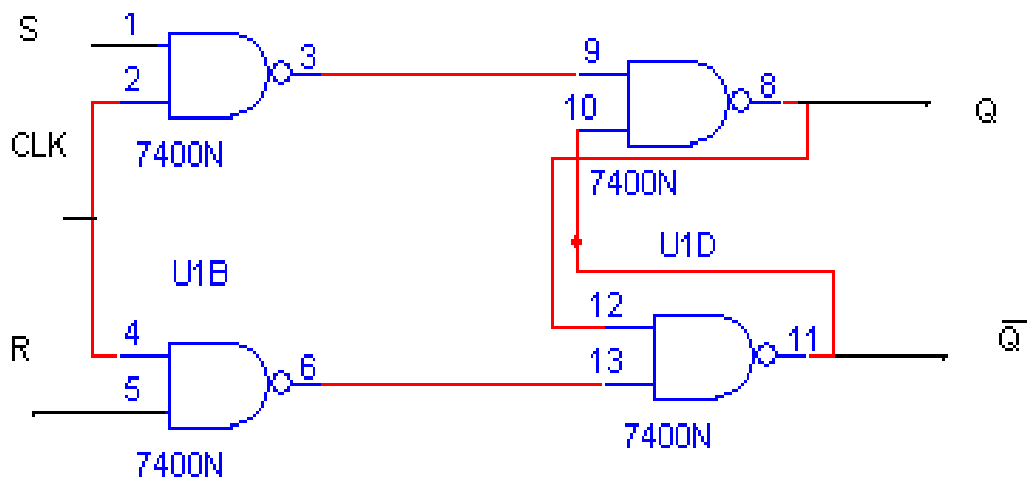
- (i) To construct and verify the truth tables of SR flip flop, JK flip flop, D and T flip - flop.
- (ii) Design a synchronous counter using JK Flip Flop to count up to 6, and verify it's truth table.

## 6.2 Apparatus

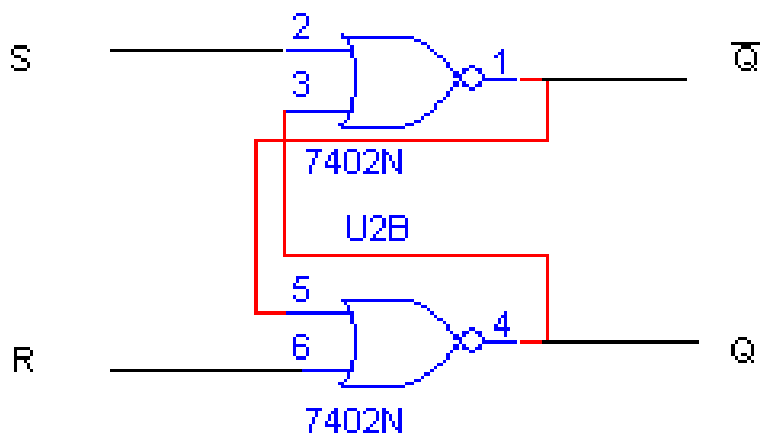
IC 7473 JK flip flop, IC 7400NAND gate , IC 7404 NOT gate, Patch cards, Connecting wires, IC bread board trainer, Power supply, AND gate (IC 7408) as required.

## 6.3 Diagram

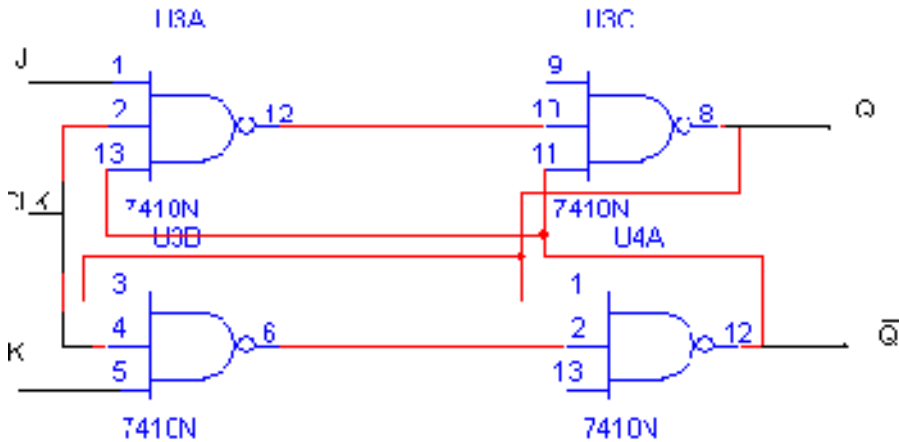
### 1. S-R FLIP-FLOP using NAND gate



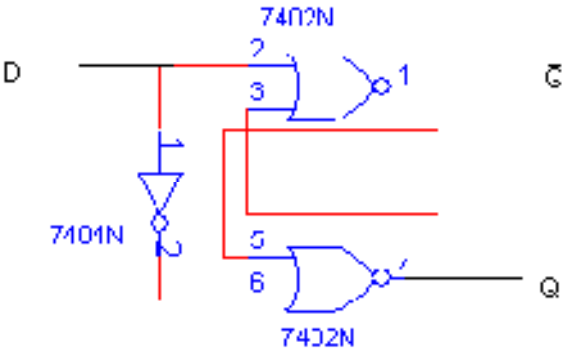
### 2. S-R FLIP-FLOP using NOR gate



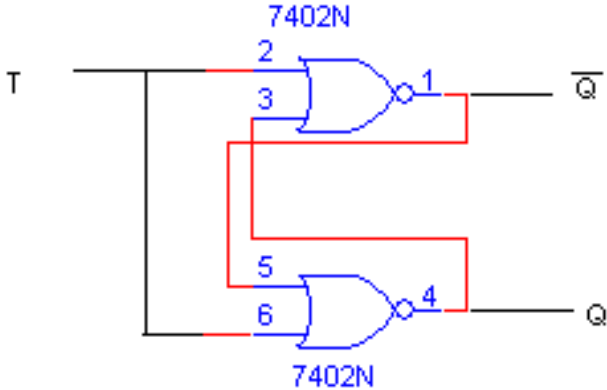
### 3. J-K FLIP- FLOP



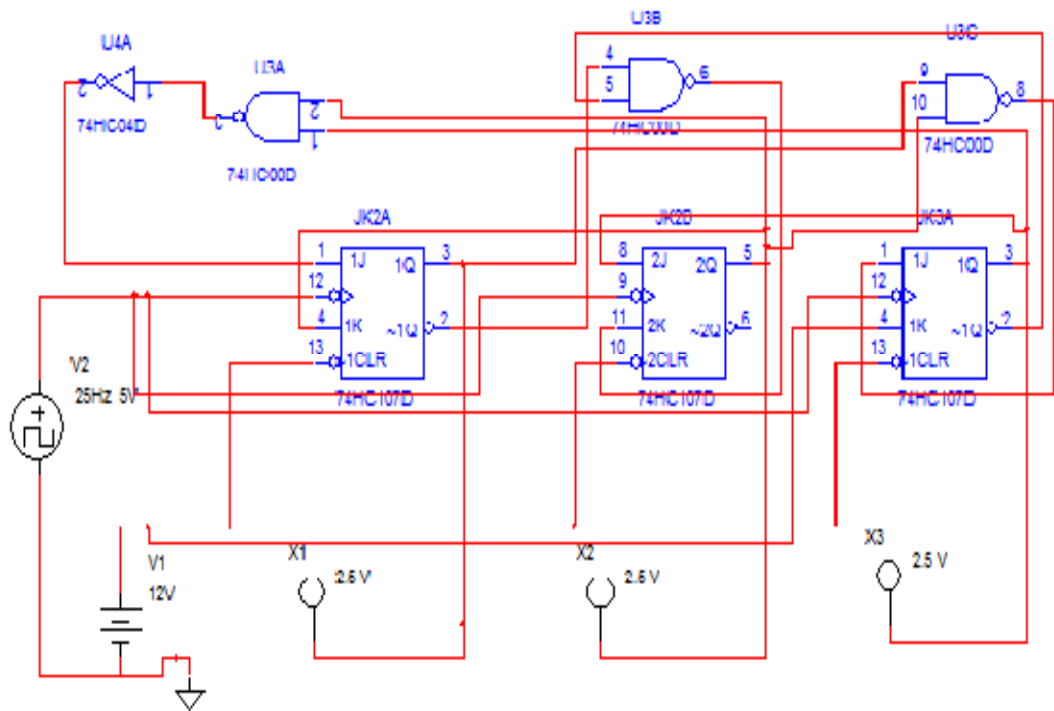
### 4. D FLIP- FLOP



### 5. T FLIP- FLOP



## 6. Synchronous counter using JK flip flop



## 6.4 Model Graph

### RS Flip-flops

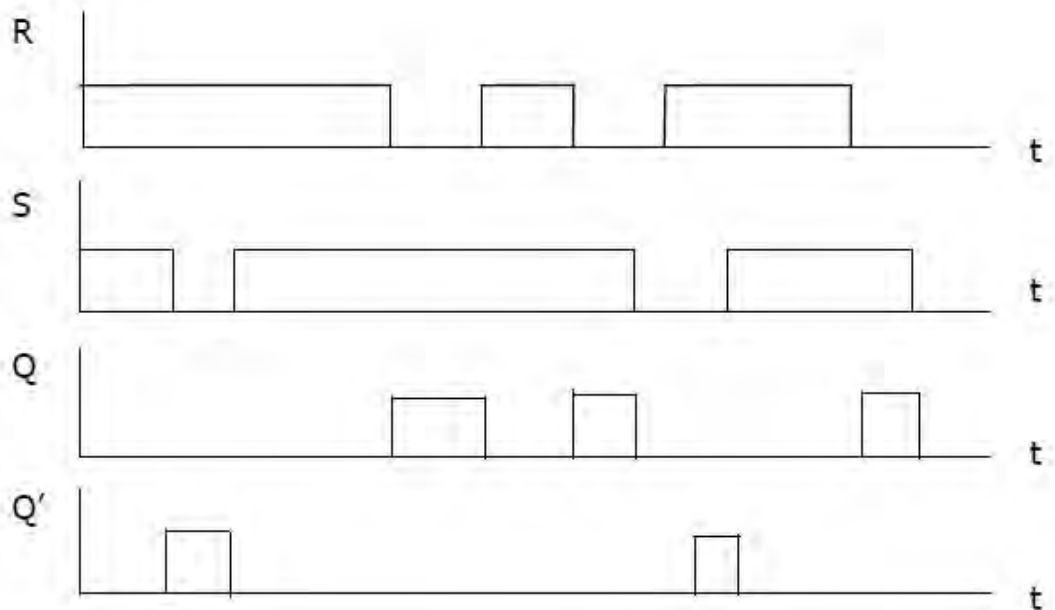


Figure 7: RS Flip-flop timing diagram

## JK Flip-flops

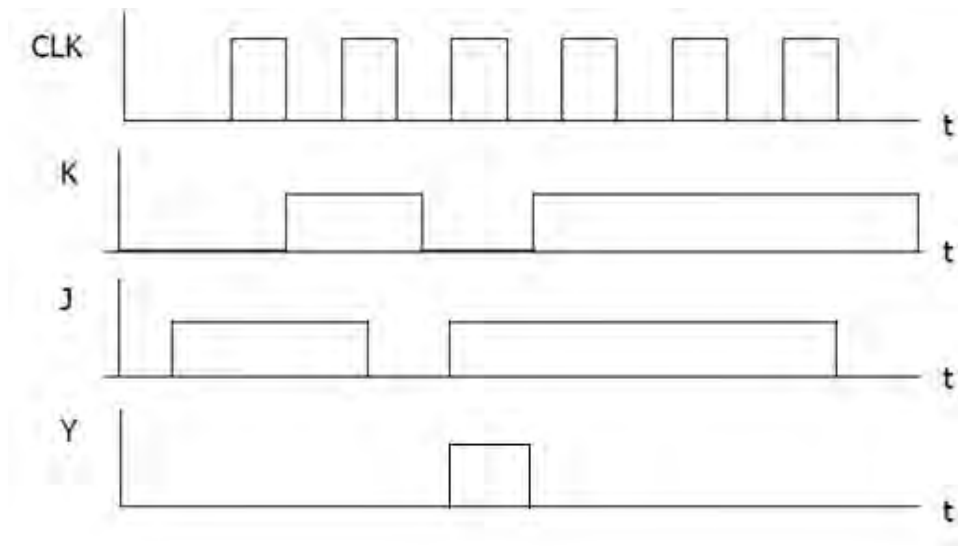


Figure 8: JK Flip-flop timing diagram

## D Flip-flops

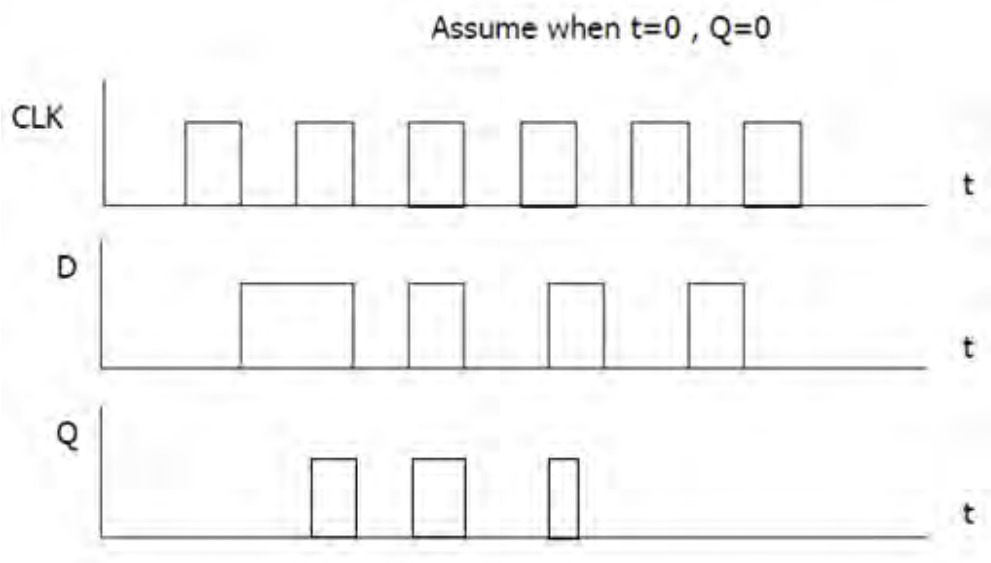


Figure 9: D Flip-flop timing diagram

## T Flip-flops

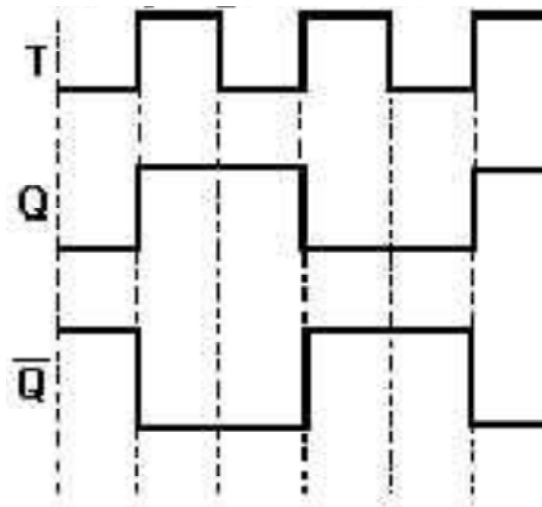
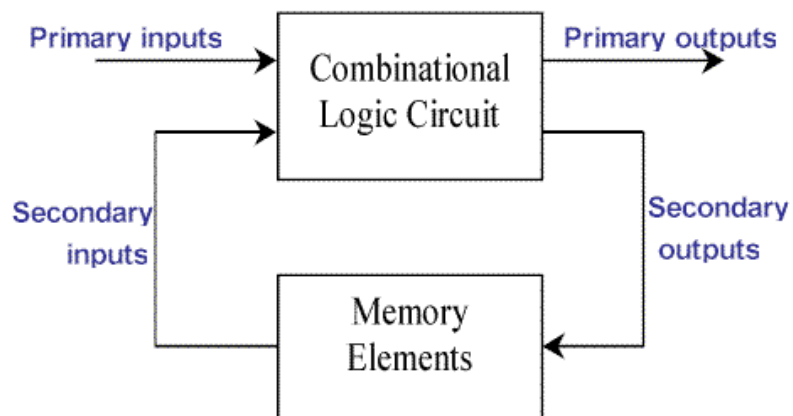


Figure 10 T Flip-flop timing diagram

### 6.5 Theory and Description

Digital electronic circuit is classified into combinational logic and sequential logic. Combinational logic output depends on the inputs levels, whereas sequential logic output depends on stored levels and also the input levels. The storage elements (Flip -flops) are devices capable of storing 1-bit binary info. The binary info stored in the memory elements at any given time defines the state of the Sequential circuit. The input and the present state of the memory element determines the output. Storage elements next state is also a function of external inputs and present state.





Flip-flops are synchronous bistable devices. The term synchronous means the output changes state only when the clock input is triggered. That is, changes in the output occur in synchronization with the clock. A flip-flop circuit has two outputs, one for the normal value and one for the complement value of the stored bit. Since memory elements in sequential circuits are usually flip-flops, it is worth summarizing the behavior of various flip-flop types before proceeding further. All flip-flops can be divided into four basic types: SR, JK, D and T. They differ in the number of inputs and in the response invoked by different value of input signals.

### SR Flip-Flop:

The fundamental latch is the simple SR flip-flop, where S and R stand for set and reset respectively. It can be constructed from a pair of cross-coupled NOR logic gates. The stored bit is present on the output marked Q.

Normally, in storage mode, the S and R inputs are both low, and feedback maintains the outputs in a constant state, with Q and the complement of Q. If S (Set) is given with high while R is held low, then the Q output is forced high, and stays high even after S returns low; similarly, if R (Reset) is given with high while S is held low, then the Q output is forced low, and stays low even after R returns low. SR not face any decision if both  $R=S=1$  So it also called undeterminant state.

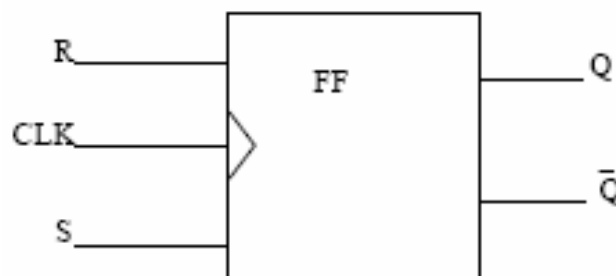
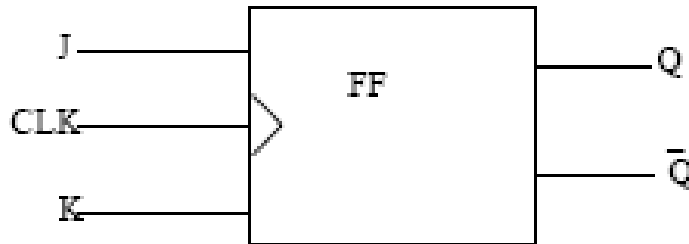


Figure 11: RS Flip-flop symbol

### JK-Flip-Flop:

The JK flip-flop augments the behavior of the SR flip-flop (J = Set, K = Reset) by interpreting the  $S = R = 1$  condition as a “flip” or toggle command. Specifically, the combination  $J = 1, K = 0$  is a command to set the flip-flop; the combination  $J = 0, K = 1$  is a command to reset the flip-flop; and the combination  $J = K = 1$  is a

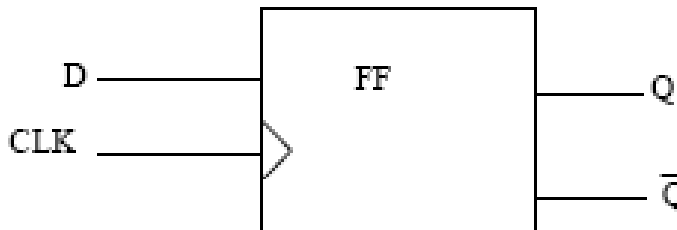
command to toggle the flip-flop, i.e., change its output to the logical complement of its current value.



**Figure 12: JK Flip-flop symbol**

### **D-Flip-Flop:**

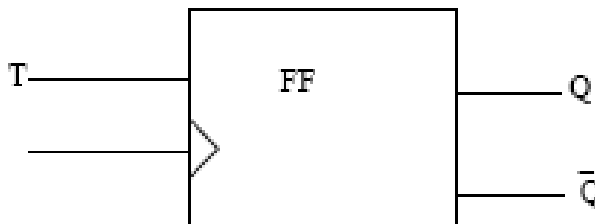
The Q output always takes on the state of the D input at the moment of a rising clock edge. (Or falling edge if the clock input is active low) It is called the D flip-flop for this reason, since the output takes the value of the D input or Data input, and Delays it by one clock count. The D flip-flop can be interpreted as a primitive memory cell, zero-order hold, or delay line.



**Figure 13: D Flip-flop symbol**

### **T-Flip-Flop:**

If the T input is high, the T flip-flop changes state ("toggles") whenever the clock input is strobed. If the T input is low, the flip-flop holds the previous value.



**Figure 14: T FlipFlop symbol**

This behavior is described by the characteristic equation: A T flip-flop can also be built using a JK flip-flop (J & K pins are connected together and act as T) or D flip-flop.

## 6.6 Self Learning Exercise-I

- Q.1 What is a latch?
- Q.2 What is a Flip- Flop?
- Q.3 Differentiate between a latch and a Flip-Flop.
- Q.4 Differentiate between combinational and sequential circuits.
- Q.5 What happens to the JK Flip Flop if the J input is treated as an inverter is wired between J and K inputs?

## 6.7 Procedure

### **S R flip-flop:**

1. The input S, R is given to NAND gates and clock pulse is applied between the other two terminals and NAND gates.
2. The input of the one NAND gate is connected to the other gate and vice versa to form SR latch.
3. The output of the NAND gate whose input is S, is connected to the input of the other NAND gate.
4. The output of the NAND gate whose input is R, is connected to the input of the other NAND gate whose output is 'Q1'.

### **J K flip-flop:**

1. Connections are made as per the circuit diagram.
2. The inputs J1 and K1 are given to the pin numbers 14 and 3 of IC 7473.
3. Clock pulse CP1 is applied at the pin 1.
4. Vcc and ground connections are given to the pin 4 and 11.
5. The outputs Q1 and Q1 bar are connected to pin 12 and 13.

### D flip-flop

1. Connections are made per the circuit diagram.
2. A NOT gate is connected between the inputs J and K.
3. From JK flip flop we can obtain the D flip flop.

### T flip-flop

1. Connections are made as per the circuit diagram.
2. From J K flip flop, we can obtain the T flip-flop by shorting the two inputs J and K.

### Synchronous counter using JK Flip Flop

1. Connect the circuit as per diagram shown.
2. Switch ON the Kit and note down the output.
3. Verify output sequence with truth table.

## 6.8 Observation

### (TRUTH TABLES)

#### S-R FLIP-FLOP

Q(t)	Inputs		Outputs Q(t+1)
	S	R	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

## J-K FLIP-FLOP

Q(t)	Inputs		Outputs Q(t+1)
	J	K	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

## D FLIP-FLOP

Q(t)	D	Q(t+1)
0	0	
0	1	
1	0	
1	1	

## T FLIP-FLOP

Q(t)	D	Q(t+1)
0	0	
0	1	
1	0	
1	1	

## Synchronous Counter Using JK Flip Flop

Present state			Next state			Output					
$Q_A$	$Q_B$	$Q_C$	$Q_A$	$Q_B$	$Q_C$	$J_A$	$K_A$	$J_B$	$K_B$	$J_C$	$K_C$

### 6.9 Graph

Plot Timing diagrams.

### 6.10 Result

- (i) Study and verify truth tables of various Flip Flops.
- (ii) Designed synchronous counter using JK Flip flop and verify it's truth table.

### 6.11 Discussion

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### 6.12 Precautions and Sources of Error

1. All the IC's should be checked before use the apparatus.

2. All LED's should be checked
3. All connection should be tight.
4. Always connect ground first and then  $V_{CC}$ .
5. The circuit should be off before change the connections.
6. After completing the experiment switch off the supply to apparatus.

### 6.13 Self Learning Exercise-II

- Q.1 How is a JK Flip Flop made to toggle?
- Q.2 When a JK Flip Flop is in a no change condition?
- Q.3 Which is the basic sequential building block in which the output follows the data input as long as the enable input is active?
- Q.4 Define the characteristic of a Flip Flop.
- Q.5 How many stable states a Flip Flop has?

### 6.14 Glossary

**Binary** : The basic counting system used in computer logic. Two values are available - 0 and 1. A zero is normally represented by a 0 Volt signal, and a one by a voltage of approximately 5 Volts - these levels are dependent upon the type of logic used.

**Binary Code** : A coding scheme that communicates information by using a series of "1s" and "0s" that are represented, respectively, by the digital "ON" and "OFF" states.

**Scan Design** :A design technique in which the internal registers or flip-flops of a circuit can be chained, to allow an external circuit to easily read and write their contents. When internal memory elements are not directly accessible from the circuit's outside pins, testing is difficult because their state is unknown. With scan design, a signal reconfigures the elements into a "scan chain" and their contents can be read and if desired, altered.

**Shift Register** :Two or more bistable elements (flip-flops) connected in series. With each tick of the clock, the output of stage n is shifted to stage n+1.

Applications include clock or signal delays, delay lines, linear-feedback shift registers.

**Clear** : An input to a latch or FF used to make  $Q = 0$ . **Clear State** The  $Q = 0$  state of a flip-flop.

**Clock** : Digital signal in the form of a rectangular pulse train or a square wave.

**Clock Skew** :Arrival of a clock signal at the clock inputs of different flip-flops at different times as a result of propagation delays.

**Clock Transition Times** : Minimum rise and fall times for the clock signal transitions used by a particular IC, specified by the IC manufacturer.

**Clocked D Flip-Flop** : Type of flip-flop in which the  $D$  (data) input is the synchronous input.

**Clocked Flip-Flops** : Flip-flops that have a clock input.

**Clocked J-K FLip-Flop** : Type of flip-flop in which inputs  $J$  and  $K$  are the synchronous inputs.

**Trigger** : Input signal to a flip-flop or one-shot that causes the output to change states depending on the conditions of the control signals.

**Truth Table** : Logic table that depicts a circuit's output response to the various combinations of the logic levels at its inputs.

## 6.15 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-1*

**Ans.1:** Storage elements that operate with signal levels are referred to as latches.

**Ans.2:** Storage elements controlled by a clock transitions are called Flip Flops.

**Ans.3:**

Latch	Flip Flops
A latch checks all its inputs continuously and changes its outputs accordingly at any time.	Flip Flop samples its inputs and changes its outputs only at a time as determined by a clocking signal.
No Clock is used	A clock is used



**Ans.4:**

Combinational circuits	Sequential circuits
A circuit whose output is dependent only on the inputs at that instant	It depends on present and past history of the inputs.
Easy to design	Harder to design
More expensive circuit	Cheaper circuit

**Ans.5:** Answer 5 JK Flip Flop becomes D Flip Flop

### ***Answers to Self Learning Exercise-II***

**Ans.1:** If  $J=1$  and  $K=1$

**Ans.2:** If  $J=0$  and  $K=0$

**Ans.3:** D latch

**Ans.4:** The algebraic description of the next state of the Flip Flop is called characteristic equation.

**Ans.5:** 2 stable states.

## **6.16 Viva Questions**

**Q.1** Define synchronous counters.

**Q.2** What is truth table?

**Q.3** What is flip-flop?

**Q.4** How many types of flip-flop are used?

**Q.5** What is disadvantage of SR flip-flop?

**Q.6** What is disadvantage of JK flip-flop?

**Q.7** To remove race around condition what we use?

**Q.8** What is race around condition?

**Q.9** What are the characteristic equation for T flip-flop?

- Q.10** What does meant by memory?
- Q.11** Define sequential circuits?
- Q.12** Define combinational circuits?
- Q.13** What is the difference between RS and JK flip flops
- Q.14** Which gate is used in SR flip flops to a JK flip-flop?
- Q.15** D flip-flop is used for?
- Q.16** What is full form of T flip-flop?
- Q.17** Explain about setup time and hold time, what will happen if there is setup time and hold time violation, how to overcome this?

## 6.17 Answers to Viva Questions

- Ans.1:** A counter in which each flip-flop is triggered by the output goes to previous flip-flop. As all the flip-flops do not change states simultaneously in asynchronous counter, spike occur at the output. To avoid this, strobe pulse is required. Because of the propagation delay the operating speed of asynchronous counter is low. This problem can be solved by triggering all the flip-flops in synchronous with the clock signal and such counters are called synchronous counters.
- Ans.2:** Truth table is a table from which we can get output of different gates.
- Ans.3:** Flip-flop is a 1 bit storing element.
- Ans.4:** 4 types of flip –flop, S-R, J-K, D, T
- Ans.5:** When both the input is one then it gives invalid output.
- Ans.6:** Race around condition.
- Ans.7:** Master slave Flip-flop.
- Ans.8:** When pulse width is more than signal width then for signal change of pulse width many number of times signal changes its state that is called race around condition.
- Ans.9:**  $Q = TQ' + QT'$
- Ans.14:** NAND Gates
- Ans.15:** Providing delay.

**Ans.16:** Toggle flip-flop

**Ans.17:** For Synchronous flip-flops, we have special requirements for the inputs with respect to clock signal input there are Setup Time

Setup Time: Minimum time Period during which data must be stable before the clock makes a valid transition. E.g. for a positive edge triggered flip-flop having a setup time of 2ns so input data should be Stable for 2ns before the clock makes a valid transaction from zero to one.

Hold Time: Minimum time period during which data must be stable after the clock has made a valid transition. E.g. for a posedge triggered flip-flop, with a hold time of 1 ns. Input Data (i.e. R and S in the case of RS flip-flop) should be stable for at least 1 ns after clock has made transition from 0 to 1 Hold time is the amount of time after the clock edge that same input signal has to be held before changing it to make sure it is sensed properly at the clock edge.

Whenever there are setup and hold time violations in any flip-flop, it enters a state where its output is unpredictable: this state is known as metastable state (quasi stable state); at the end of metastable state, the flip-flop settles down to either '1' or '0'. This whole process is known as metastability.

## References and Suggested Readings

1. Laboratory experiments and pspice simulations in analog electronics by L. K. Maheshwari, M. M. S. Anand, 2006.
2. Linear Integrated circuits by D.Roy Choudhury, 2015.
3. Power Electronics by P.C.Sen, 1987.
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5. Digital Principles and Applications by D. P. Leach and A. P. Malvino, 1975.
6. Modern Digital Electronics By- R.P.Jain, 1984.
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# **UNIT-7**

## **Schmitt Trigger Circuit using IC-741**

### **Structure of the unit**

- 7.1 Aim
- 7.2 Apparatus
- 7.3 Diagram
- 7.4 Formula
- 7.5 Theory And Description
- 7.6 Self Learning Exercise-I
- 7.7 Procedure
- 7.8 Observation
  - 7.8.1 Readings For Hysteresis Determination
  - 7.8.2 Reduction Of Hysteresis
  - 7.8.3 For Rise And Fall Times
- 7.9 Calculations
  - 7.9.1 For Determination Of Hysteresis Curve
  - 7.9.2 For Rise And Fall Times
- 7.10 Result
- 7.11 Discussion
- 7.12 Precautions And Sources Of Error
- 7.13 Self Learning Exercise –II
- 7.14 Glossary
- 7.15 Answers To Self Learning Exercises
- 7.16 Viva Questions
- 7.17 Answers To Viva Questions

## References and Suggested Readings

### 7.1 Aim

To study a Schmitt trigger circuit using IC-741.

### 7.2 Apparatus

Two transistors connected in circuit as Schmitt Trigger, two power supplies, C.R.O., A.F.(Audio frequency function) generator, rheostat resistance base multimeter and connections wire.

### 7.3 Diagram

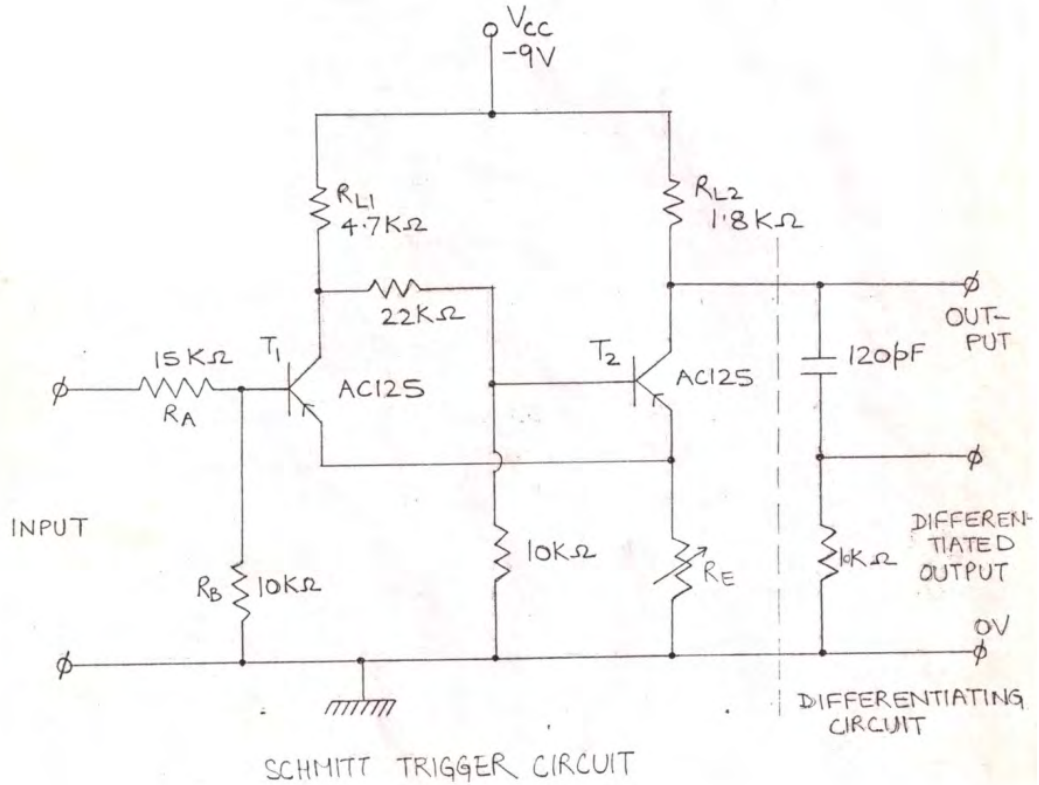


Figure 7.1: Experimental Schmitt Trigger Circuit Used.

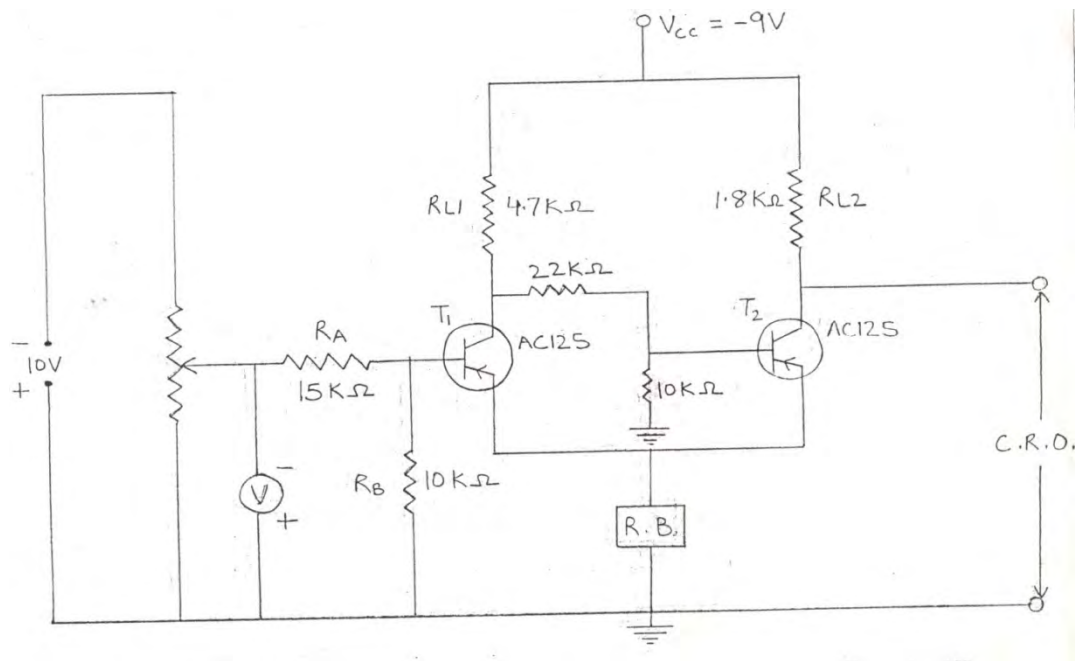


Figure 7.2: Circuit diagram for U.T.P. and L.T.P. measurement.

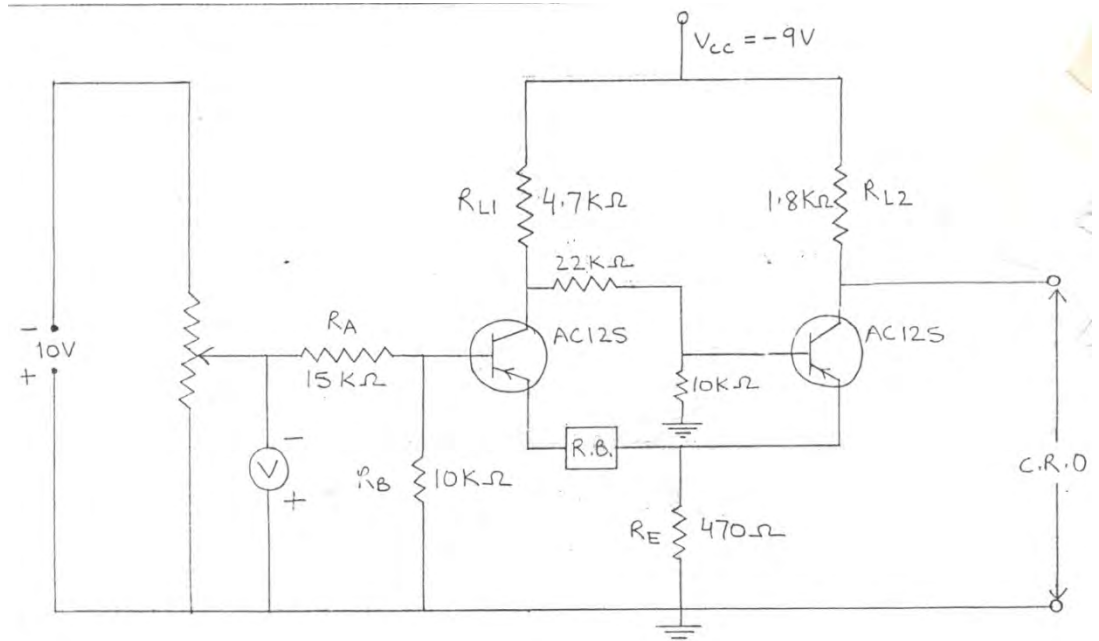


Figure 7.3: Circuit diagram for measuring L.T.P. ( $R.B.$  in  $T_1$  circuit).

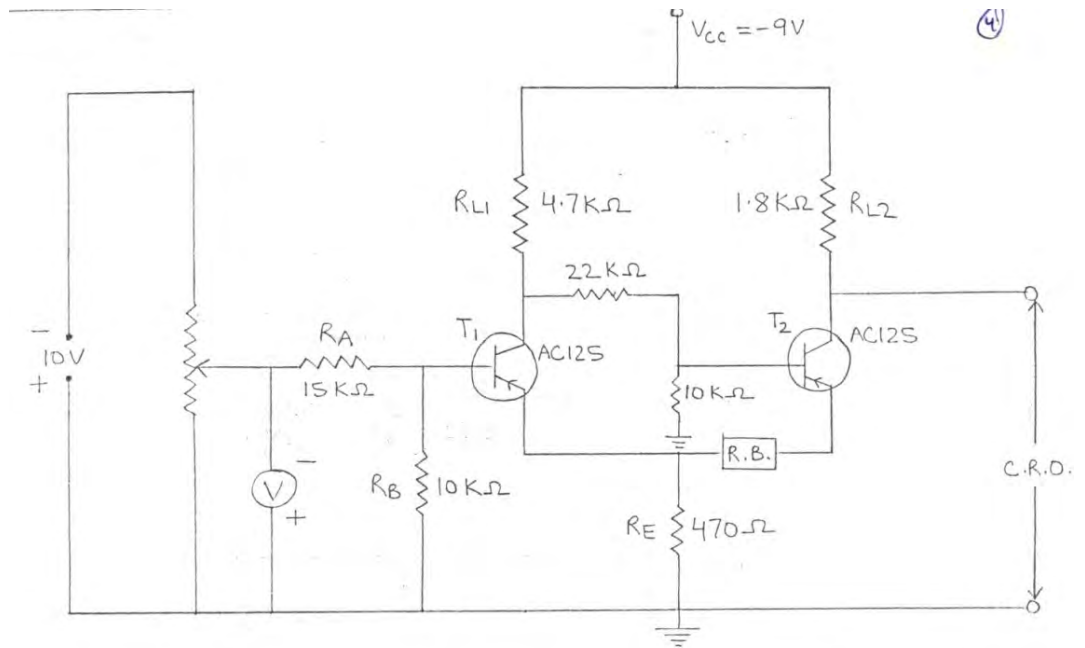


Figure 7.4: Circuit diagram for decreasing U.T.P. (R.B. in  $T_2$  circuit).

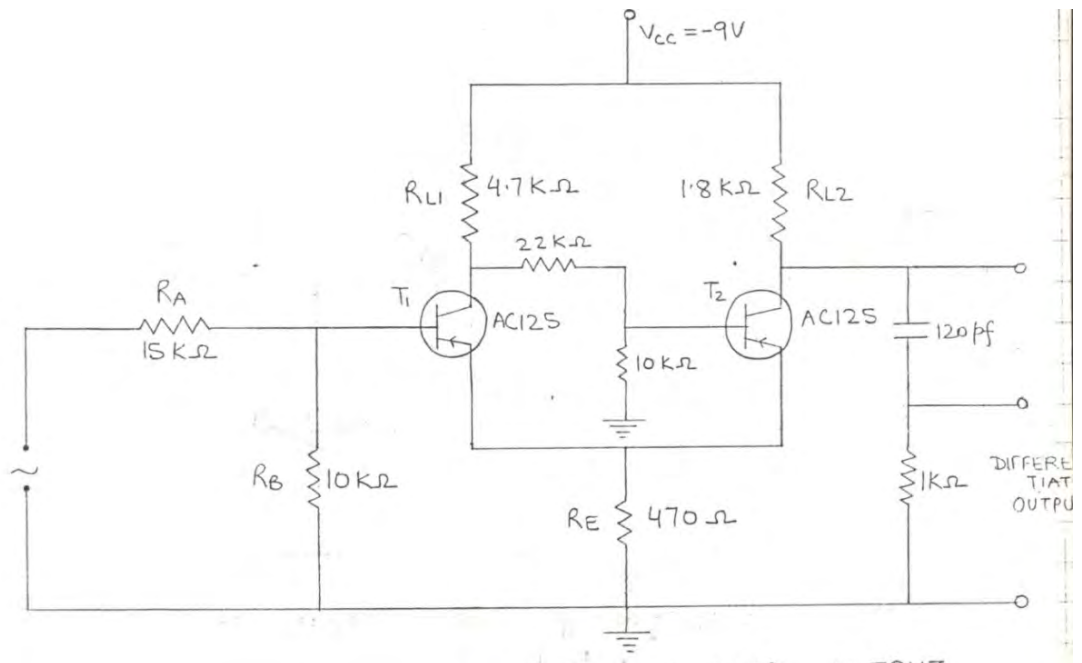


Figure 7.5: Circuit diagram for differentiated output.

$V_o$  versus  $V_{in}$  plot of Hysteresis Voltage

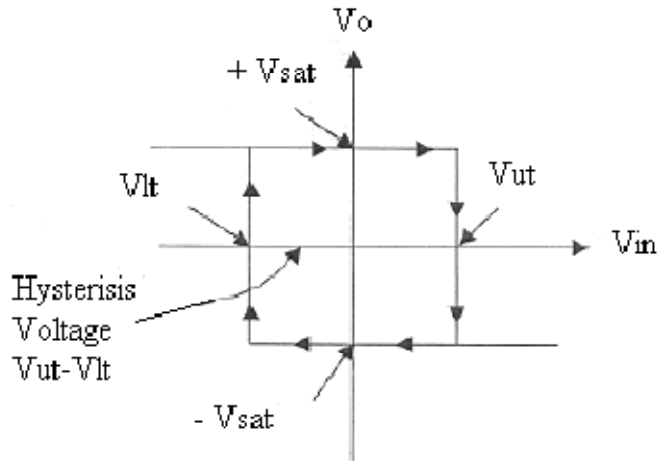


Figure 7.6: Curve for the hysteresis voltage

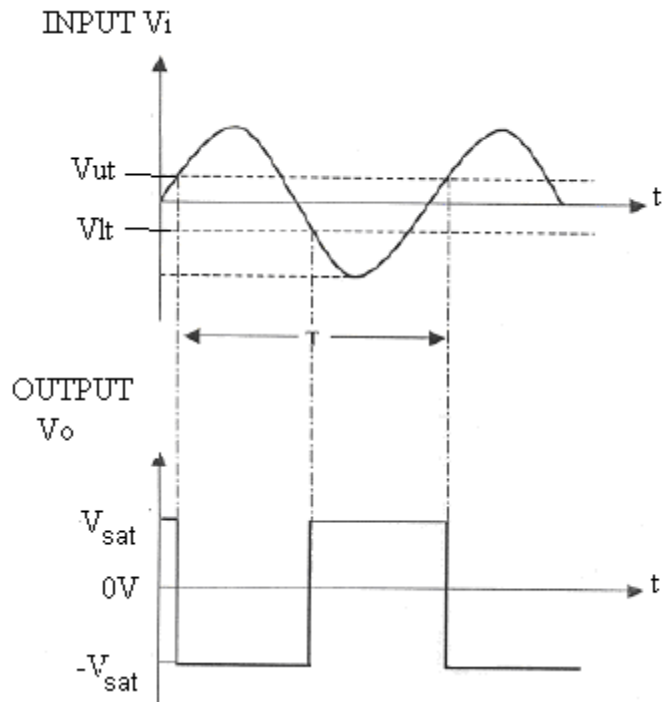


Figure 7.7: Expected Waveforms.

## 7.4 Formula

The formula which is established for determining the Hysteresis is given by:

$$V_H = V_{i_1} - V_{i_2}$$



$$V_H = \frac{V_1 - V_2}{A} (A_\beta - 1)$$

Here  $V_H$  = Hysteresis.

Now after solving this above given formula, we have a new formula, which is given as:

$$V_H = \frac{V_{CC} R_E (R_{L1} - R_{L2})}{(R_{L1} + R_E)(R_{L2} + R_E)}$$

Here  $V_{CC}$  = Common collector voltage

## 7.5 Theory And Description

An emitter coupled bistable circuit triggered by the amplitude of the input voltage is called a Schmitt Trigger. The principle can be understood as follows:

The two curves on the facing page represent idealized of an amplifier, (i) without feedback, (ii) with feedback (positive). But here we are interested in the case (iii) for the input voltage between  $V_{i1}$  and  $V_{i2}$ . There are three possible values of  $V_{out}$ , one of which lies on dotted curve. Since this state is not stable, the circuit assumes a bistable character. The difference between the thresholds is known as hysteresis  $V_H$ .

$$V_H = V_{i1} - V_{i2}$$

$$V_H = \frac{V_1 - V_2}{A} (A_\beta - 1)$$

Figure 7.1 shows one way of building a Schmitt trigger, when  $Q_1$  is on  $Q_2$  is off and vice versa. Assume that the input voltage,  $V_{in}$ , is initially set to 0V under this condition, no base current flows in  $Q_1$ . Therefore there is no collector current in  $Q_1$ . The base of  $Q_2$  is connected to a voltage divider formed by 33k and 25k resistors. The voltage at the top of the divider is enough to turn on the base and saturate  $Q_2$ . With  $Q_2$  saturated, the collector and emitter are effectively shorted together as shown in above given figure 7.2. The voltage drop across  $R_E$  is  $\frac{V_{CC} R_E}{R_{L2} + R_E}$ . If we raise the input voltage above the value of voltage drop across  $R_E$ , base current follows in  $Q_1$ . The voltage at  $Q_1$  collector decreases. This drop is coupled to the base of  $Q_2$  bringing  $Q_2$  out of saturation. The drop

in  $Q_2$  current reduces the voltage from emitter to ground. Since  $V_C$  has been reduced,  $Q_1$  is turned "ON" hard. The increase in  $Q_1$  current drops the collector voltage even further and we have regeneration. The regeneration continues until  $Q_1$  saturates and  $Q_2$  cuts off. With  $Q_2$  cut off, there is no current through  $R_{L2}$ . With  $Q_1$  saturated emitter to ground voltage is  $\frac{V_{CC}R_E}{R_{L1}+R_E}$ . To bring  $Q_2$  out of saturation the input voltage must be reduced to about  $\frac{V_{CC}R_E}{R_{L2}+R_E}$ . This causes  $Q_2$  to conduct. With  $Q_2$  conducting emitter voltage increases causing the current in  $Q_1$  to decrease further. This raises the collector voltage of  $Q_1$  and again we have regenerative action. The circuit is back in the original state. The value of  $V_{in}$  that causes output to jump from a low to a high state is called U.T.P. and the value of  $V_{in}$  that causes the output to jump from high state to low state is called L.T.P.

Formula which is used for the hysteresis is given below:

The critical input level where output stage changes are

$$\frac{V_{CC}-0.2}{(R_{L1}+R_E)} (R_E) \text{ and } \frac{V_{CC}-0.2}{(R_{L2}+R_E)} (R_E)$$

Their difference is  $V_H$

$$V_H = \frac{V_{CC}R_E(R_{L1}-R_{L2})}{(R_{L1}+R_E)(R_{L2}+R_E)}$$

Here  $V_{CC}$  = supply voltage

$R_{L1}, R_{L2}$  is the load resistances and

$V_H$  = Hysteresis

## 7.6 Self Learning Exercise -I

**Q.1** Give the brief description of transistor which is used in Schmitt trigger biasing.

**Q.2** Give the brief theory of Operational amplifier.

## 7.7 Procedure

1. Connect the equipment as shown in the circuit diagram.

2. Now switch ON the power supply.
3. After that we apply the input sine wave using function generator.
4. Now connect the channel-1 of CRO at the input terminals and Channel-2 at the output terminals.
5. Now observe the output square waveform corresponding to input sinusoidal signal.
6. Overlap both the input and output waves and note down reading of voltages at positions on sine wave where output changes its state. These voltages denote the Upper threshold voltage and the Lower threshold voltage (see EXPECTED WAVEFORMS above).
7. Verify that these practical threshold voltages are almost same as the theoretical threshold voltages calculated using formulas given in the THEORY section above.
8. On the basis of the readings sketch the waveforms by noting down the amplitude and the time period of the input  $V_{in}$  and the output  $V_o$ .
9. At last we compare the theoretical & practical values of UTP & LTP.

## 7.8 Observation

### 7.8.1 Readings For Hysteresis Determination

S.NO.	$R_E$ (in $\Omega$ )	L.T.P. VOLTS	U.T.P. VOLTS	$V_B$ (VOLTS)		OUTPUT VOLTAGE	
				$T_2$ 'ON'	$T_2$ 'OFF'	on	off
				1	300		
2	400						
3	500						
4	600						
5	700						
6	800						
7	900						

## 7.8.2 Reduction Of Hysteresis

$$R_E = 470 \Omega$$

$$V_{out} \text{ when } T_2'ON' =$$

$$V_{out} \text{ when } T_2'OFF' =$$

S.NO.	$R_E$ (in $\Omega$ )	When R is in emitter of $T_1$			When R is in emitter of $T$		
		U.T.P.	L.T.P.	$V_H$	U.T.P.	L.T.P.	$V_H$
1	300						
2	400						
3	500						
4	600						
5	700						
6	800						
7	900						

## 7.8.3 For Rise And Fall Times

$$R =$$

$$C =$$

$$R_c =$$

$$R_E =$$

$$V_i =$$

$$V_{on} =$$

$$V_{of} =$$

These all values for 1 kHz sine wave input.

## 7.9 Calculation

### 7.9.1 For Determination Of Hysteresis Curve

S.No.	$R_E$ (in $\Omega$ )	$V_H$ (Volts)	$\frac{R_E}{(R_{L_1} + R_E)(R_{L_2} + R_E)}$
1			
2			

3			
4			

### 7.8.2 For Rise And Fall Times:

$$t_r = 0.8 RC \left( \frac{V_i}{V_o} \right)$$

$$t_f = 0.8 RC \left( \frac{V_i}{V_o} \right)$$

## 7.10 Result

For a 1 kHz input pulse rise time  $t_r = \quad \mu s$

Pulse fall time  $t_f = \quad \mu s$

### Percentage Error:

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

## 7.11 Discussion

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## 7.12 Precautions And Sources Of Errors

1. The burnt sections of the bread-board should not be used.
2.  $V_{cc}$  and ground terminals of the power-supply should be connected correctly to the respective pins of the 741 IC.
3. The IC should not be inserted and taken out carefully from the bread-board to avoid the breaking of its legs.
4. Avoid the use of long connections wires. Make the connections neatly on a bread-board.
5. The input offset voltage is very small, and therefore has been ignored. It can, however, introduce error if input signal is small.

## 7.13 Self Learning Exercise -II

- Q.1** Give the Characteristics of an ideal OP-Amp.
- Q.2** Give short note on the IC-741.
- Q.3** In an inverting OP-Amp the input resistor ( $R_1$ ) is  $3\text{ k}\Omega$  and feedback resistor ( $R_f$ ) is  $12\text{ k}\Omega$ . The input is  $500\text{ mV}$ . Calculate the input impedance of the amplifier.

## 7.14 Glossary

**Slew-rate:** It is defined as the maximum rate of change of the output voltage with respect to time.

**CMRR:** The ability of an OP-AMP to reject a common mode signal is expressed by its common mode rejection ratio. It is the ratio of differential voltage gain to the common-mode voltage gain.

**Output offset voltage:** It is the *d. c.* voltage appearing at the output terminal when the two input terminals are grounded.

**Input offset current:** The input offset current is the difference between the two input currents driven from a common source.

**Input offset voltage:** The input offset voltage is the differential input voltage that must be provided between the inverting and non-inverting terminals of the OP-AMP to get zero output voltage.

## 7.15 Answer to Self Learning Exercises

### *Answer to Self Learning Exercise-I*

**Ans.1:** In the circuit of Schmitt Trigger, we have a transistor biasing which is Common Base configuration. The description of common base configuration is given below:

Common Base Configuration: Here the base terminal is common to both input as well as output terminals as shown in below given figure .

Here the emitter terminal is taken as input and collector terminal is taken as Output terminal.

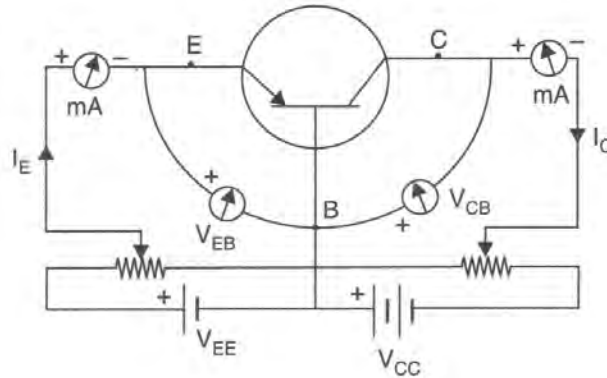


Figure : Circuit for drawing characteristics of  $(p - n - p)$  transistor in Common Base Configuration.

### Characteristics Properties of Common Base Configuration:-

The circuit to draw the characteristics of the transistor in C.B. configuration is shown in Figure2. Here the emitter base junction is forward bias by battery  $V_{EE}$  and the corresponding input voltage and current is read by the voltmeter  $V_{EB}$  and ammeter  $I_E$ .

The collector base junction is reverse biased by battery  $V_{CC}$  and the corresponding output voltage and current is read by the voltmeter  $V_{CB}$  and ammeter  $I_C$ .

### Input Characteristics:-

The input characteristics are drawn by noting voltage  $V_{EB}$  and corresponding current  $I_E$  keeping  $V_{CB}$  constant. The characteristics curves are as shown in figure 3.

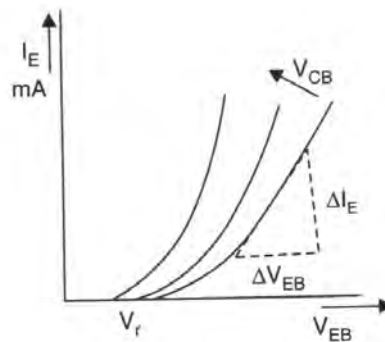


Figure 2: Input characteristic Common Base Configuration

They are similar to the forward biased diode characteristics and represented by

$$[V_{EB}, I_E]_{V_{CB} = \text{constant}}$$

The inverse of slope of characteristics curve

$$\frac{1}{\text{slope}} = \left[ \frac{\Delta V_{EB}}{\Delta I_E} \right]_{V_{CB} = \text{constant}}$$

This above formula gives input resistance which is denoted as  $h_i$ .

The shape of the curve can be easily explained by the fact here the emitter junction behaves like forward biased diode so on slightly increasing the voltage  $V_{EB}$ , the  $I_E$  increases a lot. Also there exists a cut in voltage  $V_r$  below which emitter current is very small or zero. Cut in voltage is 0.2 V and 0.7 V for germanium and silicon respectively. The curves shift towards left on increasing  $V_{CB}$  it is due to the early effect that current increases for same value of  $V_{EB}$ .

Output Characteristics:-

The output characteristics are drawn by noting the output voltage  $V_{CB}$  and corresponding collector current  $I_C$  keeping emitter current  $I_E$  constant.

The characteristics are as shown in Fig 3 and represented by

$$(V_{CB}, I_C)_{I_E = \text{constant}}$$

So we get different curves for different values of  $I_E$ .

The family of characteristics is divided into three regions.

Active Region: -

It is customary to plot  $V_{CB}$  along (+ve) X-axis although the values of  $V_{CB}$  are negative for  $p - n - p$  transistor.

For the first curve when  $I_E = 0$  i.e. when input terminals are open, the collector current  $I_C = I_{CO}$  the reverse saturation current (which is of the order of  $\mu A$  for Germanium and nano Amperes for Silicon) of the diode.

Suppose now that input circuit is on and  $I_E$  is flows then a fraction of it  $\alpha I_E$  would flow towards collector region and collector current  $I_C$  in this case is  $(\alpha I_E + I_{CO})$  and it is independent of collector voltage since  $I_{CO}$  is very small and  $I_C$  mainly depends on  $I_E$  so that curves appear to be practically flat. They have very small slope and its value gives ' $h_o$ ' or ' $(1/r_o)$ '.



The forward biasing of emitter-base junction allows the series combination of  $V_{EE}$  and  $V_{CC}$  to inject majority carriers (holes in case of  $p-n-p$  transistor) through the emitter towards base. The doping of the base is less than that of emitter or collector so that not many recombination's of holes with electrons take place. This property is further reduced by the fact that then base region is much narrower than either emitter or collector region, so the majority carriers (holes) do not remain for long time in the base region. Roughly 98% of the holes injected into the base are swept away towards the collector region. Thus the collector current is relatively independent of the voltage  $V_{CB}$  applied across the collector-base junction and thereby the output characteristics representing collector current in C.B. configuration appear flat or parallel to  $x$  - axis.

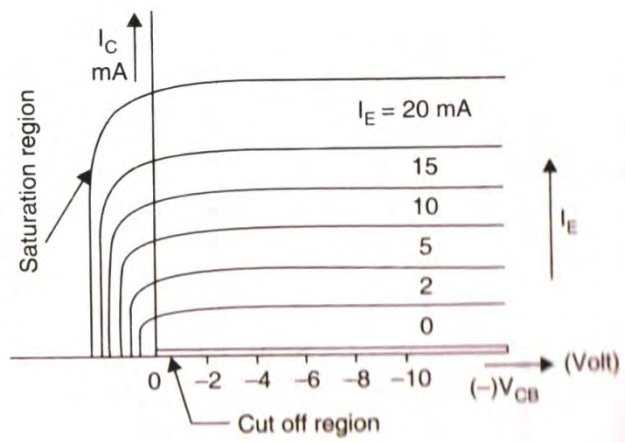


Figure 3: Output characteristics Common Base Configuration.

*Saturation region:* The region to the left of origin O,  $V_{CB} = 0$  and above  $I_E = 0$  region characteristics is known as Saturation region, where both emitter and collector junctions are forward biased. Here  $V_{CB}$  is positive for  $p-n-p$  transistor so the forward biasing of collector is responsible for large change in collector current for small change in  $V_{CB}$  and  $I_C$  increases exponentially with voltage.

*Cut off Region:* The characteristic for  $I_E = 0$  passes through origin but is otherwise similar to the other characteristics. This characteristic does not coincide with (X-axis) voltage axis although in diagram it appears so, since, the gap is too small to be shown on the scale chosen. The region below the  $I_E = 0$  characteristic for which both emitter and collector junctions are reverse biased is called cut off region.

**Ans.2:** The use of diodes, transistors, tubes, etc. in various electronic devices has been made quite efficiently. In spite of their efficient use, the circuits using these devices have become quite cumbersome and costly. With the introduction of space technology and computers the need for miniaturization has been strongly felt. These days it has been possible to have a package of large number of transistors, diodes, resistors etc. on a very small chip. For this a monolithic integrated circuitry has been introduced with which a complete circuit without the use of external connecting wires can be manufactured on a silicon chip. In addition to reduction in size and weight, it has been possible to have high accuracy and reduction in cost price also.

An important setup in this direction is the introduction of operational amplifiers better known as OP-Amp which is the fundamental building block. It is high gain direct amplifier whose response characteristics are extremely controlled by negative feedback. It is used for accomplishing many linear as well as nonlinear functions and that is why it is called operational amplifier.

The basic block of the operational amplifier is Differential Amplifier.

*Differential Amplifier:-*

It is a basic unit of operational amplifier having differential input. In a broad way it amplifies the difference of two input signals. If the two input signals are  $v_1$  and  $v_2$  respectively then an ideal differential amplifier will give an output voltage

$$v_o = A_d(v_1 - v_2)$$

Where  $A_d$  is the gain of the amplifier.

The efficiency of a differential amplifier is measured by the ratio of  $A_d$  and  $A_c$ . This ratio is called common mode rejection ratio (CMRR).

We can write as

$$CMRR = \frac{\text{voltage gain for difference signal}}{\text{voltage gain for common mode signal}}$$

$$\rho = \left| \frac{A_d}{A_c} \right|$$

In Decibel we can written as

$$CMRR (\rho) = 20 \log_{10} \left[ \left| \frac{A_d}{A_c} \right| \right]$$

An OP-Amp is a high gain, direct coupled differential linear amplifier whose response characteristics are externally controlled by negative feedback. It is widely used in Computers, can perform mathematical operations such as summing, integration, and differentiation. Its input impedance is very high and output impedance is extremely low and its gain depends on the feedback ratio only.

### ***Answer of Self Learning Exercises-II***

**Ans.1:** The characteristics of an ideal OP-Amp is given below:

1. Infinite Voltage Gain,
2. Infinite Bandwidth,
3. Infinite Input Impedance,
4. Zero Output Impedance,
5. Perfectly Balanced Circuit,
6. Perfectly Stable Circuit.

Symbolically it has three leads other than power supply and ground leads as shown in given figure Terminal leads 1 and 2 function as two input ports and terminal 3 acts as output.

Infinite input impedance means that none of the input terminals draw any current i.e.  $I_1 = I_2 = 0$ . Zero output impedance means that any amount of current can be drawn without disturbing input voltage.

Terminal 1 is labeled (-)ve and terminal 2 is labeled as (+)ive. It means that if a signal is applied at terminal 1, it will be inverted at output i.e., appear as  $180^\circ$  out of phase with respect to input whereas if a signal is applied at terminal 2, it will not be inverted at output. On account of this function terminal 1 is called *Inverting Terminal* and terminal 2 is called *Non – Inverting Terminal*. Its symbol is shown in below given figure

Its equivalent circuit is shown in figure

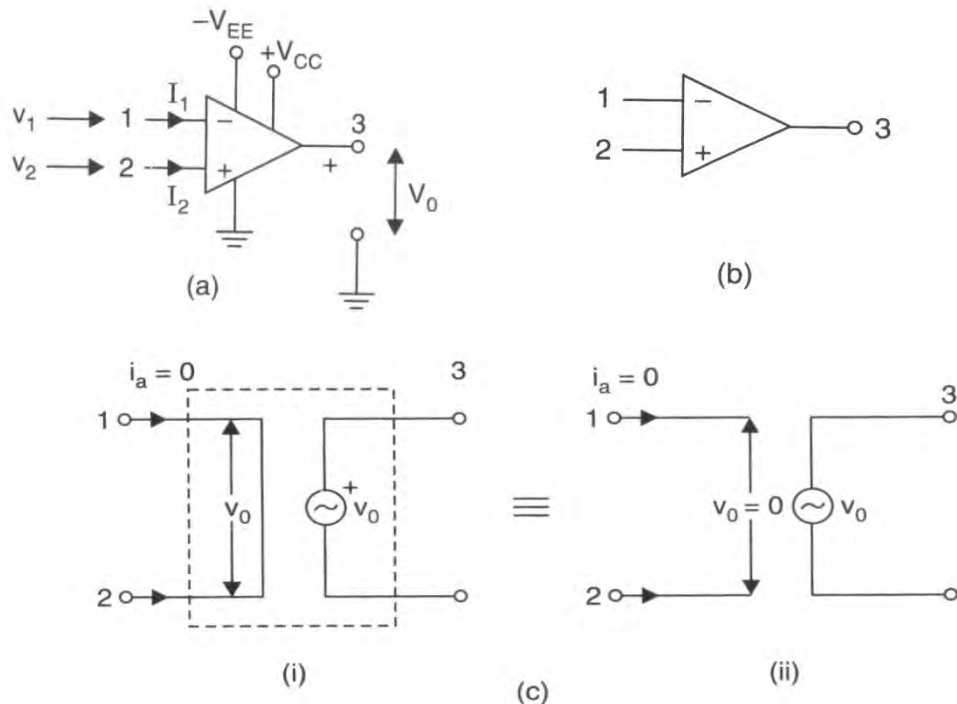


Figure : OP – Amp (a) Symbol OP – Amp (b) (c) (i) (ii) Equivalent circuit

The magnitude of the output signal at terminal 3 is proportional to the difference of the two-input signals which is also called error voltage applied between terminals 1 and 2 respectively.

$$v_o \propto (v_1 - v_2)$$

$$v_o = A(v_1 - v_2) = A(v_i) \text{ where } v_i = (v_1 - v_2)$$

Here constant 'A' is called the voltage gain of the OP-Amp and  $v_i = (v_1 - v_2)$  is called the error voltage.

Now another figure of OP-Amp is given below (fig 1 d) which is known as Block Diagram. The input stage is a differential amplifier followed by more stages of gain and a class B pushpull Emitter Follower because a differential amplifier is the first stage.

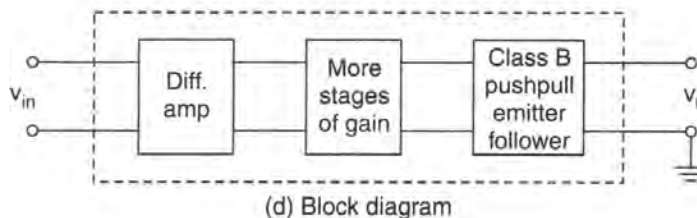


Figure 1 (d) Block diagram

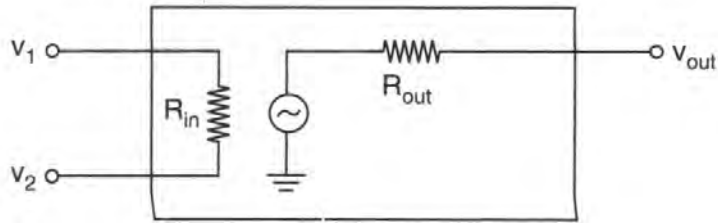


Figure 1 (e) Equivalent Circuit

It determines the input characteristics of the OP-Amp. In most OP-Amps the output is single ended as shown in figure 1(e). With positive and negative supplies, the single ended output is designed to have a quiescent value of zero. This way a zero input results into a zero output.

**Ans.2:** The 741 OP-Amp has become an industry standard due to its varied uses some modifications. It has different versions numbered 741, 741 A, 741 C, 741 E and 741 N. These differ in their voltage gain, temperature range, noise level etc. the 741 C stands is the least expensive and most widely used. (Here C stands for commercial grade)

Figure 1(d) shows a block diagram of an OP-Amp in general. The input stage is a differential amplifier followed by more stages of gain, and a class B push pull Emitter follower.

The first stage is differential amplifier. So it determines the input characteristics of the OP-Amp. In most OP-Amp the output is single ended as shown figure 1(a). With positive and negative supplies, the single ended output is designed to have a quiescent value of zero. This way the zero input voltage ideally in zero output.

For instance some do not use a class B push pull output and others may have double ended output. The block diagram 1(d) a shown captures two important features that apply to typical OP-Amps, the differential input and single ended output.

**Ans.3:(i)** Voltage gain of inverting amplifier

$$A_f = -\frac{R_f}{R_1}$$

$$A_f = (-) \frac{12 \text{ k}\Omega}{3 \text{ k}\Omega} = (-)4 = 4 \text{ in magnitude}$$

Where (-)ive sign represents phase of 180°.

(ii) Output voltage of the amplifier

$$v_o = A_f(v_i)$$
$$v_o = 4 \times 500 \times 10^{-3} = 2 \text{ V}$$

(iii) Input Impedance  $R_{if} = R_1 = 3 \text{ k}\Omega$

## 7.16 Viva Questions

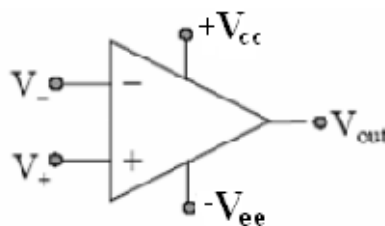
- Q.1 What is an op-amp?
- Q.2 Mention some applications of op-amp.
- Q.3 Draw the symbol of op-amp.
- Q.4 What are the packages of IC-741?
- Q.5 Mention some features of op-amp.
- Q.6 What is the value of supply voltage for an op-amp stand for?
- Q.7 What is the output voltage of an inverting amplifier?
- Q.8 What is the output voltage of a non-inverting voltage?
- Q.9 When is the open-loop gain and closed loop gain of an OP-AMP?
- Q.10 What is the input offset current?

## 7.17 Answer to Viva Questions

**Ans.1:** An OP-AMP is a high-gain, direct-coupled differential amplifier whose gain is controlled by negative feedback from the output to the input.

**Ans.2:** Op-amps are used as video and audio amplifiers, oscillators, and in communication electronics; for example Non-inverting amplifier, Inverting amplifier, Integrator Differentiator Low Pass, High Pass, Band pass and Band Reject Filters etc.

**Ans.3:** The symbol of op-amp is given below:



**Ans.4:** IC741 is available in three packages: 8-pin metal can, 10-pin flat pack and 8 or 14-pin DIP.

**Ans.5:** The features of operational amplifier are given as:

1. No External frequency compensation is required.
2. Short circuit Protection,
3. off Set Null Capability,
4. Large Common mode and differential Voltage ranges,
5. Low Power Dissipation, No-Latch up Problem.

**Ans.6:** The value of supply voltage is  $\pm 18$  V.

**Ans.7:** The output voltage of an inverting amplifier is given below:

$$V_o = -\frac{R_f}{R_i} V_i.$$

Here negative sign represents the  $180^\circ$  phase shift which occurs between the input and output.

**Ans.8:** The output-inverting amplifier is

$$V_o = \left(1 + \frac{R_f}{R_i}\right) V_i.$$

**Ans.9:** The open-loop gain of an OP-AMP is the gain without negative feedback while the closed loop gain is the gain with negative feedback.

**Ans.10:** The input offset current is the difference between the two input currents driven from a common source. The input offset current for IC741 is 20nA.

## References And Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook,
2. B. Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires.

# UNIT-8

## Half Wave & Full Wave Rectifiers

### Structure of the Unit

- 8.1 Aim
  - 8.2 Apparatus
  - 8.3 Diagram
  - 8.4 Formula
  - 8.5 Model Graph
  - 8.6 Theory and Description
  - 8.7 Self Learning Exercise-I
  - 8.8 Procedure
  - 8.9 Observations
  - 8.10 Graph
  - 8.11 Calculations
  - 8.12 Result
  - 8.13 Discussion
  - 8.14 Precautions and Sources of Error
  - 8.15 Self Learning Exercise-II
  - 8.16 Glossary
  - 8.17 Answers to Self Learning Exercises
  - 8.18 Viva Questions
  - 8.19 Answers to Viva Questions
- References and Suggested Readings

### **8.1 Aim**

Implement the following circuits to study the ripple characteristics:

- (i) half wave rectifier



- (ii) full wave rectifier
- (iii) half wave rectifier with shunt capacitor filter
- (iv) full wave rectifier with shunt capacitor filter

## 8.2 Apparatus

Diodes (say 1N4001), resistors (of about  $2.2k\ \Omega$ ), capacitors (of about  $100\mu F$ ), center tapped transformer (about 12V AC step down transformer), bread board (or trainer board for rectifier circuit with capacitor filter), connecting wires, oscilloscope, multimeter etc.

## 8.3 Diagram

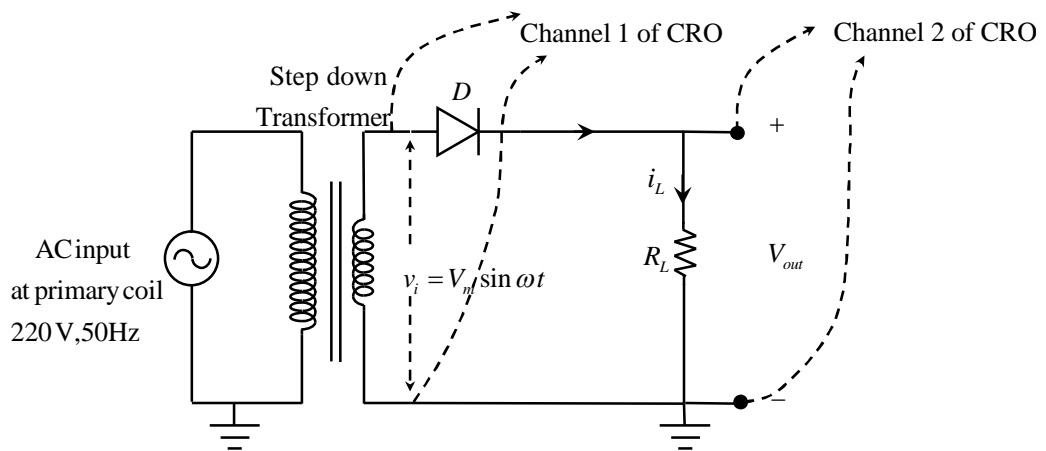


Figure 8.1: Half wave rectifier

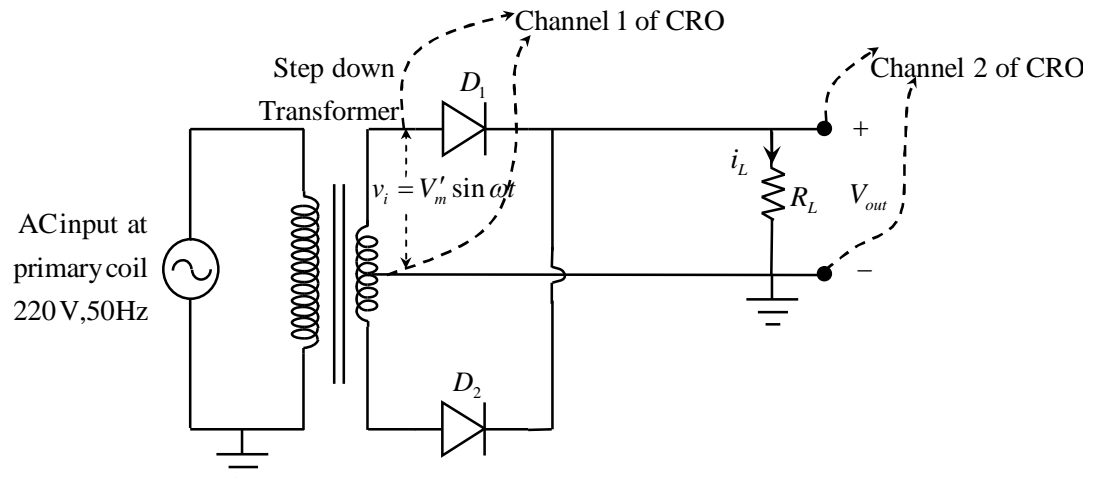
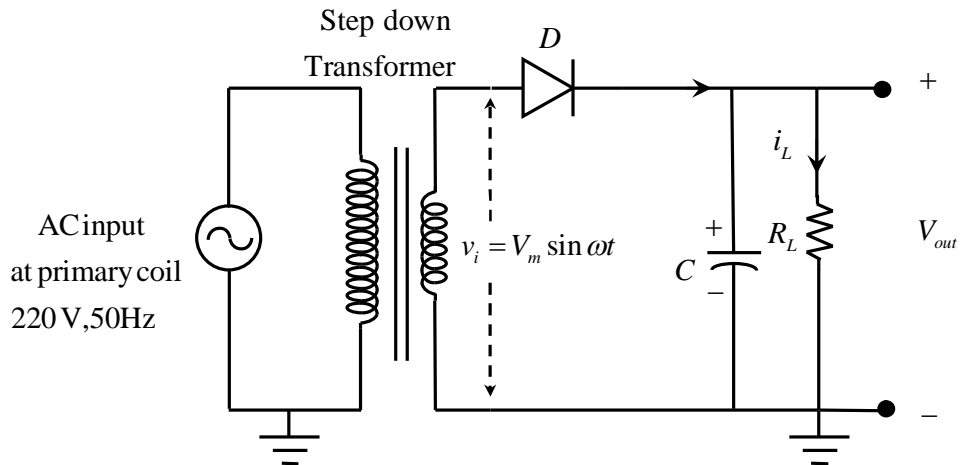
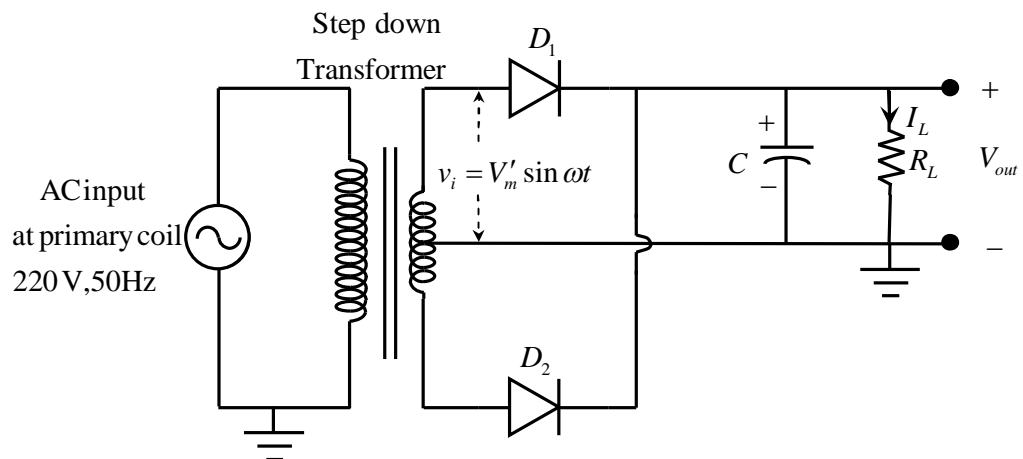


Figure 8.2 Full wave center tapped rectifier



**Figure 8.3: Half wave rectifier with shunt capacitor filter**



**Figure 8.4: Full wave center tapped rectifier with shunt capacitor filter**

## 8.4 Formula

### (i) Half Wave rectifier without Filter

DC output voltage across the Load resistor ( i.e. ideal theoretical value of calculated dc output voltage) =  $(V_{dc})_T = \frac{V_m}{\pi}$  (in Volts)

where  $V_m = \sqrt{2}(V_{rms})_{in}$  is the peak value of the input voltage to half wave rectifier at the secondary side of the transformer.

Theoretical value of ripple factor  $r_t = 1.21$

Value of ripple factor (practical value)  $r_p = \frac{V_{ac}}{V_{dc}}$

where  $V_{ac}$  = Output ac Voltage at the load &  $V_{dc}$  = Output dc Voltage at the load

Ripple frequency (in Hz) of output voltage at load  $f_r = \frac{1}{T_r}$

Here  $T_r$  = Time period (in seconds) of ripple output voltage at load.

### **(ii) Full Wave Rectifier Circuit without Filter**

DC output voltage across the Load resistor ( i.e. ideal theoretical value of calculated dc output voltage) =  $(V_{dc})_T = \frac{2V'_m}{\pi}$  (in Volts)

where  $V'_m = \sqrt{2}(V_{rms})_{in}$  is the peak value of the input voltage to full wave rectifier at the secondary side of the transformer from one end to center tap.

Theoretical value of ripple factor  $r_t = 0.48$

Value of ripple factor (practical value)  $r_p = \frac{V_{ac}}{V_{dc}}$

Ripple frequency (in Hz) of output voltage at load  $f_r = \frac{1}{T_r}$

Here  $T_r$  = Time period (in seconds) of ripple output voltage at load .

### **(iii) Half Wave Rectifier Circuit with Shunt Capacitor Filter**

Theoretical value of ripple factor  $r_t = \frac{1}{2\sqrt{3}} \frac{1}{f R_L C} = \frac{0.289}{f R_L C}$

where C is the capacitance of shunt capacitor (in Farads)

$R_L$  is the load resistance (in Ohms).

$f$  = frequency(in Hz) of the input voltage at the input side of the rectifier i.e. at the secondary side of the transformer.

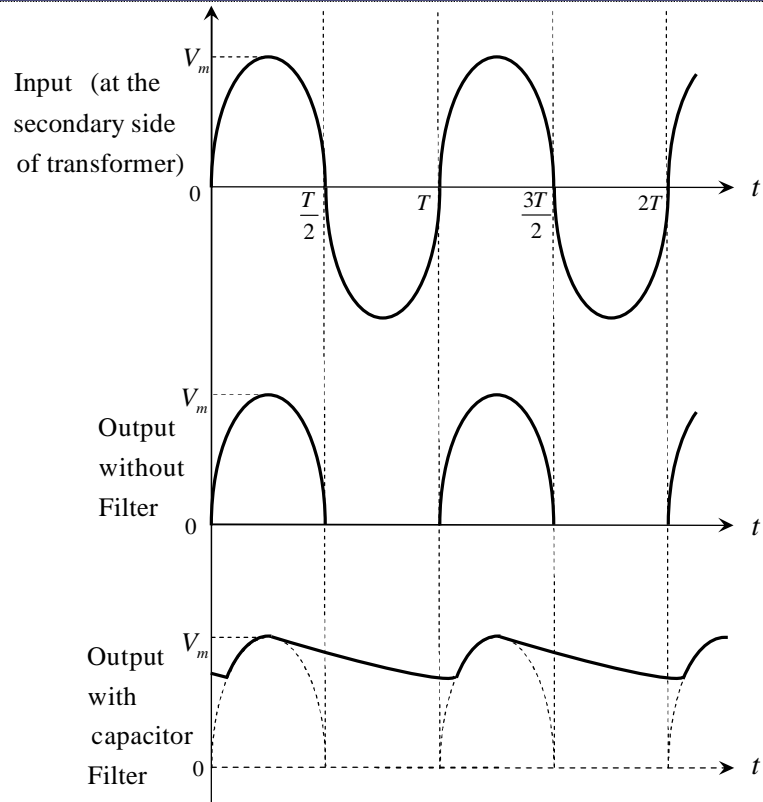
$T = \frac{1}{f}$  = Time period of the input voltage at the input side of the rectifier i.e. at the secondary side of the transformer.

### **(iv) Full Wave Rectifier Circuit with Shunt Capacitor Filter**

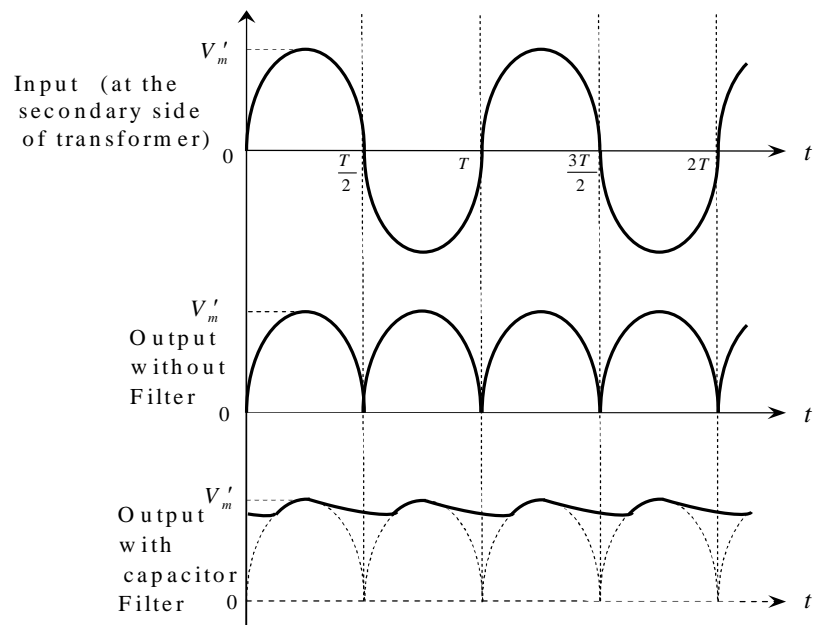
Theoretical value of ripple factor  $r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C} = \frac{0.144}{f R_L C}$

$f$  = frequency(in Hz) of the input voltage at the input side of the rectifier i.e. at the secondary side of the transformer

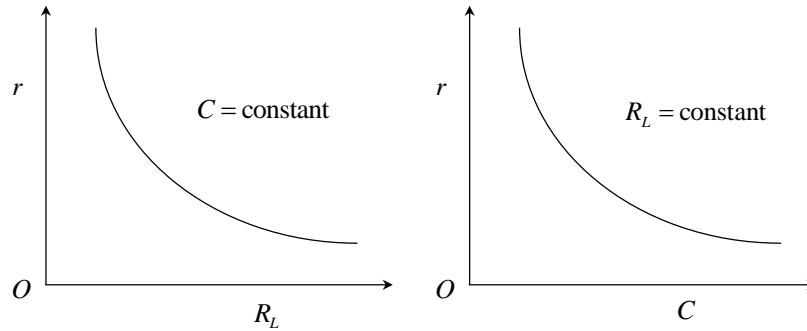
## 8.5 Model Graph



**Figure 8.5: Half wave rectifier input -output waveforms**

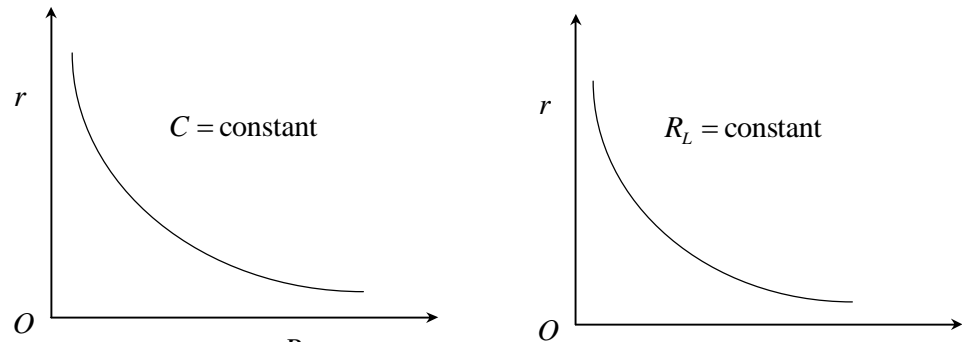


**Figure 8.6: Full wave rectifier input -output waveforms**



**Figure 8.7(a)** Ripple factor versus load resistance graph for half wave rectifier circuit with shunt capacitor filter at a given value of shunt capacitance.

**Figure 8.7(b)** Ripple factor versus shunt capacitance graph for half wave rectifier circuit with shunt capacitor filter at a given value of load resistance.



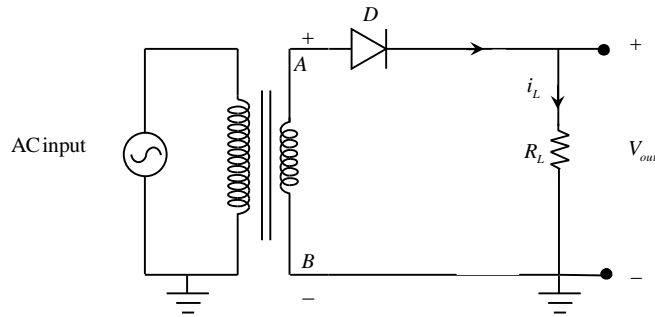
**Figure 8.8(a)** Ripple factor versus load resistance graph for full wave rectifier circuit with shunt capacitor filter at a given value of shunt capacitance

**Figure 8.8(b)** Ripple factor versus shunt capacitance graph for full wave rectifier circuit with shunt capacitor filter at a given value of load resistance.

## 8.6 Theory and Description

### Half Wave Rectifier:

By rectification process, ac current is converted into dc current (unidirectional current though it may not be constant). AC mains voltage is connected to the primary side of a step down transformer (figure 8.9). During the positive half cycle, potential of point A with respect to B is positive and diode is in forward bias. The diode D is in conducting state in forward bias, current flows through the load resistor and there will be output voltage across the load (figure 8.5).



**Figure 8.9**

DC output of the half wave rectifier is given by

$$V_{dc} = \frac{V_m}{\pi} \cong 0.318V_m \cong 31.8\% \text{ of input peak voltage } V_m$$

If we consider cut off voltage of Si diode, then  $V_{dc} \cong 0.318(V_m - 0.7)$

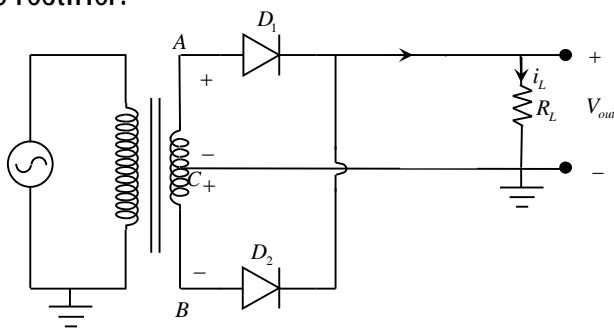
The output frequency is the same as the input frequency i.e.  $f_{out} = f_{in}$

Negative half cycles of the input signal to rectifier are absent from the output signal at load resistor. Waveforms have half wave signals and there is unidirectional load current.

### Full Wave Rectifier:

Full wave rectifier is equivalent to two half wave rectifiers as shown in figure (8.10). During the positive half cycle, diode  $D_1$  is forward biased and diode  $D_2$  is reverse biased. Hence conduction takes place through the diode  $D_1$  during positive half cycle and diode  $D_2$  is nonconducting. During negative half cycle diode  $D_1$  is nonconducting and  $D_2$  is conducting.

The load current always flows in the same direction in each half cycle of the input voltage and load voltage has the same polarity. For same peak output voltage, the output dc current through the load in a full wave rectifier is twice that in a half wave rectifier.



**Figure 8.10**

DC output of full wave rectifier is given as

$$V_{dc} = \frac{2V'_m}{\pi} \cong 0.636V'_m \cong 63.6\% \text{ of input peak voltage } V'_m$$

where  $V'_m$  is the peak value of the input voltage to full wave rectifier at the secondary side of the transformer from one end to center tap. If we consider the cut off voltage of Si diode, then

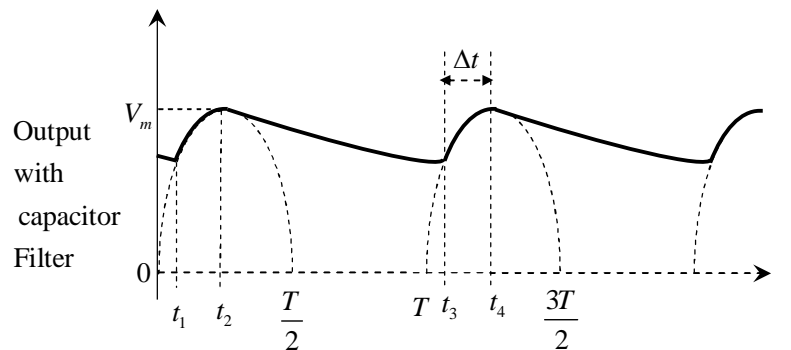
$$V_{dc} \cong 0.636(V'_m - 0.7)$$

The output frequency of the full wave signal is double the input frequency i.e.

$$f_{out} = 2f_{in}$$

### Half Wave Rectifier with shunt capacitor filter:

The outputs of a half wave and full wave rectifier have ripple components. The fluctuations of the output voltage across the load resistor is reduced by using filters. A shunt capacitor filter can be constructed by placing a capacitor C in shunt with the load resistor (figure 8.3).



**Figure 8.11**

The conduction of the diode and charging up of the capacitor continue from time  $t_1$  to  $t_2$  (figure 8.11). The charging continues until the input voltage reaches its peak value  $V_m$ . Thus the diode allows charge to flow into the capacitor when the input voltage (at the secondary side of the transformer) exceeds the capacitor voltage. After reaching the input voltage at its maximum value, it starts to decrease. When the input voltage falls below that of the capacitor, and diode gets nonconducting state and capacitor discharges through the load resistor from  $t_2$  to  $t_3$  (see figure 8.11). If  $R_L C$  time constant is much greater than the time period  $\Delta t$  (see figure 8.11), rate of discharging of capacitor is negligible and ripple becomes small. From  $t_3$  to  $t_4$  capacitor charges again and process is repeated for forthcoming input cycles. Here almost all the ripple components are bypassed by the capacitor. Usually electrolytic capacitor having large capacitances (of the order of  $\mu F$ ) are

used for this purpose. Full wave rectifier with capacitor filter can also be explained.

## 8.7 Self Learning Exercise-I

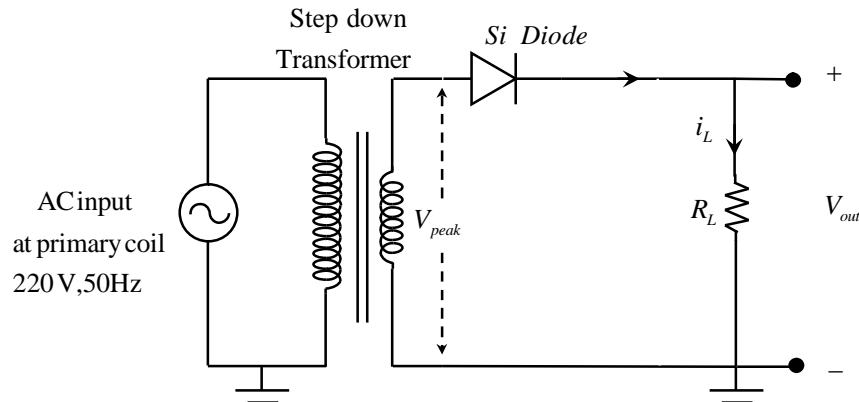
**Q.1** What is the theoretical value of ripple factor for an ideal full wave rectifier ?

**Q.2** Identify the polarity of the following capacitor for biasing.



**Q.3** What symbol is used for an oscilloscope ?

**Q.4** Consider the voltage drop across the Si diode for the given circuit and peak voltage across the secondary ends is  $V_{peak}$ . What is the peak value of the output voltage for given half wave rectifier ?



## 8.8 Procedure

### (i) Half Wave Rectifier Circuit without Filter

1. Make the connections as per circuit diagram 8.1 to implement the half wave rectifier.
2. Connect the circuit to an appropriate power source and turn it on.
3. Measure the ac voltage at the input side of the half wave rectifier (i.e. voltage at the secondary side of the transformer) by multimeter.
4. Measure the output dc voltage  $V_{dc}$  and ac voltage  $V_{ac}$  by connecting digital multimeter across the load resistor.
5. Trace the waveform of the input voltage (at the secondary side of the transformer) using CRO.



6. Trace the waveform of the output voltage at load resistor using CRO.
7. Vary the load resistance (say  $1k\Omega$  to  $10k\Omega$ ) and note down  $V_{dc}$  and  $V_{ac}$ , then trace the output waveforms with CRO. Note that Power supply should be turned off while connecting or disconnecting the components, wires etc.

**(ii) Full Wave Rectifier Circuit without Filter**

1. Make the connections as per circuit diagram 8.2 to implement the full wave rectifier and repeat the step 2 of part (i).
2. Measure the ac voltage at the input side of the full wave rectifier (i.e. at secondary side of transformer from one end to center tap).
3. Repeat the procedural steps from 4 to 7 of the part (i).

**(iii) Half Wave Rectifier Circuit with Shunt Capacitor Filter**

**(iii) A:** *Half wave rectifier circuit with shunt capacitor filter at a given value of shunt capacitance:-*

1. Make the connections as per circuit diagram 8.3 to implement the half wave rectifier with shunt capacitor filter at given value of capacitance.
2. Repeat the procedural step 2 and from 4 to 7 of the part (i) at the given value of shunt capacitance.

**(iii) B:** *Half wave rectifier circuit with shunt capacitor filter at a given value of load resistance:-*

1. Make the connections as per circuit diagram 8.3 to implement the half wave rectifier with shunt capacitor filter at given value of load resistance.
2. Repeat the procedural steps 2, 4, 5 and 6 of the part (i)
3. Note down  $V_{dc}$  and  $V_{ac}$  across the load, then trace the output waveforms with CRO.
4. Repeat the above step 3 [of part (iii)B] for different values of shunt capacitance.

**(iv) Full Wave Rectifier Circuit with Shunt Capacitor Filter**

**(iv)A:** *Full wave rectifier circuit with shunt capacitor filter at a given value of shunt capacitance:-*

1. Make the connections as per circuit diagram 8.4 to implement the full wave rectifier with shunt capacitor filter at given value of capacitance.
2. Repeat the procedural steps 2 and from 4 to 7 of the part (i) at the given value

of shunt capacitance.

**(iv)B:** Full wave rectifier circuit with shunt capacitor filter at a given value of load resistance:-

1. Make the connections as per circuit diagram 8.4 to implement the full wave rectifier with shunt capacitor filter at given value of load resistance.
2. Repeat the procedural steps 2 ,4,5 and 6 of the part (i).
3. Note down  $V_{dc}$  and  $V_{ac}$  across the load ,then trace the output waveforms with CRO.
4. Repeat the above step 3 [of part (iv)B] for different values of shunt capacitance.

## 8.9 Observations

Here  $(V_{ac})_{in} = (V_{rms})_{in}$  = Input ac voltage to half wave (at the secondary side of the transformer).

$(V'_{ac})_{in} = (V'_{rms})_{in}$  = Input ac voltage to full wave rectifier (secondary voltage from the one end to the center tap).

$r$  = Ripple factor

$T_r$  = Time period of ripple output voltage at load (by CRO)

[Symbol abbreviations used in tables are given in section 8.4 (Formula)]

### (i) Half Wave Rectifier without Filter

S. No.	Input voltage (Volts)		$R_L$ ( $k\Omega$ )	Output Voltage (Volts)			$r$		$f_r$ (Hz)
	$(V_{ac})_{in}$	$V_m$		dc value		ac value	$r_t = 1.21$	$r_p = \frac{V_{ac}}{V_{dc}}$	$f_r = \frac{1}{T_r}$
				$(V_{dc})_T = \frac{V_m}{\pi}$	$V_{dc}$	$V_{ac}$			
1.									
2.									
3.									
4.									
5.									

**(ii) Full Wave Rectifier without Filter**

S. No.	Input voltage (Volts)		$R_L$ ( $k\Omega$ )	Output Voltage (Volts)			$r$		$f_r$ (Hz)
	$(V'_{ac})_{in}$	$V'_m$		dc value		ac value	$r_t = 0.48$	$r_p = \frac{V_{ac}}{V_{dc}}$	
				$(V_{dc})_T = \frac{2V'_m}{\pi}$	$V_{dc}$	$V_{ac}$			
1.									
2.									
3.									
4.									
5.									

**(iii) Half Wave Rectifier with Capacitor Filter**

Using CRO, trace for any fixed value of the input ac voltage ( at the secondary side of transformer) :

Time period of the input ac voltage  $T = \dots\dots\dots$ s

Frequency of the input ac voltage  $f = \dots\dots\dots$  Hz

**Observation at fixed value of capacitance:**

Capacitance  $C = \dots\dots\dots\mu F$

S. No.	$R_L$ ( $k\Omega$ )	Output Voltage (Volts)		$r$	
		$V_{dc}$	$V_{ac}$	$r_t = \frac{1}{2\sqrt{3}} \frac{1}{f R_L C}$	$r_p = \frac{V_{ac}}{V_{dc}}$
1.					
2.					
3.					
4.					
5.					

**Observation at fixed value of load Resistance:**

Load Resistance  $R_L = \dots\dots\dots k\Omega$

S. No.	C ( $\mu F$ )	Output Voltage (Volts)		r	
		$V_{dc}$	$V_{ac}$	$r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C}$	$r_p = \frac{V_{ac}}{V_{dc}}$
1.					
2.					
3.					
4.					
5.					

**(iv) Full Wave Rectifier with Capacitor Filter**

Using CRO Trace for any fixed value of the input ac voltage at the secondary side of transformer :

Time period of the input ac voltage  $T = \dots\dots\dots s$

Frequency of the input ac voltage  $f = \dots\dots\dots Hz$

**Observation at fixed value of capacitance:**

Capacitance  $C = \dots\dots\dots \mu F$

S. No.	$R_L$ ( $k\Omega$ )	Output Voltage (Volts)		r	
		$V_{dc}$	$V_{ac}$	$r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C}$	$r_p = \frac{V_{ac}}{V_{dc}}$
1.					
2.					
3.					
4.					
5.					

**Observation at fixed value of Load Resistance:**

Load Resistance  $R_L = \dots\dots\dots k\Omega$

S. No.	C ( $\mu F$ )	Output Voltage (Volts)		r	
		$V_{dc}$	$V_{ac}$	$r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C}$	$r_p = \frac{V_{ac}}{V_{dc}}$
1.					
2.					
3.					
4.					
5.					

**8.10 Graph**

Obtain the required traces using CRO.

***For Half wave rectifier with shunt capacitor filter:***

Plot the graph between ripple factor  $r$  (practical value) and load resistance  $R_L$  at the given value of shunt capacitance.

Plot the graph between ripple factor  $r$  (practical value) and shunt capacitance  $C$  at the given value of load resistance.

***For Full wave rectifier with shunt capacitor filter:***

Plot the graph between ripple factor  $r$  (practical value) and load resistance  $R_L$  at the given value of shunt capacitance.

Plot the graph between ripple factor  $r$  (practical value) and shunt capacitance  $C$  at the given value of load resistance.

**8.11 Calculations**

**Half Wave Rectifier with Capacitor Filter:-**

***At fixed value of capacitance:***

$$R_L = \dots\dots\dots, r_t = \frac{1}{2\sqrt{3}} \frac{1}{f R_L C} = \frac{0.289}{f R_L C} = \dots\dots\dots$$

**At fixed value of Load resistance:**

$$C = \dots\dots\dots, r_t = \frac{1}{2\sqrt{3}} \frac{1}{f R_L C} = \frac{0.289}{f R_L C} = \dots\dots\dots$$

**Full Wave Rectifier with Capacitor Filter:-**

**At fixed value of capacitance:**

$$R_L = \dots\dots\dots, r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C} = \frac{0.144}{f R_L C} = \dots\dots\dots$$

**At fixed value of Load resistance:**

$$C = \dots\dots\dots, r_t = \frac{1}{4\sqrt{3}} \frac{1}{f R_L C} = \frac{0.144}{f R_L C} = \dots\dots\dots$$

## 8.12 Result

1. Half wave rectifier without filter has been implemented.

Theoretical value of ripple factor=1.21

Theoretical and practical values of ripple factor are almost equal.

Obtained traces are consistent with theory.

- 2. ....
- 3. ....
- 4. ....

## 8.13 Discussion

.....  
.....  
.....

## 8.14 Precautions and Sources of Error

1. Circuit components (diodes, capacitors etc.) must be connected with correct polarities otherwise diodes ,capacitors may burn up. **Be careful about**

***identification of the primary and secondary sides of the transformer because at the primary side voltage is about 220 Volts which is dangerous.***

2. Current limiting series resistor should be used to prevent from damaging of components.
3. Power supply should be turned off while connecting or disconnecting the components , wires etc.
4. Circuit should be assembled in systematic and arranged manner because improper connections cause overheating of diodes due to short circuit. While taking observation with multimeter /CRO terminals should be connected properly.
5. Jumper wires used for experiment should not be in broken state. Passing of jumping wires over the circuit components and unnecessary twisting of wires should be avoided. Arrangement of components (diodes, resistors, capacitors etc.) on the bread board should be such that short wire connections are needed. Connections should be intact.
6. The ratings of the diodes , capacitors etc. must be appropriate. The peak voltage at the secondary of the transformer (i.e. at the input end of the rectifier ) must be sufficiently lower than the reverse breakdown voltage of the diode.
7. Ranges and polarities of components must be appropriate.

### **8.15 Self Learning Exercise-II**

- Q.1 What is the cut in voltage for Ge diode ?
- Q.2 What is the maximum efficiency of a half wave rectifier?
- Q.3 "Ripple factor may be decreased by increasing C in case of half wave rectifier with shunt capacitor filter". Do you agree with this statement ?
- Q.4 "For a good filter ripple factor should be high ."Is this statement true?
- Q.5 What do you mean PIV ?

### **8.16 Glossary**

**CRO:** Cathode Ray Oscilloscope

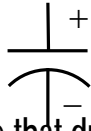
**PIV:** Peak Inverse Voltage

## 8.17 Answers to Self Learning Exercises

### ***Answers to Self Learning Exercise-I***

**Ans.1:** 0.48

**Ans.2:**



Note that due to incorrect polarity, capacitor may explode.

**Ans.3:** 

**Ans.4:**  $(V_{peak} - 0.7)V$  Here 0.7V is the voltage drop across the diode while the diode is conducting.

### ***Answers to Self Learning Exercise-II***

**Ans.1:** 0.3V

**Ans.2:** 40.6%

**Ans.3:** Yes, ripple factor  $r = \frac{1}{2\sqrt{3}} \frac{1}{f R_L C}$

**Ans.4:** No

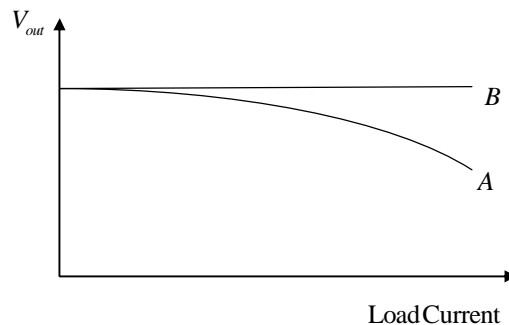
**Ans.5:** The PIV (Peak inverse voltage) is the maximum value of the voltage that the diode can withstand before breakdown when it is reverse biased.

## 8.18 Viva Questions

**Q.1** For a good filter whether circuit should be L section filter or  $\pi$  section filter ?

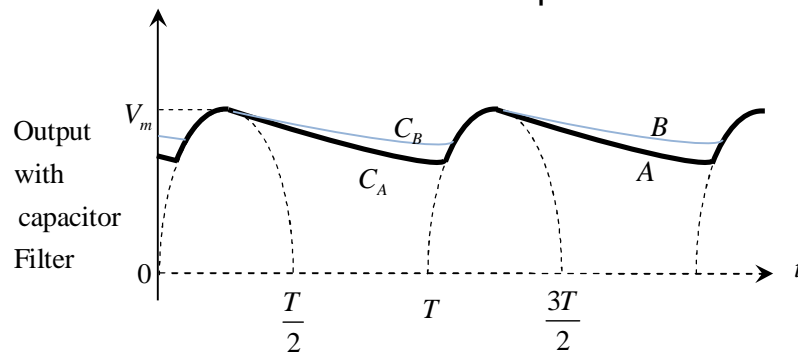
**Q.2** What is the DC output voltage for half wave rectifier if the peak voltage at the input end of the rectifier is  $V_p$  ?

**Q.3** Which curve represents the poor voltage source ?

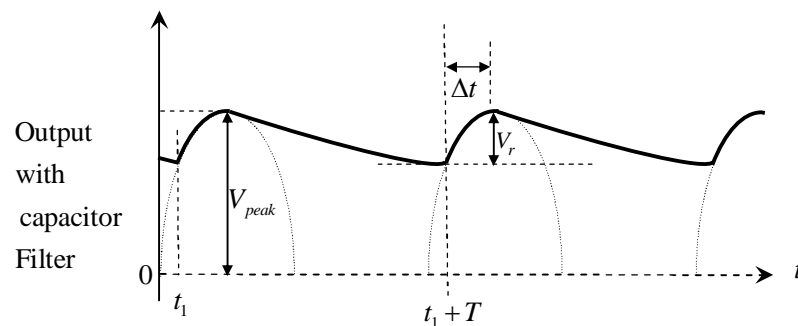




- Q.4** Explain the operation of a half wave rectifier with shunt capacitor filter.
- Q.5** Draw the circuit diagram of L -Section filter for half wave rectifier.
- Q.6** Explain the working of full wave rectifier.
- Q.7** Draw the circuit diagram of  $\pi$  section filter for full wave rectifier.
- Q.8** Draw the circuit diagram of a full wave bridge rectifier and draw its output waveforms.
- Q.9** What are the main advantages and disadvantages of bridge rectifier over the center tapped rectifier?
- Q.10** Consider output waveforms of a half wave rectifier with shunt capacitor filter where two different values of shunt capacitances are used one by one.
- (i) Which of the following circuit has low ripple?
- (ii) Which of the circuit has low value of Capacitance C.



- Q.11** What is the approximate DC value of the following pulsating output voltage waveform of a half wave rectifier with C filter .Assume that time constant  $R_L C$  is much greater than  $\Delta t$ .



- Q.12** Compare the half wave rectifier, full wave center tapped rectifier and full wave bridge rectifier on parameters such as number of diodes, ripple factor,

output ripple frequency and output peak voltage. Consider here  $V_m$  is the peak value of the input voltage at the secondary side of the transformer from its one end to another end, frequency of input voltage is  $f_{in}$  and voltage drop across the Si diode in conducting state is 0.7V.

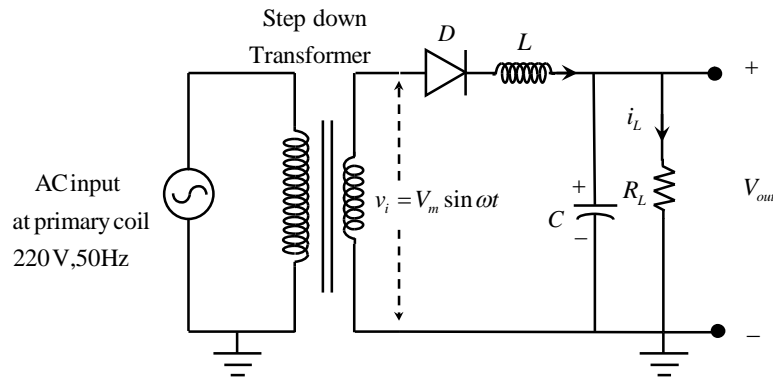
## 8.19 Answers to Viva Questions

**Ans.1:**  $\pi$  section filter

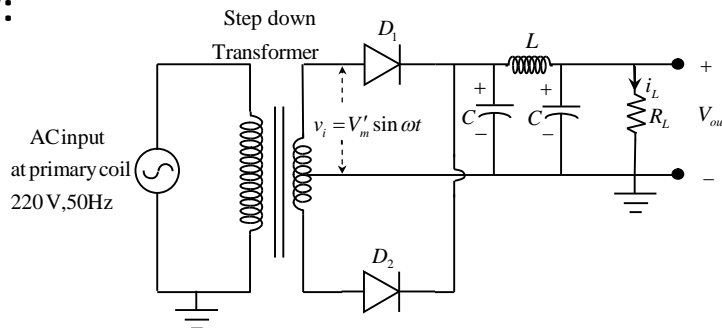
**Ans.2:**  $V_{dc} = \frac{V_p}{\pi} = 0.318V_p$

**Ans.3:** The curve A represents the poor voltage source as output at the source begins to drop as current is drawn .

**Ans.5:**

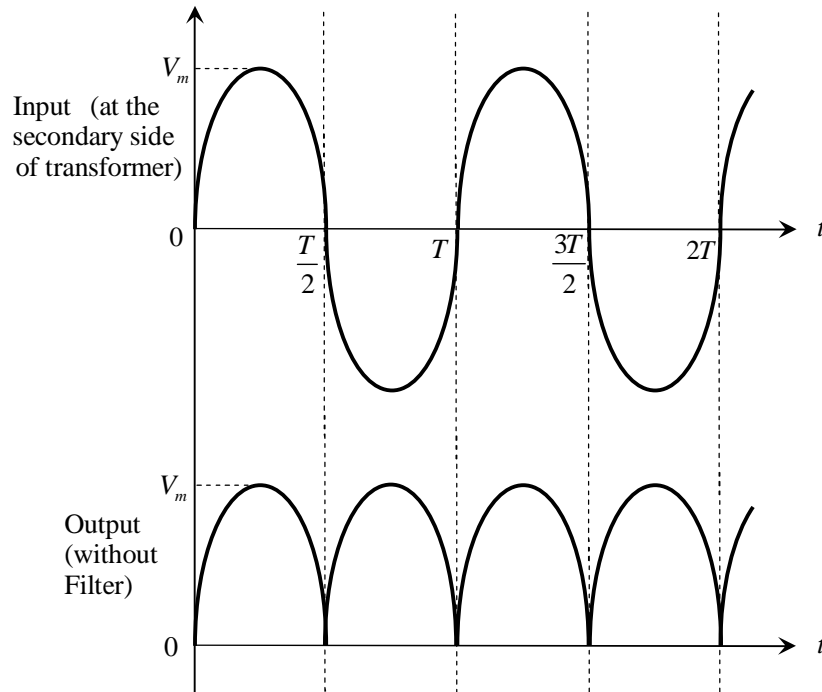
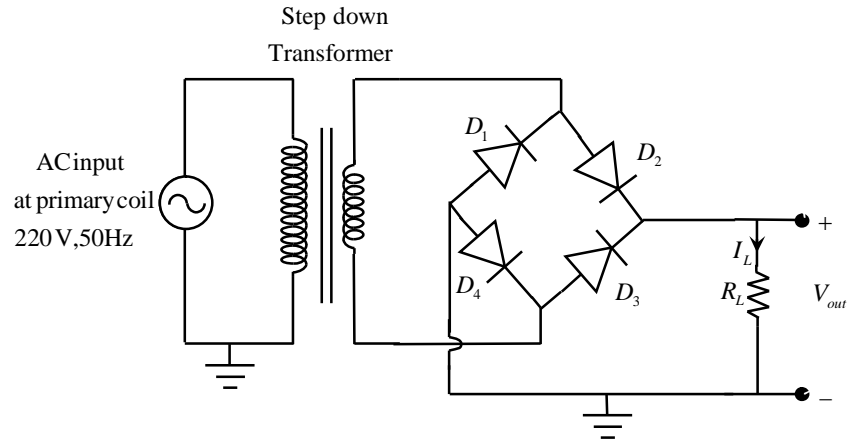


**Ans.7:**



**Ans.8:** DC output voltage across the Load resistor ( i.e. ideal theoretical value of calculated dc output voltage) =  $(V_{dc})_T = \frac{2V_m}{\pi}$  (in volt)

where  $V_m = \sqrt{2}(V_{rms})_{in}$  is the peak value of the input voltage to rectifier at the secondary side of the transformer.



Note that input to rectifier (at the secondary side of transformer) and output to the rectifier (at the load) do not have common ground point.

**Ans.9: Main advantages:** In case of Bridge rectifier, the output voltage is twice that of the center tapped rectifier for the given same value of secondary voltage. Also peak inverse voltage is one half in comparison with center tapped.

**Main disadvantages:**

Bridge rectifier requires four diodes and there is forward voltage drop across two diodes. This forward voltage drop across the diodes cannot be neglected particularly at low voltages such as below 5 Volts.

**Ans.10:** (i) Circuit B having shunt capacitance  $C_B$  has low ripple.  
(ii) Circuit A having shunt capacitance  $C_A$ .

**Ans.11:**  $V_{dc \text{ Output}} \cong V_{peak \text{ output}} - 0.5V_r$

where  $V_r \cong \frac{V_{peak \text{ output}}}{f R_L C}$  and  $f = \frac{1}{T}$  is the frequency of the input voltage.

Note that in case of full wave rectifier  $V_r \cong \frac{V_{peak \text{ output}}}{2f R_L C}$

**Ans.12:**

Type of rectifier	No. of diodes required	Ripple factor	Output ripple frequency	Peak Output Voltage(volt)
Half Wave	1	1.21	$f_{in}$	$V_m - 0.7$
Full Wave Center Tapped	2	0.48	$2f_{in}$	$\frac{V_m}{2} - 0.7$
Full Wave Bridge	4	0.48	$2f_{in}$	$V_m - 1.4$

## References and Suggested Readings

1. Albert Paul Malvino, Electronic Principle (Sixth Edition, 1999), Tata McGraw Hill.
2. Robert L. Boylestad & Louis Nashelsky, Electronic Devices and Circuit Theory (Tenth Edition 2009), Prentice-Hall India.
3. Jacob Millman & Christos C. Halkias, Integrated Electronics (Second Edition 2011), Tata McGraw Hill.
4. Gupta & Kumar, Handbook of Electronics (Twenty Seventh Edition 2000), Pragati Prakashan.
5. <http://www.canton.edu/employee/hartle/elec131/lab3.pdf>
6. D. Chattopadhyay, P.C. Rakshit, B. Saha & N.N Purkait, Foundation of Electronics, Wiley Eastern Limited.

# UNIT-9

## Solving Boolean Expressions

### Structure of the Unit

- 9.1 Aim
  - 9.2 Apparatus
  - 9.3 Diagram
  - 9.4 Formula (Truth Table)
  - 9.5 Theory and Description
  - 9.6 Self Learning Exercise-I
  - 9.7 Procedure
  - 9.8 Observations
  - 9.9 Result
  - 9.10 Discussion
  - 9.11 Precautions and Sources of Error
  - 9.12 Self Learning Exercise-II
  - 9.13 Glossary
  - 9.14 Answers to Self Learning Exercises
  - 9.15 Viva Questions
  - 9.16 Answers to Viva Questions
- References and Suggested Readings

### 9.1 Aim

- (A) Solve the following Boolean expression into simplified form and realize the logic circuit

$$R = \overline{(\bar{x} + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + y + z)}$$

- (B) Realize the logic circuit from following truth table

x	y	z	Output R
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(C) For the following K-map construct the truth table and realize the simplified logic expression .

	yz			
x	00	01	11	10
0				1
1	1			1

(D) Realize the logic diagram for Boolean function

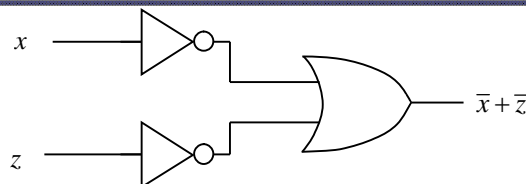
$$Y = (\bar{A} + B)(\bar{B} + C)(\bar{C} + D) \text{ by NOR Gates only.}$$

## 9.2 Apparatus

Power supply, Digital logic trainer, connecting leads [or Bread board, DC supply of 5 Volts ,connecting wires, LEDs, IC7408 (AND), IC 7404 (NOT), IC 7432 (OR) , IC 7400 (NAND) , IC 7402 (NOR) as required ]

## 9.3 Diagram

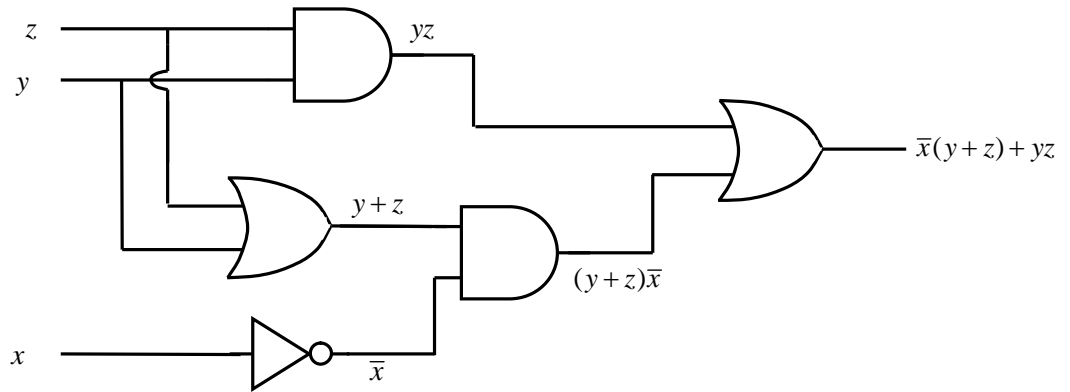
(A) Diagram:



**Figure 9.1**

$$R = (\bar{x} + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + y + z) = \bar{x} + \bar{z}$$

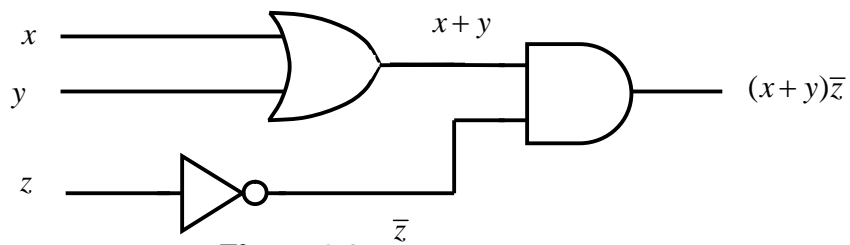
**(B) Diagram:**



**Figure 9.2**

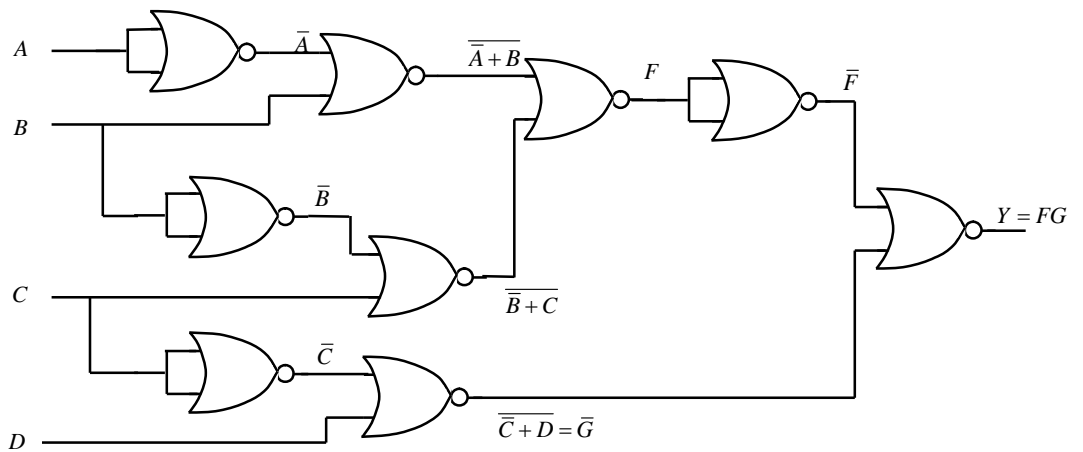
$$R = \bar{x}y + \bar{x}z + yz = \bar{x}(y+z) + yz$$

**(C) Diagram:**



**Figure 9.3**

**(D) Diagram:**



**Figure 9.4**

## 9.4 Formula/Truth Table

**(A)** From DeMorgan's Theorem

$$\begin{aligned}
 R &= \overline{(\bar{x} + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + y + z)} \\
 &= \overline{(\bar{x} + \bar{y} + z)} + \overline{(x + \bar{y} + \bar{z})} + \overline{(\bar{x} + y + z)} + \overline{(x + \bar{y} + z)} + \overline{(x + y + \bar{z})} + \overline{(x + y + z)} \\
 &= (\bar{\bar{x}} \cdot \bar{\bar{y}} \cdot \bar{\bar{z}}) + (\bar{x} \cdot \bar{\bar{y}} \cdot \bar{\bar{z}}) + (\bar{\bar{x}} \cdot \bar{y} \cdot \bar{z}) + (\bar{x} \cdot \bar{\bar{y}} \cdot \bar{z}) + (\bar{\bar{x}} \cdot \bar{y} \cdot \bar{\bar{z}}) + (\bar{x} \cdot \bar{y} \cdot \bar{z}) \\
 &= (xy\bar{z}) + (\bar{x}yz) + (x\bar{y}\bar{z}) + (\bar{x}y\bar{z}) + (\bar{\bar{x}}\bar{y}\bar{\bar{z}}) + (\bar{x}\bar{y}\bar{z}) \\
 &= (x + \bar{x})y\bar{z} + \bar{x}z(y + \bar{y}) + \bar{y}\bar{z}(x + \bar{x}) \\
 &= y\bar{z} + \bar{x}z + \bar{y}\bar{z} \\
 &= (y + \bar{y})\bar{z} + \bar{x}z \\
 &= \bar{z} + \bar{x}z = \bar{z}(1 + \bar{x}) + \bar{x}z \\
 &= \bar{z} + \bar{z}\bar{x} + \bar{x}z \\
 R &= \bar{z} + \bar{x}
 \end{aligned}$$

Truth Table

x	z	Output $R = \bar{x} + \bar{z}$
0	0	1
0	1	1
1	0	1
1	1	0

**(B)** K-map is shown in diagram corresponding to table (of Aim B).

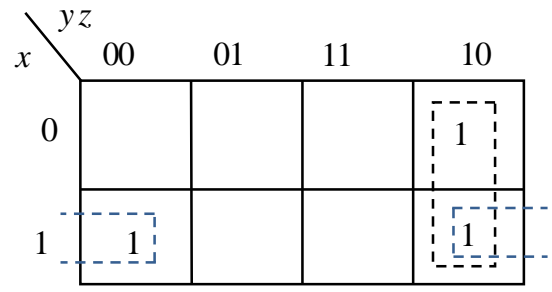
	$yz$	00	01	11	10
$x$	0		1	1	1
1				1	

Output  $R = yz + \bar{x}z + \bar{x}y$



(C) Truth Table (From th given K map of Aim C )

x	y	z	Output R
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



From K-map output  $R = y\bar{z} + x\bar{z}$

(D) Truth Table

A	B	C	D	Output Y $(\bar{A} + B)(\bar{B} + C)(\bar{C} + D)$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0

0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$Y = (\bar{A} + B)(\bar{B} + C)(\bar{C} + D)$$

$$\Rightarrow Y = \overline{\overline{(\bar{A} + B)(\bar{B} + C)(\bar{C} + D)}} \because \bar{\bar{x}} = x$$

Using De Morgan's theorem

$$Y = \overline{\overline{\bar{A} + B} + \overline{\overline{\bar{B} + C}} + \overline{\overline{\bar{C} + D}}}$$

$$Y = \overline{\overline{\bar{A} + B} + \overline{\overline{\bar{B} + C}} \cdot \overline{\overline{\bar{C} + D}}}$$

$$= F \cdot G$$

$$= \overline{\overline{F} + \overline{G}}$$

$$Y = \overline{\overline{F} + \overline{G}}$$

$$\left\{ \begin{array}{l} \text{Let } F = \overline{\overline{\bar{A} + B} + \overline{\overline{\bar{B} + C}}} \\ G = \overline{\overline{\bar{C} + D}} = \overline{\overline{\bar{C} + D}} \end{array} \right.$$

## 9.5 Theory and Description

The laws, theorems of Boolean Algebra and Karnaugh maps (K maps) are used for simplification of Boolean expressions. Circuit designers usually use a simplified logic expression for implementation of logic circuit. De Morgan's theorems are useful for simplification of logic expressions.

### De Morgan's Theorems

$$(i) \quad \overline{A + B} = \bar{A} \cdot \bar{B}$$

The complement of the sum of variables is equal to the product of the complements of the variables.

$$\overline{A + B + C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

$$\overline{A + B + C + \dots + F} = \bar{A} \cdot \bar{B} \cdot \bar{C} \dots \bar{F}$$

(ii)  $\boxed{A \cdot B = \bar{A} + \bar{B}}$

The complement of the product of variables is equal to sum of complements of the variables.

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{ABC\dots F} = \bar{A} + \bar{B} + \bar{C} + \dots + \bar{F}$$

There are integrated circuits (IC) for logic functions. The Institute of Electrical and Electronics Engineers (IEEE) along with American National Standards Institute (ANSI) provides the different types of symbols for logic gates.

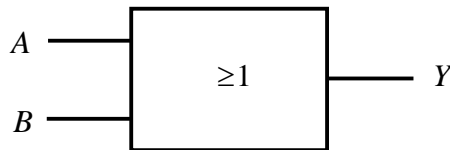
**IEEE/ANSI Standards:-**

In this convention, a rectangular symbol is used for all devices, For inversion operation, an small triangle is used instead of an small bubble.



NOT Gate : IEEE/ANSI Standard

The symbol '1' inside the block means that NOT Gate has only one input.



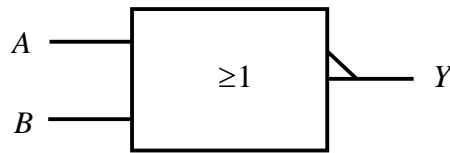
OR Gate : IEEE/ANSI Standard

The symbol  $\geq 1$  inside the block means that output is 1 when one or more than one input is 1.



AND Gate : IEEE/ANSI Standard

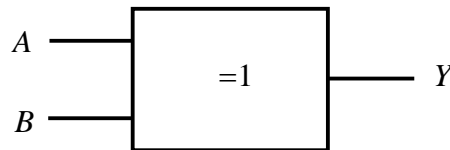
The symbol '&' means that output is 1 when all the inputs are 1.



NOR Gate : IEEE/ANSI Standard

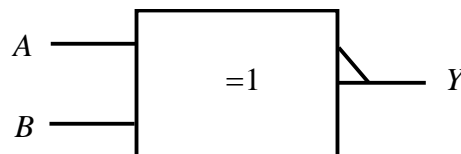


NAND Gate : IEEE/ANSI Standard



XOR Gate : IEEE/ANSI Standard

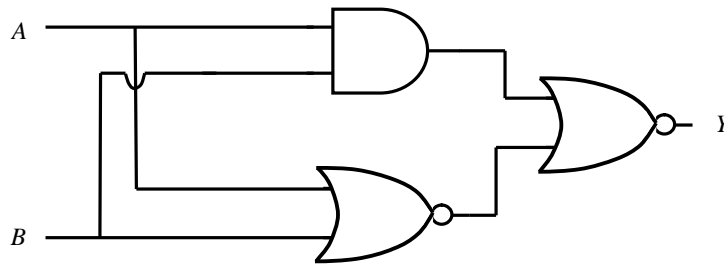
Symbol = 1 indicates that the output is 1 only when one of its input is 1



XNOR Gate : IEEE/ANSI Standard

## 9.6 Self Learning Exercise-I

- Q.1** How many row entries would a six variable logic function require ?
- Q.2** Show that  $CD + \overline{CD}F = CD + F$
- Q.3** Realize a digital circuit for logic operation  $Y = \overline{A}B + \overline{A}.\overline{B} + \overline{A+B}$  . Also write the truth table.
- Q.4** Implement the NAND function using NOR Gates.
- Q.5** State the expression for Y in terms of input variables for given logic diagram.



## 9.7 Procedure

(A) For implementation of the Boolean expression

$R = \overline{(\bar{x} + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + y + z)}$  into simplified form and realization the logic circuit :

### For Trainer board Arrangement

1. Obtain the simplified form of the given Boolean function by Boolean algebra or K map
2. Make the connections as per the circuit diagram.
3. Connect the circuit to an appropriate power source and turn it on.
4. Observe the output on LED for each input combination.
5. Write down the corresponding logic level in the observation table.

### For Breadboard Arrangement

1. Obtain the simplified form of the given Boolean function by Boolean algebra or K map.
2. Insert ICs 7404 (Hex inverter Gate) and IC 7432 (Quad 2 input OR Gate) in the breadboard.
3. Before you wire the circuit for logic gate, we should label the associated Chip Pin Number on the circuit diagram.
4. Assemble the connection (using components, connecting wires etc.) to implement the circuit illustrated in figure 9.1
5. Connect output to LED with correct polarity and connect a series current limiting resistor (about  $270\Omega$ ) to LED to limit the current and prevent burn out.
6. For IC 7404 (14 pin IC) and IC 7432 (14pin IC), Pin 7 corresponds to ground

(0V) and Pin 14 corresponds to  $V_{cc}$  (+5V DC supply).

7. Turn on the power and observe output at LED indicator corresponding to all possible combinations of inputs (via input switches). Write down corresponding outputs in the observation table and verify the truth table.
8. If there is any mismatched result, troubleshoot result.
9. Turn off the power and disconnect the circuit connections.

For other parts of experiment, use appropriate circuit diagrams (appropriate ICs, etc.) to demonstrate remaining experiments of this unit.

## 9.8 Observation

### (A) Observation:

Truth Table (Figure 9.1)

x	z	Output $R = \bar{x} + \bar{z}$

### (B) Observation:

Truth Table (Figure 9.2)

x	y	z	Output R

**(C) Observation:**

Truth Table (Figure 9.3)

x	y	z	Output R

**(D) Observation:**

Truth Table (Figure 9.4)

A	B	C	D	Output Y $(\bar{A} + B)(\bar{B} + C)(\bar{C} + D)$


**9.9 Result**

(A) The simplified form of the Boolean function

$R = (\bar{x} + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + y + z)$  is  $R = \bar{x} + \bar{z}$ .

Logic circuit of the Boolean expression  $R = \bar{x} + \bar{z}$  has been implemented and truth table has been verified.

- (B) .....
- (C).....
- (D).....

**9.10 Discussion**

.....  
 .....  
 .....  
 .....

**9.11 Precautions and Sources of Errors**

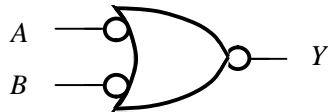
1. Circuit components (LED etc.), power supply must be connected with correct polarities.
2. DC supply voltage must not exceed the value of 5 Volts otherwise it will damage the ICs used during the experiment.
3. LED indicator should be used with series current limiting resistor.
4. Power supply should be turned off while connecting or disconnecting the components, wires etc.
5. Insertion and removal of the ICs should be done gently to avoid the bending of the pins of ICs.



6. Arrangement of IC chips on the breadboard should be such that short wire connections are needed.
7. Circuit should be assembled in systematic and organized fashion because improper connections cause overheating of ICs due to short circuit.
8. Jumper wires used for experiment should not be in broken state.
9. All connection should be tight before switching on the power supply. Avoid using long wires.

## 9.12 Self Learning Exercise-II

**Q.1** Identify the operation of following logic circuit



**Q.2** What is the final simplified expression for following Boolean expression

$$A(B + \overline{AB}) + \overline{ABC} + \overline{AB}$$

**Q.3** Realize the logic diagram corresponding to the following Boolean expression without simplifying them. Use only "two input AND Gates, two input OR Gates, and NOT Gates".

$$Y = (\overline{A} + \overline{B})(ACD)(\overline{CD} + \overline{CD})$$

**Q.4** Simply the following expression

$$R = xyz + x\overline{y}z + \overline{x}yz + \overline{x}\overline{y}z$$

## 9.13 Glossary

**LED** : Light Emitting Diode

**IC**: Integrated Circuit

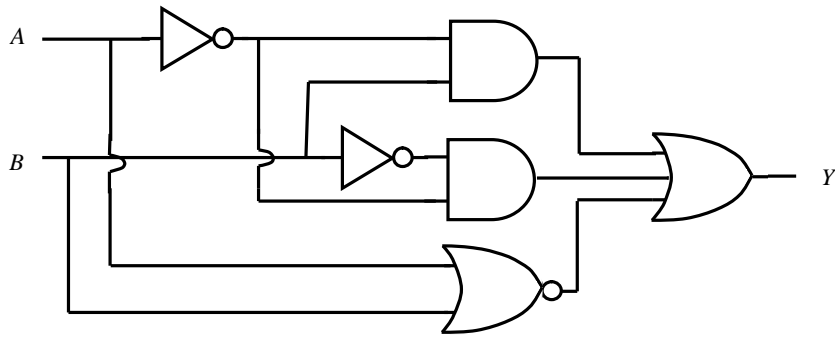
## 9.14 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:**  $2^6 = 64$

**Ans.2:** Use Boolean algebra rule  $A + \overline{A}B = A + B$

**Ans.3:**

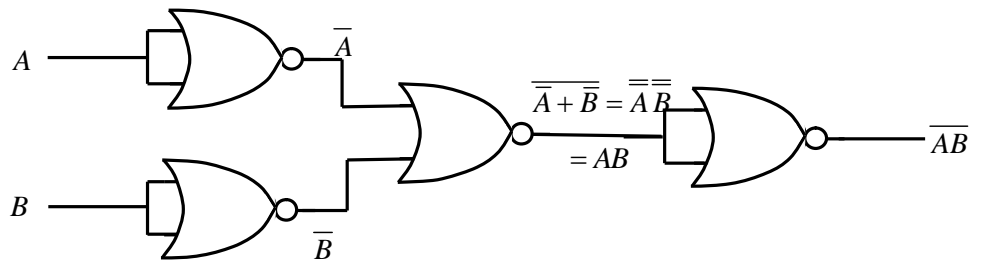


Here we can also write

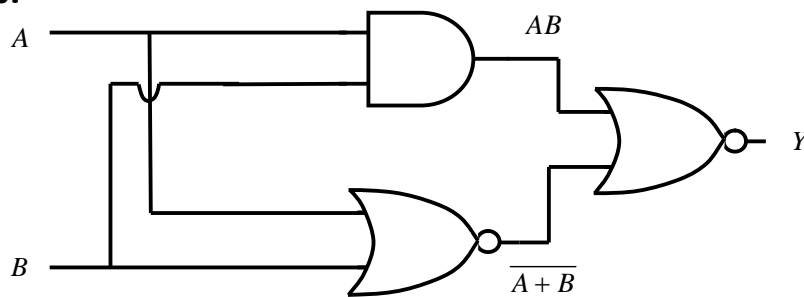
$$\begin{aligned}
 Y &= \bar{A} B + \bar{A} \bar{B} + \overline{A+B} = \bar{A} B + \bar{A} \bar{B} + \bar{A} \bar{B} \\
 &= \bar{A} B + \bar{A} \bar{B} = \bar{A} (B + \bar{B}) = \bar{A} \cdot 1 = \bar{A}
 \end{aligned}$$

Inputs		Output
A	B	$Y = \bar{A}$
0	0	1
0	1	1
1	0	0
1	1	0

**Ans.4:**



**Ans.5:**



$$Y = \overline{A}B + \overline{A+B} = \overline{A}B + \overline{A+B}$$

$$Y = \overline{A}B(A+B) = A \oplus B$$

### Answers to Self Learning Exercise-II

**Ans.1:**  $Y = \overline{A+B} = \overline{A} \overline{B} = AB$  i.e. AND Gate

**Ans.2:**  $A(B + \overline{A}B) + \overline{A}BC + \overline{A}B$

$$= AB + A\overline{A}B + \overline{A}(BC + \overline{B})$$

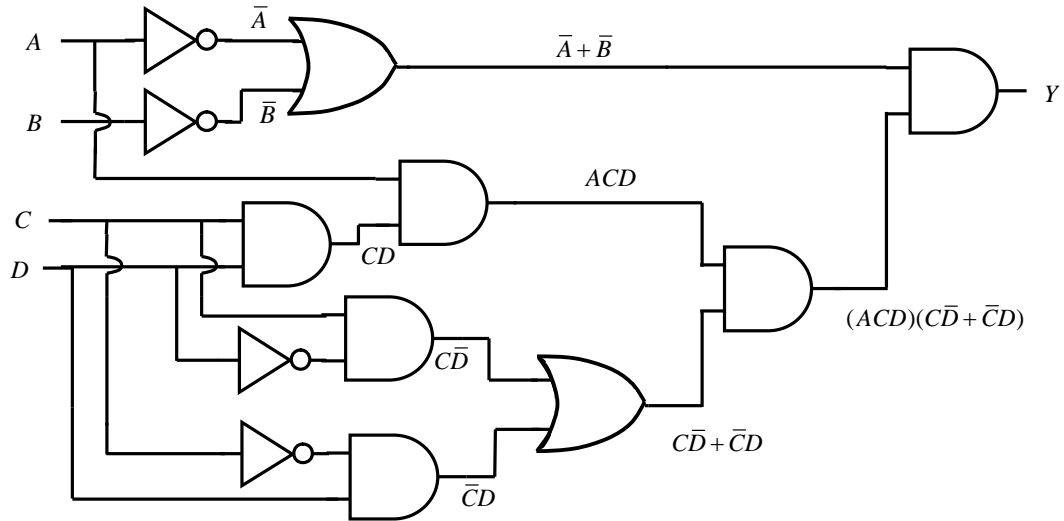
$$= AB + A\overline{B} + \overline{A} + \overline{A}(BC + \overline{B})$$

$$= A(B + \overline{B}) + \overline{A} + \overline{A}BC + \overline{A}\overline{B}$$

$$= A.1 + \overline{A} + \overline{A}BC + \overline{A}\overline{B}$$

$$= 1 + \overline{A}BC + \overline{A}\overline{B} = 1$$

**Ans.3:**  $Y = (\overline{A} + \overline{B})(ACD)(\overline{C}D + \overline{C}\overline{D})$



**Ans.4:** K-map is given below

Output  $R = z$

	$yz$	00	01	11	10
$x$	0		1	1	
1			1	1	

## 9.15 Viva Questions

**Q.1** Logic state 1 in a positive logic system corresponds to

- (A) Zero Voltage
- (B) Positive Voltage
- (C) Relatively higher voltage level
- (D) Relatively lower voltage level

**Q.2** Show that  $A \oplus B = \overline{\overline{A} \overline{AB}} \overline{\overline{AB} B}$

**Q.3** Apply De Morgan's theorem on following Boolean Expression

$$\overline{\overline{\overline{A} B C D}}$$

**Q.4** Simplify the following Boolean expression  $Y = A' B C + A B C + A' B$

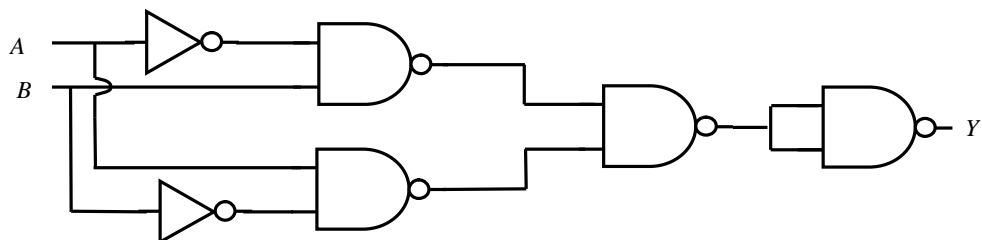
**Q.5** Simplify the following expression

$$x y \bar{z} \bar{t} + x y z \bar{t} + \bar{x} y z \bar{t} + \bar{x} y \bar{z} \bar{t}$$

**Q.6** Realize the logic diagram corresponding to the following Boolean expressions without simplifying them. Use only two input AND Gates, two input OR Gates, and NOT Gates.

$$Y = (AB + C) (\bar{C} + A\bar{B})$$

**Q.7** For given logic diagram, express the output Y in terms of input variables A and B.



**Q.8** Draw the Pinout diagram of IC 7400 quad 2 input NAND Gate

**Q.9** Write the simplified expression for following K-map and implement the logic circuit.

		yz			
		00	01	11	10
x	0	1			
	1	1	1		

**Q.10** Write the main logic gate IC Pin out numbers.

## 9.16 Answers to Viva Questions

**Ans.1:** (C) Relatively higher voltage level

$$\begin{aligned}
 \text{Ans.2: } \overline{\overline{A} \overline{AB}} \overline{\overline{AB} \overline{B}} &= \overline{\overline{A} \overline{AB}} + \overline{\overline{AB} \overline{B}} \\
 &= \overline{A \overline{AB}} + \overline{AB \overline{B}} \\
 &= (A + B)(\overline{AB}) = A \oplus B
 \end{aligned}$$

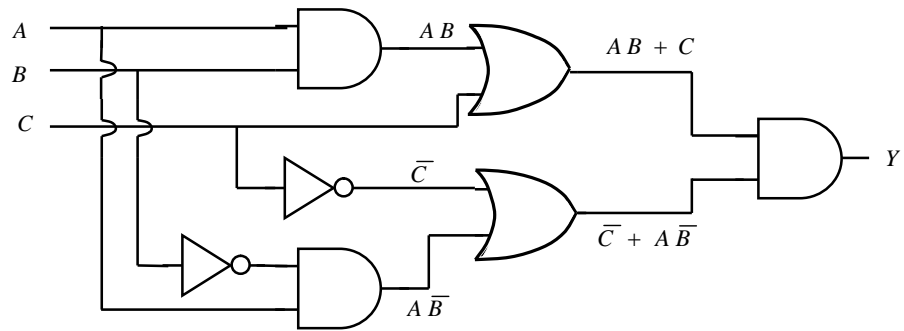
$$\begin{aligned}
 \text{Ans.3: } \overline{\overline{ABCD}} &= \overline{(\overline{A + B}) C D} \\
 &= \overline{(A + \overline{B}) C D} \\
 &= \overline{(A + \overline{B} + \overline{C}) D} \\
 &= \overline{(\overline{A} \cdot \overline{B} + \overline{C}) D} \\
 &= \overline{(\overline{A} \overline{B} + \overline{C}) D} \\
 &= \overline{(\overline{A} \overline{B} + \overline{C})} + \overline{D} \\
 &= \overline{\overline{A} \overline{B} \overline{C}} + \overline{D} \\
 &= \overline{(\overline{A} + \overline{B}) C} + \overline{D} \\
 &= (A + \overline{B}) C + \overline{D} \\
 &= AC + \overline{B} C + \overline{D}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ans.4: } Y &= \overline{A} B C + A B C + \overline{A} B \\
 &= B C (\overline{A} + A) + \overline{A} B \\
 &= B C + \overline{A} B
 \end{aligned}$$

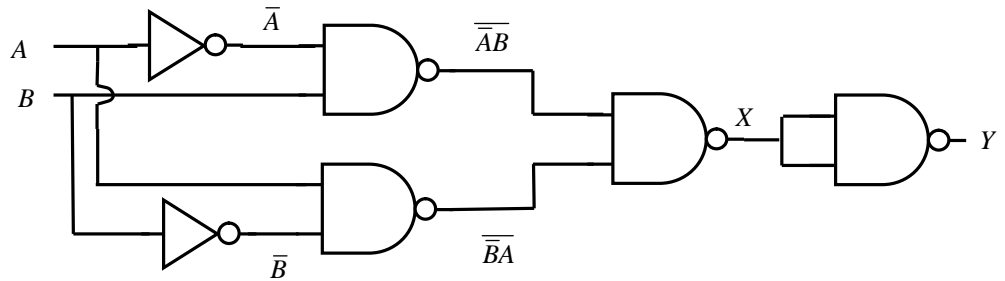
$$\begin{aligned}
 \text{Ans.5: } R &= x y \overline{z} \overline{t} + x y z \overline{t} + \overline{x} y z \overline{t} + \overline{x} y \overline{z} \overline{t} \\
 &= x y \overline{t} (z + \overline{z}) + \overline{x} y \overline{t} (z + \overline{z}) \\
 &= x y \overline{t} + \overline{x} y \overline{t} \\
 &= y \overline{t} (x + \overline{x}) = y \overline{t}
 \end{aligned}$$

**Ans.6:**

$$Y = (AB + C)(\overline{C} + \overline{AB})$$

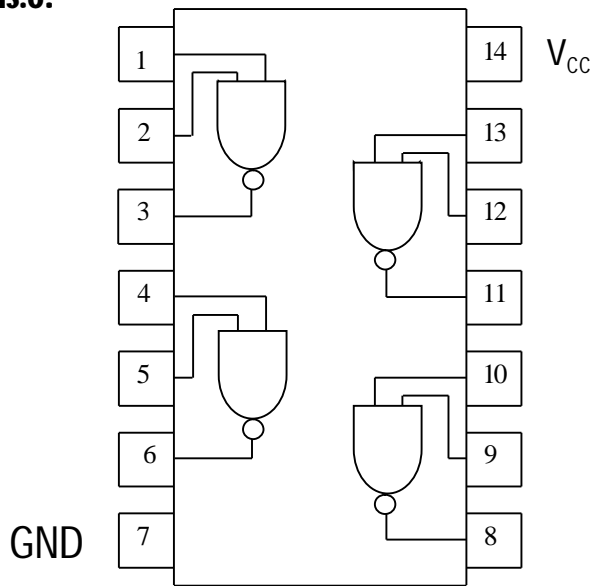


**Ans.7:**



Here  $X = \overline{A} B \quad A \overline{B}$  &  $Y = \overline{A} B \quad A \overline{B} = (A + \overline{B})(\overline{A} + B) = AB + \overline{A} \overline{B}$

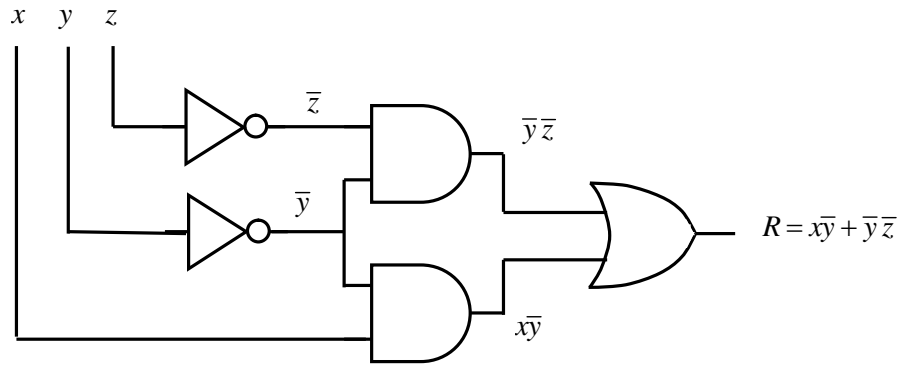
**Ans.8:**



**Ans.9:**

x \ yz	00	01	11	10
0	1			
1	1	1		

$$R = x \overline{y} + \overline{y} \overline{z}$$



**Ans.10:** Some of available IC Gates

IC No.	Description
7400	Quad 2 input NAND Gate
7402	Quad 2 input NOR Gate
7404	Hex inverter Gate
7408	Quad 2 input AND Gate
7432	Quad 2 input OR Gate
7486	Quad XOR Gate

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# UNIT-10

## Half Adder & Full Adder

### Structure of the Unit

#### (A) Half Adder

- 10.1A Aim
- 10.2A Apparatus
- 10.3A Diagram
- 10.4A Formula (Truth Table)
- 10.5A Theory and Description
- 10.6A Procedure
- 10.7A Observations
- 10.8A Result
- 10.9A Discussion
- 10.10A Precautions and Sources of Error
- 10.11A Self Learning Exercise-I

#### (B) Full Adder

- 10.1B Aim
- 10.2B Apparatus
- 10.3B Diagram
- 10.4B Formula (Truth Table)
- 10.5B Theory and Description
- 10.6B Procedure
- 10.7B Observations
- 10.8B Result
- 10.9B Discussion
- 10.10B Precautions and Sources of Error



- 10.11B Self Learning Exercise-II
  - 10.12 Glossary
  - 10.13 Answers to Self Learning Exercises
  - 10.14 Viva Questions
  - 10.15 Answers to Viva Questions
- References and Suggested Readings

## Part A : Half Adder

### 10.1A Aim

Implement and verify the half adder circuit using

- (I) XOR gate and AND gate
- (II) NAND gates only
- (III) NOR gates only

### 10.2A Apparatus

Power supply, Digital logic trainer ,connecting leads [or Breadboard , DC supply of 5 Volts, connecting wires, LEDs, IC 7486 (XOR), IC7408 (AND), IC7400 (NAND), IC7402 (NOR) as required].

### 10.3A Diagram

- (I) Implementation of half adder using XOR and AND gates

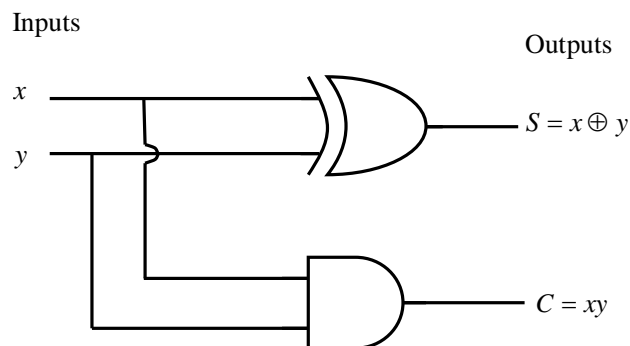
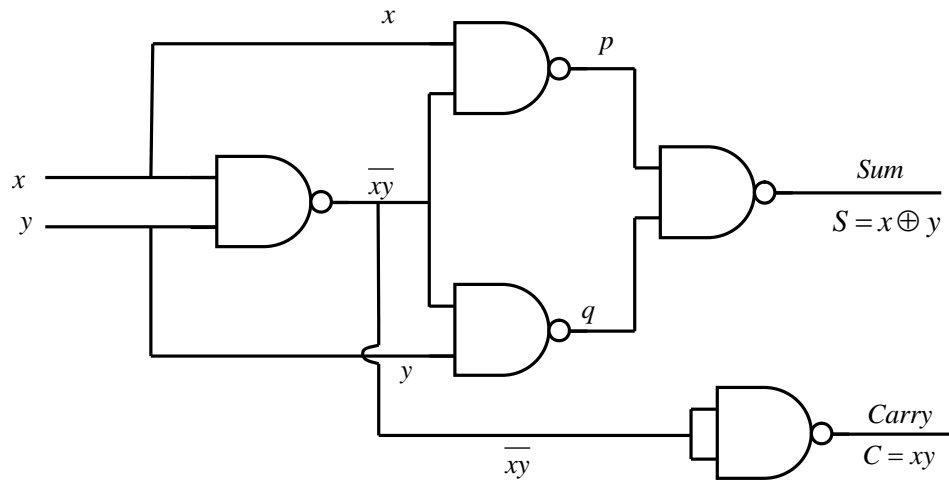


Figure 10.1A

(II) Implementation of half adder using NAND gates



**Figure 10.2A**

$$p = \overline{x\overline{xy}} = \overline{x} + \overline{\overline{xy}} = \overline{x} + xy = \overline{x} + y \quad [\because \overline{A} + AB = \overline{A} + B]$$

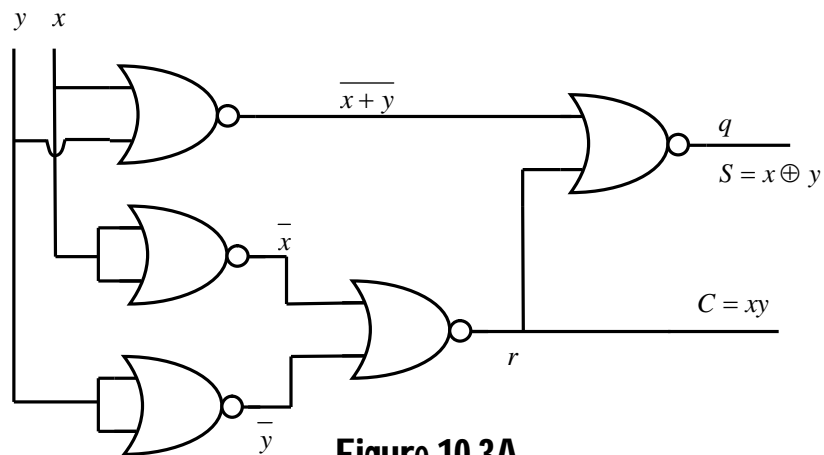
$$q = \overline{y\overline{xy}} = \overline{y} + \overline{\overline{xy}} = \overline{y} + xy = \overline{y} + x$$

$$\begin{aligned} S &= \overline{pq} = \overline{(\overline{x} + y)(x + \overline{y})} \\ &= \overline{x\overline{x} + \overline{x}\overline{y} + xy + y\overline{y}} \\ &= \overline{\overline{x}\overline{y} + xy} = \overline{\overline{x}\overline{y}} \overline{xy} = (\overline{\overline{x} + \overline{y}})(\overline{xy}) \end{aligned}$$

$$S = (x + y)(\overline{xy}) = x \oplus y$$

$$C = \overline{\overline{xy}} = xy$$

(III) Implementation of Half adder using NOR gates



**Figure 10.3A**

$$r = \overline{\overline{x + y}} = \overline{\overline{x} \cdot \overline{y}} = \overline{\overline{xy}} = xy = C$$

$$q = \overline{\overline{x + y + r}} = \overline{\overline{x + y + xy}}$$

$$= (\overline{x + y})(\overline{xy})$$

$$= (\overline{x + y})(\overline{xy}) = x \oplus y$$

$$S = x \oplus y$$

## 10.4A Formula (Truth table)

Truth Table of Half Adder

Inputs		Outputs	
x	y	C (Carry)	S (Sum)
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$S = \text{sum} = x \oplus y = x \overline{y} + \overline{x} y$$

$$C = \text{carry} = xy$$

## 10.5A Theory and Description

### Half Adder :

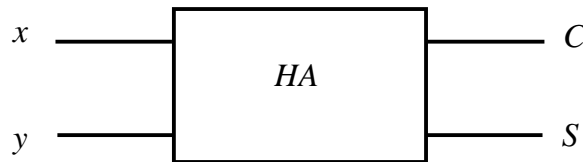
A half adder is a combinational circuit that performs the addition of two single bit numbers and produces a 2-bit numbers.

We usually denote symbols x and y to the inputs. Symbols S (or  $\Sigma$ ) for sum and C (or  $C_{out}$ ) for carry to the outputs can be assigned. The truth table of the half adder is shown in section 10.4A. The S represents the least significant bit (LSB) of the sum and C represents the most significant bit (MSB). The simplified Boolean function for the two outputs can be expressed as

$$S = \text{sum} = x \oplus y = x \overline{y} + \overline{x} y$$

$$C = \text{carry} = xy$$

Half adder block symbol is shown below



**Figure 10.4A**

### 10.6A Procedure

(I) For implementation of Half Adder using XOR gate and AND gate (see figure 10.1A)

6. Make the connections as per the circuit diagram.
7. Connect the circuit to an appropriate power source and turn it on.
8. Observe the outputs on LEDs for each input combinations.
9. Write down the corresponding logic levels in the observation table.

For Breadboard Arrangement

1. Insert IC 7408 (AND Gate) and IC 7486 (XOR gate) in the breadboard.
2. Before you wire the circuit for logic gate, we should label the associated Chip Pin Number on the circuit diagram.
3. Assemble the connection (using components, connecting wires etc.) to implement the circuit illustrated in figure 10.1A.
4. Connect outputs to LEDs with correct polarities and connect a series current limiting resistor (about  $270\Omega$ ) to LED to limit the current and prevent burn out.
5. For IC 7408 (14 pin IC) and IC 7486 (14pin IC), Pin 7 corresponds to ground (0V) and Pin 14 corresponds to  $V_{cc}$  (+5V DC supply).
6. Turn on the power and observe outputs at LED indicator corresponding to all possible combinations of inputs (via input switches). Write down corresponding outputs in the observation table and verify the truth table.
7. If there is any mismatched result, troubleshoot result.
8. Turn off the power and disconnect the circuit connections.

For other parts of experiment, repeat the above steps using appropriate circuit

diagrams (appropriate ICs, etc.) to demonstrate remaining experiments of this unit.

### 10.7A Observation

(I) Implementation of half adder using XOR and AND gates (see figure 10.1A)

Truth Table of Half Adder

Inputs		Outputs	
x	y	C (Carry)	S (Sum)
0	0		
0	1		
1	0		
1	1		

(II) Implementation of half adder using NAND Gates only (see figure 10.2A)

Truth Table of Half Adder

Inputs		Outputs	
x	y	C (Carry)	S (Sum)
0	0		
0	1		
1	0		
1	1		

(III) Implementation of half adder using NOR Gates only (see figure 10.3A)

Truth Table of Half Adder

Inputs		Outputs	
x	y	C (Carry)	S (Sum)
0	0		
0	1		

1	0		
1	1		

**10.8A Result**

- (I) Half adder has been implemented using XOR gate and AND gate and truth table of Half adder has been verified.
- (II) .....
- (III).....

**10.9A Discussion**

.....  
 .....  
 .....  
 .....  
 .....

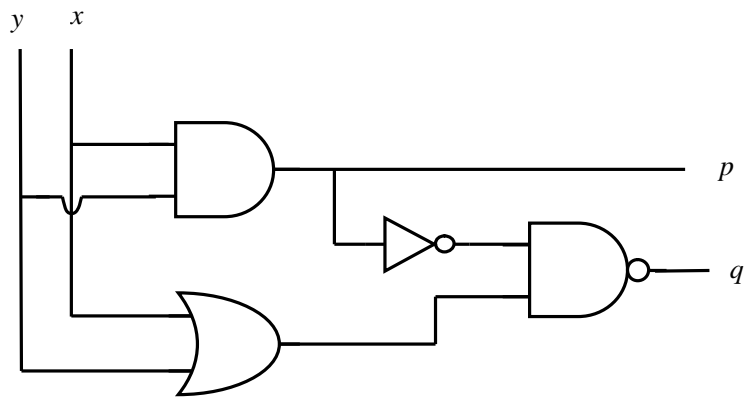
**10.10A Precautions and Sources of Error**

1. Circuit components (LED etc.), power supply must be connected with correct polarities.
2. DC supply voltage must not exceed the value of 5 Volts otherwise it will damage the ICs used during the experiment.
3. LED indicator should be used with series current limiting resistor.
4. Arrangement of IC chips on the breadboard should be such that short wire connections are needed.
5. Power supply should be turned off while connecting or disconnecting the components, wires etc.
6. Circuit should be assembled in systematic and organized fashion because improper connections cause overheating of ICs due to short circuit.
7. Jumper wires used for experiment should not be in broken state.

8. Insertion and removal of the ICs should be done gently to avoid the bending of the pins of ICs
9. All connection should be tight before switching on the power supply.

### 10.11A Self Learning Exercise-I

- Q.1 Write the basic rules for binary addition.
- Q.2 Write the truth table of a half adder.
- Q.3 What are outputs  $p$  and  $q$  for given circuit ?



## Part II: Full Adder

### 10.1B Aim

Implement and verify the full adder circuit using

- (I) Two AND gates, two XOR gates and one OR gate
- (II) Two half adders and one OR gate
- (III) NAND gates only

### 10.2B Apparatus

Power supply, Digital logic trainer, connecting leads [or Bread board, DC supply of 5 Volts ,connecting wires, LEDs, IC7486 (XOR), IC7408 (AND), IC 7400 (NAND) , IC 7432 (OR) as required ]

### 10.3B Diagram

(I) Implementation of full adder using two AND gates, two XOR gates and one OR gate

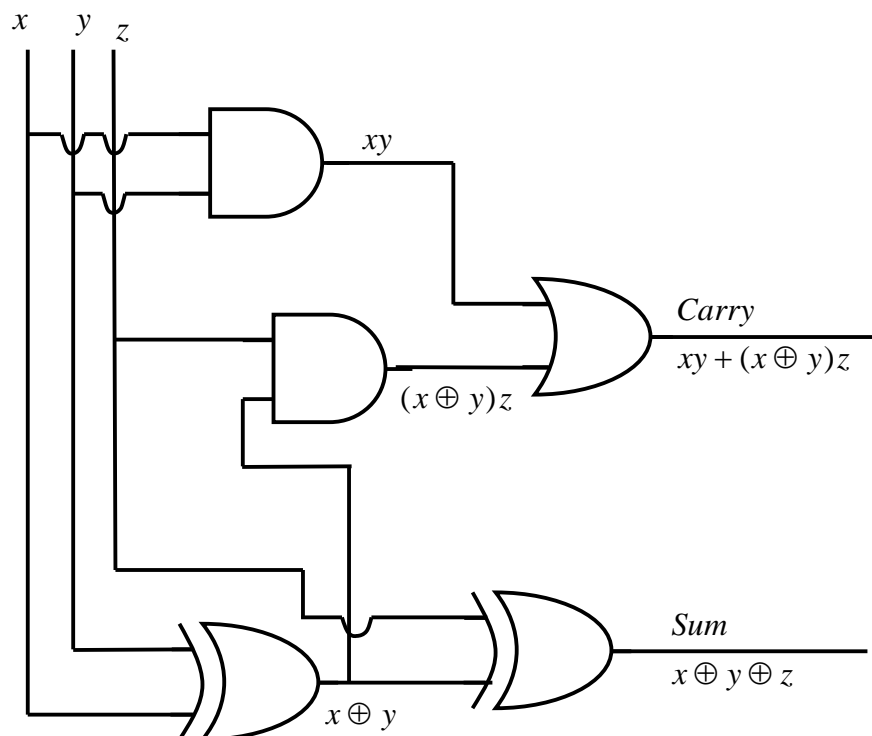
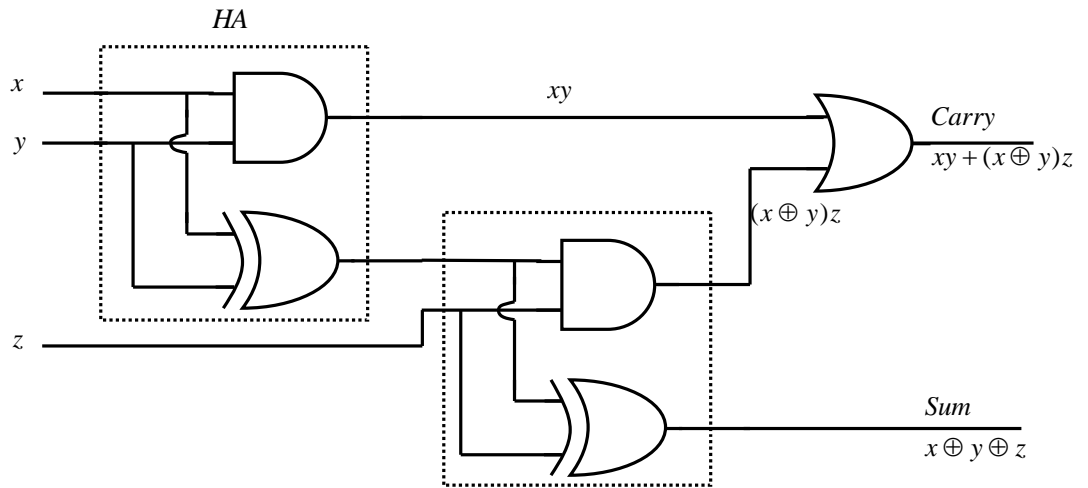
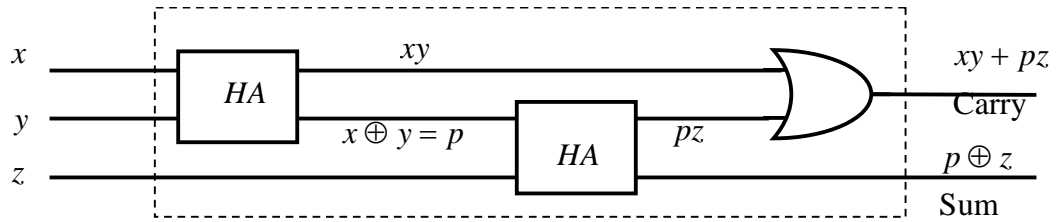


Figure 10.1B



(II) Implementation of full adder using two half adders and OR gate



HA  
Figure 10.2B

(III) Implementation of full adder using NAND gates only

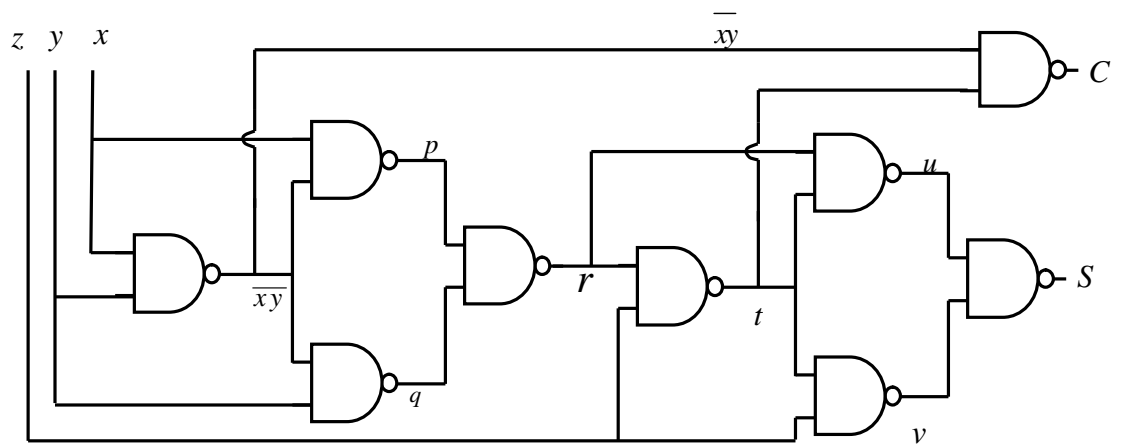


Figure 10.3B

## 10.4B Formula (Truth table)

The truth table of the full adder is shown below

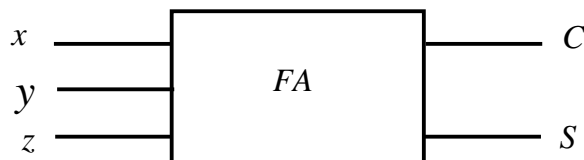
Inputs			Outputs	
x	y	z (C <sub>in</sub> )	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$\text{Carry } C = xy + yz + xz = xy + \bar{x}yz + x\bar{y}z$$

$$\text{Sum } S = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz = x \oplus y \oplus z$$

## 10.5B Theory and Description

Full adder is a combinational circuit that performs the addition of three bits at a time and this circuit needs three binary inputs and two binary outputs. We assign symbols x, y and z to three inputs. Input z is also called 'Carry In' because it comes from a previous lower significant position. The two outputs are denoted by the symbols S for sum and C for carry out. The S represents the least significant bit (LSB) of the sum. Block diagram of full adder is given below.



The truth table of the full adder is shown in section 10.4B

### K Map for Sum S

	yz	00	01	11	10
x	0		1		1
1	1	1		1	

$$S = \bar{x} \bar{y} z + \bar{x} y \bar{z} + x \bar{y} \bar{z} + x y z$$

### K Map for Carry C

	yz	00	01	11	10
x	0			1	
1	1		1	1	1

$$C = xy + yz + xz$$

**Implementation of full adder (see figure 10.3B) using NAND gates can be explained as:**

$$\begin{aligned} p &= \overline{\overline{xy} \overline{x}} = \overline{\overline{xy} + \bar{x}} = xy + \bar{x} \\ &= xy + \bar{x}(1+y) = xy + \bar{x} + \bar{x}y = y(x + \bar{x}) + \bar{x} = y + \bar{x} \end{aligned}$$

Similarly  $q = \bar{y} + x$

$$\begin{aligned} r &= \overline{pq} = \overline{(y + \bar{x})(\bar{y} + x)} \\ &= \overline{y + \bar{x} + \bar{y} + x} = \bar{y} \bar{x} + \bar{y} \bar{x} \\ r &= \bar{y} x + y \bar{x} \\ t &= \overline{rz} = \overline{(\bar{y} x + y \bar{x})z} \\ &= \overline{(\bar{y} x + y \bar{x})} + \bar{z} \\ &= (\bar{y} x) (y \bar{x}) + \bar{z} \end{aligned}$$

$$\begin{aligned}
&= (\bar{y} + \bar{x})(\bar{y} + \bar{x}) + \bar{z} \\
&= (y + \bar{x})(\bar{y} + x) + \bar{z} \\
&= y\bar{y} + xy + \bar{x}\bar{y} + \bar{x}x + \bar{z}
\end{aligned}$$

$$t = x\bar{y} + \bar{x}y + \bar{z}$$

$$C = \overline{(xy)t} = \overline{xy + t}$$

$$= \overline{xy + \overline{xy + \bar{x}\bar{y} + \bar{z}}}$$

$$= \overline{xy} + \overline{\overline{xy} \overline{\bar{x}\bar{y}} \overline{\bar{z}}}$$

$$= \overline{xy} + (\bar{x} + \bar{y})(\bar{x} + \bar{y})\bar{z}$$

$$= \overline{xy} + (\bar{x} + \bar{y})(x + y)\bar{z}$$

$$= \overline{xy} + (\bar{x}\bar{x} + \bar{x}y + \bar{y}x + \bar{y}\bar{y})\bar{z}$$

$$= \overline{xy} + \bar{x}y\bar{z} + x\bar{y}\bar{z}$$

$$= x(\bar{y} + \bar{y})\bar{z} + \bar{x}y\bar{z}$$

$$= x(\bar{y} + y)\bar{z} + \bar{x}y\bar{z} \quad [ \because A + \bar{A}B = A + B ]$$

$$= xy + xz + \bar{x}y\bar{z}$$

$$= xy + z(x + \bar{x}y)$$

$$= xy + z(x + y)$$

$$C = xy + xz + yz$$

Here  $\overline{u} = \overline{tr}$  &  $\overline{v} = \overline{tz}$

$$S = \overline{\overline{tr} \overline{tz}} = \overline{tr + tz} = \overline{tr} + \overline{tz} = \bar{t}(r + z)$$

$$= \overline{(x\bar{y} + \bar{x}y + \bar{z})} \overline{(x\bar{y} + y\bar{x} + z)}$$

$$= \overline{xy} \overline{x\bar{y}} + \overline{xy} \overline{y\bar{x}} + \overline{xy} \overline{z} + \overline{\bar{x}y} \overline{x\bar{y}} + \overline{\bar{x}y} \overline{y\bar{x}} + \overline{\bar{x}y} \overline{z} + \overline{x\bar{y}} \overline{\bar{z}} + \overline{y\bar{x}} \overline{\bar{z}} + \overline{z} \overline{\bar{z}}$$

$$S = \overline{xyz} + \overline{\bar{x}\bar{y}}\bar{z} + \overline{\bar{x}y}\bar{z} + \overline{x\bar{y}}\bar{z}$$

## 10.6B Procedure

**(I)** For implementation of Full Adder using two AND gates , two XOR gates and one OR gate (refer figure 10.1B)

1. Make the connections as per the circuit diagram.
2. Connect the circuit to an appropriate power source and turn it on.
3. Observe the outputs on LEDs for each input combination.
4. Write down the corresponding logic levels in the observation table.

For Breadboard Arrangement

1. Insert IC 7408 (AND Gate) ,IC 7486 (XOR gate) and IC 7432 (OR gate) into the breadboard.
2. Before you wire the circuit for logic gate, we should label the associated Chip Pin Number on the circuit diagram.
3. Assemble the connection (using components, connecting wires etc.) to implement the circuit illustrated in figure 10.1B
4. Connect output to LEDs with correct polarities and connect a series current limiting resistor (about  $270\Omega$ ) to LED to limit the current and prevent burn out.
5. For IC 7408 (14 pin IC), IC 7486 (14 pin IC), IC 7432 (14 pin IC), Pin 7 corresponds to ground (0V) and Pin 14 corresponds to  $V_{cc}$  (+5V DC supply).
6. Turn on the power and observe outputs at LED indicators corresponding to all possible combinations of inputs (via input switches). Write down corresponding outputs in the observation table and verify the truth table.
7. If there is any mismatched result, troubleshoot result.
8. Turn off the power and disconnect the circuit connections.

For other parts of experiment, repeat the above steps using appropriate circuit diagrams (appropriate ICs, etc.) to demonstrate remaining experiments of this unit.

## 10.7B Observations

(I) Implementation of full adder using two AND gates, two XOR gates and one OR gate (figure 10.1B):

The truth table of the full adder

Inputs			Outputs	
x	y	z (C <sub>in</sub> )	C	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(II) Implementation of full adder using two half adders and OR gate (figure 10.2B):

The truth table of the full adder

Inputs			Outputs	
x	y	z (C <sub>in</sub> )	C	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		

1	1	0		
1	1	1		

(III) Implementation of full adder using NAND gates only (figure 10.3B):

The truth table of the full adder

Inputs			Outputs	
x	y	z (C <sub>in</sub> )	C	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

**10.8B Result**

- (I) Full adder has been implemented using two XOR gates, two AND gates and one OR gate and truth table of the Full adder has been verified .
- (II) .....
- (III).....

**10.9B Discussion**

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## 10.10B Precautions and Sources of Errors

1. Circuit components (LEDs etc.), power supply must be connected with correct polarities.
2. DC supply voltage must not exceed the value of 5 Volts otherwise it will damage the ICs used during the experiment.
3. LED indicator should be used with series current limiting resistor.
4. Arrangement of IC chips on the breadboard should be such that short wire connections are needed.
5. Power supply should be turned off while connecting or disconnecting the components, wires etc.
6. Circuit should be assembled in systematic and organized fashion because improper connections cause overheating of ICs due to short circuit.
7. Jumper wires used for experiment should not be in broken state.
8. Insertion and removal of the ICs should be done gently to avoid the bending of the pins of ICs
9. All connection should be tight before switching on the power supply.

## 10.11B Self Learning Exercise-II

- Q.1** How many inputs and outputs are used in full adder circuit ?
- Q.2** What do you understand by nibble ?
- Q.3** Identify the IC that is used for logic function XOR.

## 10.12 Glossary

**ALU** : Arithmetic Logic Unit

**IC** : Integrated circuit

**Full adder** : A logic circuit which can add 3 bits at a time

**Half adder** : A logic circuit that performs the addition of 2 bits

**LED**: Light Emitting Diode

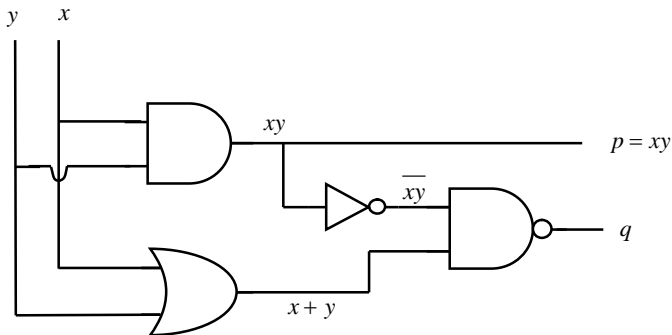


## 10.13 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:**  $0+0=0, 0+1=1, 1+0=1, 1+1=10$

**Ans.3 :**



$$q = \overline{(x+y)} \cdot \overline{xy} = \overline{x+y} + \overline{xy} = \overline{x} \overline{y} + x y$$

### *Answers to Self Learning Exercise - II*

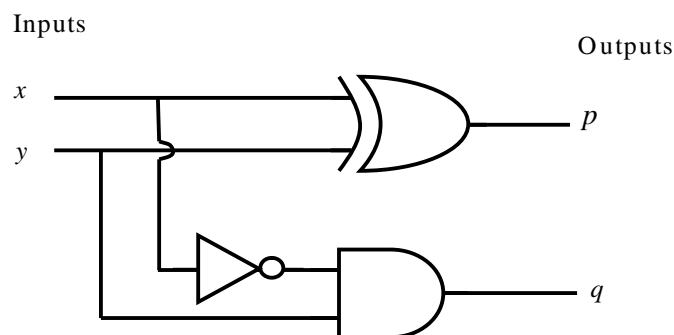
**Ans.1:** Three inputs and 2 outputs

**Ans.2:** It is a combination of four bits.

**Ans.3:** IC 7486

## 10.14 Viva Questions

- Q.1** State the main drawback of half adder.
- Q.2** Write the truth table of full adder.
- Q.3** What will be outputs of a half adder when both inputs are 1s?
- Q.4** Sum S for a half adder is given by  $S = \overline{(C + \overline{x} \overline{y})}$  where carry  $C = xy$ . Is this statement true?
- Q.5** Write the outputs p and q of the following circuit



- Q.6** Identify the IC which is used for logic function AND.
- Q.7** What is full form of ALU ?
- Q.8** Is full adder combinational circuit ?
- Q.9** What is the order of typical supply voltage that is applied to IC ?
- Q.10** Full adder has output sum  $S = x \oplus y \oplus z$ .

Then prove that  $x \oplus y \oplus z = x \bar{y} \bar{z} + \bar{x} y \bar{z} + \bar{x} \bar{y} z + x y z$

### 10.15 Answers to Viva Questions

**Ans.1:** Carry bit cannot be added from previous stage.

**Ans.3:** Sum equals 0 with a carry of 1

**Ans.4:** Yes,

$$\begin{aligned} S &= \overline{xy + \bar{x}\bar{y}} = \overline{xy} \overline{\bar{x}\bar{y}} \\ &= \overline{xy} (\bar{\bar{x}} + \bar{\bar{y}}) = \overline{xy} (x + y) = x \oplus y \end{aligned}$$

**Ans.5:**  $p = x \oplus y$ ,  $q = \bar{x} y$

**Ans.6:** IC 7408

**Ans.7:** Arithmetic logic Unit

**Ans.8:** Yes ,because combinational circuit produces the same output irrespective of the order of inputs.

**Ans.9:** About 5 volt

### References and Suggested Readings

1. M. Morris Mano, Digital design (Fourth edition 2008), Pearson Education
2. M. Morris Mano, Digital logic and Computer Design (2002) Prentice - Hall India
3. Malvino and Brown, Digital computer Electronics (Third edition 2001), Tata McGraw Hill
4. Albert Paul Malvino, Donald P. Leach, Goutam Saha, Digital Principles and Applications (Eighth Edition 2014), McGraw Hill Education.
5. <http://ammini.edu.in/content.aspx?pageid=362>

# UNIT-11

## Decoder, Multiplexer and Demultiplexer

### Structure of the Unit

- 11.1 Aim
- 11.2 Apparatus
- 11.3 Diagram
- 11.4 Theory and Description
  - 11.4.1 Theory For Decoder
  - 11.4.2 Theory for Multiplexer and Demultiplexer
  - 11.4.3 Circuit Description for Multiplexer and Demultiplexer
- 11.5 Self Learning Exercise-I
- 11.6 Procedure
  - 11.6.1 For Decoder
  - 11.6.2 For Multiplexer and Demultiplexer
- 11.7 Observation
  - 11.7.1 For Decoder
  - 11.7.2 For Multiplexer
  - 11.7.3 For Demultiplexer
- 11.8 Result
- 11.9 Discussion
- 11.10 Precautions and Sources of Error
- 11.11 Self Learning Exercise -II
- 11.12 Glossary
- 11.13 Answers to Self Learning Exercises
- 11.14 Viva Questions
- 11.15 Answers to Viva Questions

## References and Suggested Readings

### 11.1 Aim

- To study (a) The operation of decoder using IC 74138,  
 (b) The operation of multiplexer and demultiplexer.

### 11.2 Apparatus

Decoder 74138 Trainer kit (for decoder operation), Multiplexer and demultiplexer trainer kit (for their operation), Adapter (+5V / 350mA), Set of Patch Cords.

### 11.3 Diagram

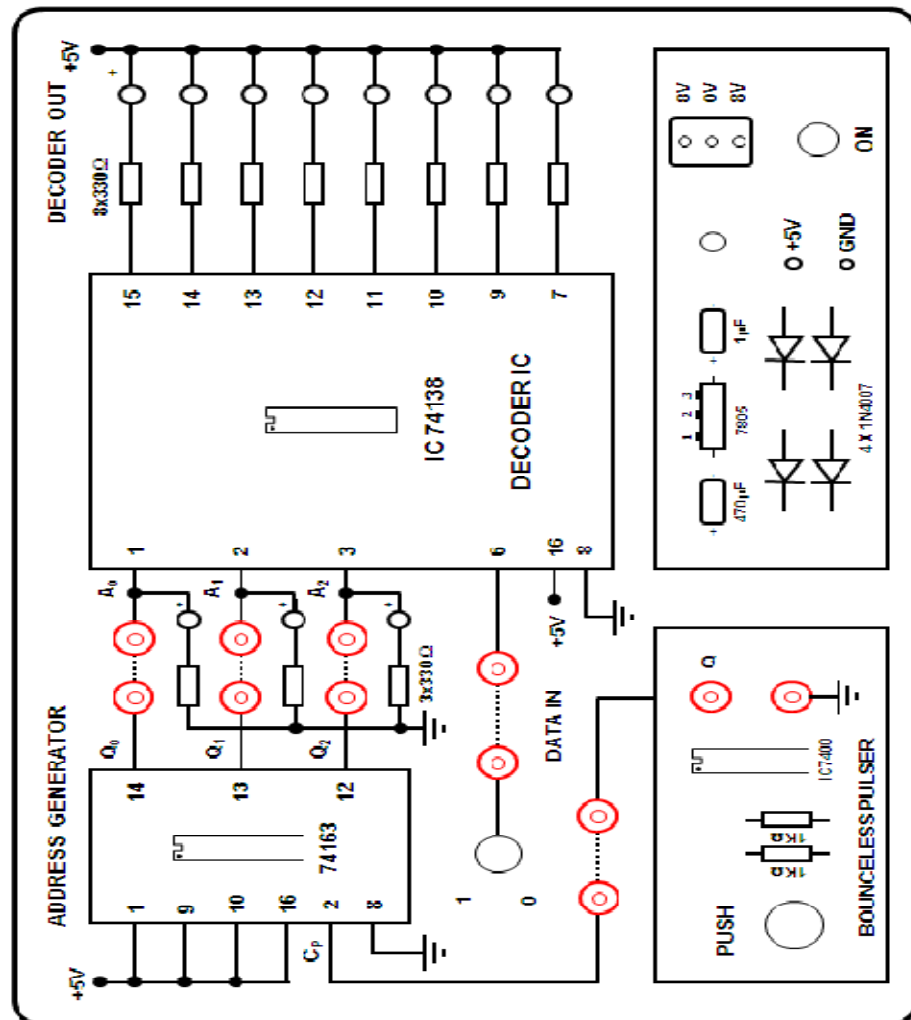


Figure 11.1: Circuit Diagram of Decoder.

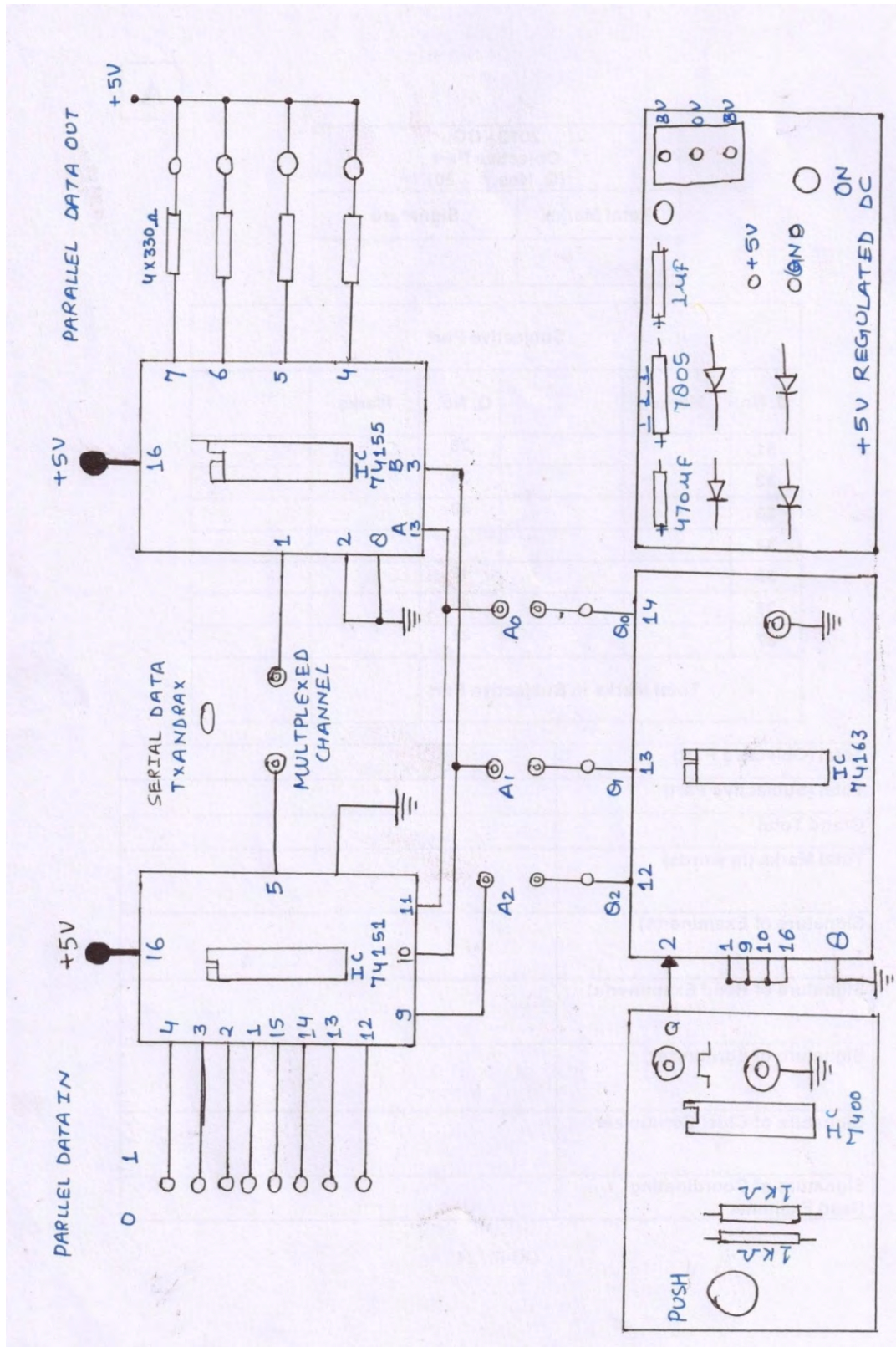


Figure 11.2: Circuit Diagram of Multiplexer and Demultiplexer.

## 11.4 Theory and Description

### 11.4.1 Theory for Decoder

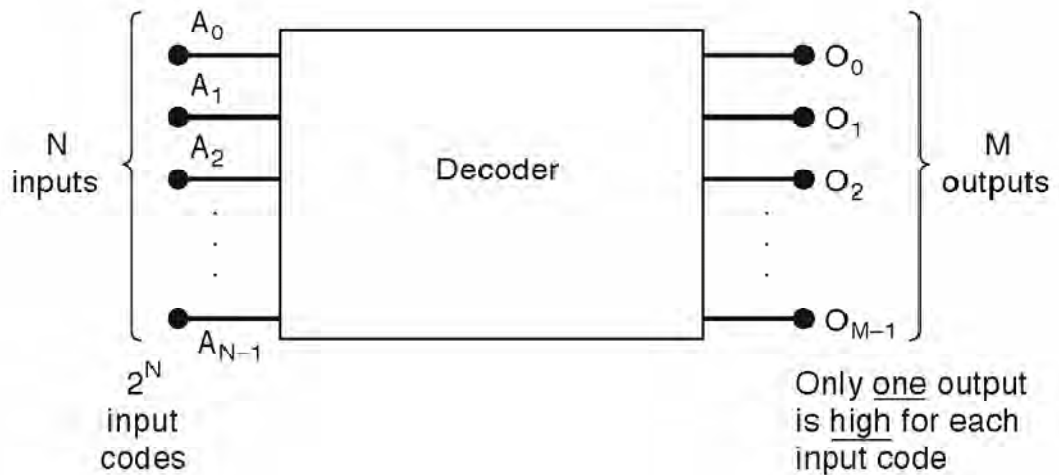
The process of converting binary number into its corresponding decimal number is called decoder; *i. e.* decoder can identify or recognize a particular code. A variety of decoder circuits can be designed for specific purposes, e.g. (i) Decoder can be used to indicate the number of input variables that are true or false, (ii) Decoder can be used to indicate majority or minority variables states, (iii) Decoders can be used as code converters.

A binary code of  $n$  bits is capable of representing up to  $2^n$  distinct elements of coded information. A decoder is a combinational circuit that converts binary information from  $n$  input lines to a maximum of  $2^n$  unique output lines. If the  $n$ -bit coded information has unused combinations, the decoder may have fewer than  $2^n$  outputs.

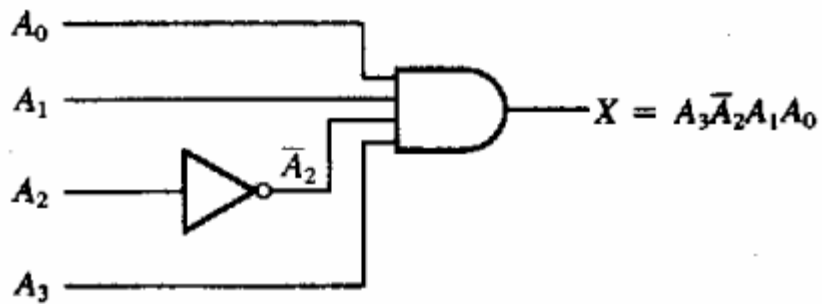
A group of bits among which the legal combinations of values are only those with a single high '1' bit and all the others low '0'. A similar implementation in which all bits are '1' except one '0' is sometimes called one-cold. A simple decoder circuit is a binary to one-hot or one-cold converter. One-to-four-line demultiplexer when E is taken as a data input line and A and B are taken as the selection inputs.

The single input variable E has a path to all four outputs, but the input information is directed to only one of the output lines, as specified by the binary combination of the two selection lines

The 74138 is a high speed 1 of 8 Decoder. This device is ideally suited for high speed bipolar memory chip select address decoding. The multiple input enables Parallel expansion to a 1 of 24 decoder using three 74138 devices or to a 1 of 32 decoder using four 74138 devices and demultiplexer. Demultiplexer outputs are indicated by the LEDs. By synchronizing the Multiplexer address and demultiplexer address the input Channel on the multiplexer is connected directly to the corresponding output Channel. A multiplexer or demultiplexer is something like a 1 pole 8 way Switches.



**Figure 11.3: General Decoder Diagram.**



**Decoding logic for producing a HIGH output when 1011 is on the inputs**

**Figure 11.4: Decoder for Binary pattern 1001.**

### 11.4.2 Theory for Multiplexer and Demultiplexer

To send multiple Data streams from one place to other, multiplexing of signals process is used. Multiplexing is the process of sending of number of separate signals over the same channel, simultaneously without interference between them.

The basic idea of multiplexing is to transmit two or more analog messages or digital signals concurrently over a single communication channel, thus sharing what might be an expensive resource. As an example in the telephone industry, a number of phone calls can be carried on a single wire. Another example is a home stereo system remote control that allows one to select between a CD player, a DVD

player, or cable TV. Sound systems having digital output can carry several channels over a single fiber optic cable. In electronics, a multiplexer, or MUX for short, is a device that performs this multiplexing function— forwarding the selected input into a single channel. In this lesson we will study digital multiplexing in which the number of inputs is a power of two (2, 4, 8, 16, ...) and there is one output. We will see how to select one of the inputs and send it down the single output channel.

A multiplexer has  $2^n$  data inputs,  $n$  control inputs, 1 output. A multiplexer routes (or connects) the selected data input to the output. The value of the control inputs determines the data input that is selected.

Demultiplexer is always found in 1 of 2 and 1 of 4 line demultiplexer, 1 of 8 Decoder from demultiplexer. A demultiplexer is a circuit that gives the binary information from one end to another as an output at a unique channel (one-line or multiple one) according to a unique combination of the channel selector inputs at an input at one-line.

### **11.4.2.1 Circuit Description for Multiplexer and Demultiplexer**

The 74151 is a High Speed 8 Input Digital Multiplexer. It provides in one package, the ability to select one line of data from up to eight sources. 8 logic input switches are provided on the Trainer to Set either zero or 1 level on the corresponding input channel. At any time, which input is selected in the output depends upon the output of address generator output at that instant. The 74161 / 74163 are high speed Synchronous Modulo -16 Binary Counters. They are Synchronously Resettable for application in Programmable dividers and have two types of Count Enable inputs plus Terminal Count Output for versatility in forming synchronous multistage counters. 3 output bits are used as address inputs to the 8 - to - 1 Line Multiplexer 74151.

The 74138 is a high speed 1 of 8 Demultiplexer. This device is ideally suited for high speed bipolar memory chip select address decoding. The multiple input enables Parallel expansion to a 1 - of - 24 decoder using three 74138 devices or to a 1 - of - 32 decoder using four 74138 devices and demultiplexer. Demultiplexer outputs are indicated by the LEDs. By synchronizing the Multiplexer address and demultiplexer address the input Channel on the multiplexer is connected directly to



the corresponding output Channel. A multiplexer or demultiplexer is something like a 1 pole 8 way Switches.

## 11.5 Self Learning Exercise-I

**Q.1** Give the brief description of Decoder.

**Q.2** Give the simple theory of Multiplexer.

## 11.6 Procedure

### 11.6.1 For Decoder

1. First of all connect the adopter to the circuit and switch on the trainer.
2. Then connect patch chord at the line between Bounce less pulser and CP of address generator (IC 74163).
3. Connect the address outputs  $Q_0, Q_1$  and  $Q_2$  to inputs of  $A_0, A_1$  and  $A_2$  of Decoder (IC 74138).
4. Now we change the output of address generator from 000 to 111 in a definite sequence. By pressing the push button switch from Bounce less pulsar switch. Observe the status on address LEDs  $A_0, A_1$  and  $A_2$ .
5. Now we will select data in switch to '1' position.
6. Suppose if the output of the address generator is 000, now channel '0' of Decoder (IC74138) is activated. This can be observed by LED indicators of decoder output.
7. By the changing the address generator output and observes output of decoder respectively.
8. Now we will verify the table which indicates for each address location, which output is activated and also indicates how the state of decoder output changes.(In Observation)

### 11.6.2 For Multiplexer

1. First of all connect the adapter and Switch on the trainer.
2. Now we set all input switches of IC 74151 multiplexer in '0' position.
3. Here the clock is internally connected to the address generator.

4. The output of the address generator changes from 000 to 111 in definite sequence by pressing the push button switch and then observes the status on Address LEDs  $A_0$ ,  $A_1$  and  $A_2$ .
5. Now we consider that if the output of the address generator is 000, channel 0 of IC 74151(Multiplexer) is activated. This can be observed by LED indication at the output of IC 74151.
6. Now if we change the status on Channel 1 that will change the output status of the multiplexer output. In this position, all other channels are inactive. Change the input status of other channels which are not reflected in the output.
6. The same action will occur for different address settings.
7. So, change the address to corresponding to Channel 4 (100). Observe that whatever input (1 or 0) in Channel 4 is reflected at the output. Any data (1 or 0) at all other channels are ignored.
8. Now we will verify the table which indicates for each address location, which output is activated and also indicates how the state of decoder output changes. (In Observation)

### 11.6.3 For Demultiplexer

1. First of all we connect  $A_0$  to  $A_0$ ,  $A_1$  to  $A_1$  and  $A_2$  to  $A_2$  and then  $A_2$  must be low through ground.
2. Connect multiplexer output to input of demultiplexer.
3. Now verify the table which indicated for each address which channel is activated and how the state of channel changes. (In Observation)

## 11.7 Observation

### 11.7.1 For Decoder

ADDRESS			CHANNEL SELECTED
$A_2$	$A_1$	$A_0$	DECODER O/P
0	0	0	0
0	0	1	1

0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

### 11.7.2 For Multiplexer

ADDRESS SELECTED			CHANNEL SELECTED
$A_2$	$A_1$	$A_0$	PARALLEL DATA INPUT
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

### 11.7.3 For Demultiplexer

ADDRESS SELECTED		CHANNEL SELECTED
$A_1$	$A_0$	PARALLEL DATA OUT
0	0	0
0	1	1

1	0	2
1	1	3

### 11.8 Result

Result is given on the basis of verification of truth tables.

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### 11.9 Discussion

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### 11.10 Precautions and Sources of Errors

- (1) The IC should be inserted and taken out carefully from the circuit because its legs are very weak and break quickly.
- (2) The input should be connecting in correct place otherwise the circuit will not give us output.
- (3) It should be remembered that the longer leg of the LED is the anode or positive and should be connected to the terminal 3 and the shorter leg or the cathode should be connected to the ground.
- (4) Care should be taken to connect the circuit because it is so typical to connect.
- (5) Circuit should checked by the teacher before connecting the adapter with circuit.

## 11.11 Self Learning Exercise -II

**Q.1** Give some uses of Decoder.

**Q.2** Give the brief theory of Demultiplexer.

## 11.12 Glossary

**Decoder:** A process which converting binary form into its corresponding decimal number is known as Decoder.

**Multiplexer:** It is used to select one out of many input data lines and to transmit data on the selected line.

**Demultiplexer :** These have dual role. They may be used to activate one of the  $2^n$  outputs by state of the  $n - bit$  control signal at the input.

**IC:** It means integrated circuit which is made on a small chip having all components such as resistance, capacitor, junction diode etc.

## 11.13 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

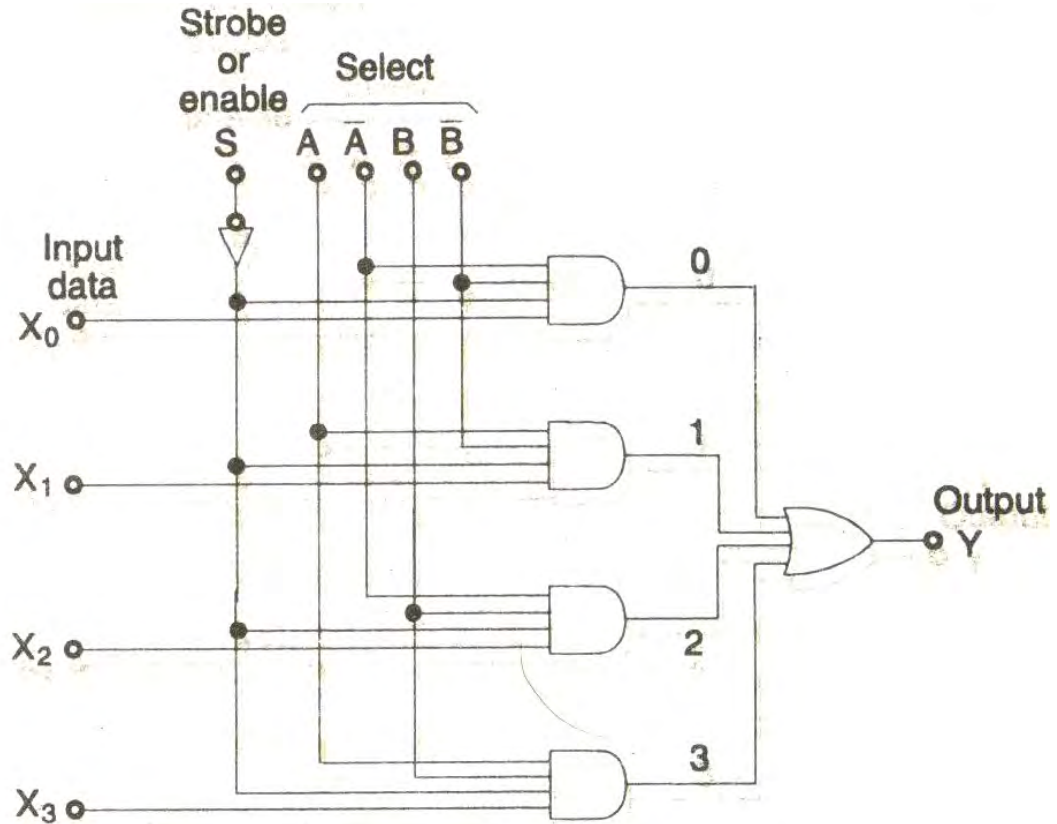
**Ans.1:** As well as numbers are conveyed by means of binary levels or pulse trains in digital systems. As an example, 4-bits of a character are reserved to convey instructions in order to cater to 16 different instructions. With the information coded in binary form with a multi-position switch operated in accordance with this code, a particular line is excited to each of the 16 codes. This process, of identifying a particular code, is known as *decoding*.

If decoding is needed only during certain intervals of time, an additional input, called a strobe, is added to each NAND gate, with all the strobe inputs connected together, and excited by a binary signal S. In this case the strobe input can be used along with a decoder having any number of inputs or outcrops.

**Ans.2:**A multiplexer is a device which would select one output of N input data sources and transmit the selected data a single information channel.

The conversion of a demultiplexer into a multiplexer by *(i)* adding a NAND gate whose inputs comprise all N outputs and *(ii)* augmenting each NAND gate

with an individual data input. The logic system for 4-to-1-line data selector multiplexer, is shown in below given figure 1.



**Figure: 4 to 1 line Multiplexer.**

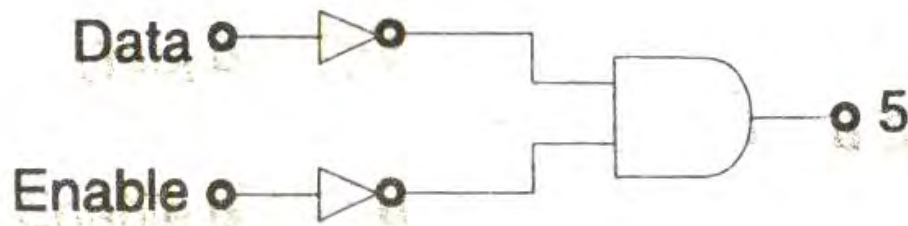
Here, the same decoding configuration is used in the multiplexer, is used in the multiplexer. Now, if the select code is 01 then  $X_1$  would be appearing at the output  $Y$ ; but, if the address is 11, then  $Y$  would be  $X_3$ , etc. As long as the system stays enabled ( $S = 0$ ). Multiplexers are also available for choosing 1 of 8 or 1 of 16 data sources; the latter comprises a 24-pin IC, 16-data inputs, a 4-bit select code, a strobe input, one output and a power-supply lead, extended from the four 4-input AND gates to the sixteen 6-input AND circuits. Selection of 1 out of 32 information sources would be possible by interconnecting two 16-data input multiplexers.

### ***Answers to Self Learning Exercise-II***

**Ans.1:** There are some uses of Decoder which is given as:

- (1) Decoder can be used to indicate the number of input variables that are true or false.
- (2) Decoders can be used to indicate majority or minority variable states.
- (3) Decoders can be used as code convertors.

**Ans.2:** In a demultiplexer, a binary signal is transmitted on one of the N lines, the particular line being selected by means of an address. Conversion of a decoder into a demultiplexer can be made by suitable connections. If the data signal is applied at S, the output would be the complement of this signal and would appear only in the addressed line, since the output is 0 if all inputs are 1.



**Figure: Conversion of Decoder into a demultiplexer.**

The system can be cascaded with that indicated in above Figure, so as to apply an enabling signal of a demultiplexer. With both the data and enable inputs being 0, the data will appear on the line with the desired code. On the other hand, if the enable input is 1, the data would be inhibited from appearing on any line, so that all inputs would remain at 1.

### 11.14 Viva Questions

- Q.1** What is IC ?
- Q.2** What component use in an IC as a capacitor ?
- Q.3** What is RC ?
- Q.4** In an IC, what will you measure in output ?
- Q.5** What are electrolytes ?
- Q.6** What is the topic about LED at which scientists awarded from Nobel prize?
- Q.7** What is LED ?
- Q.8** What is Decoder ?

- Q.9** Give the disadvantages of IC ?
- Q.10** Give the reason for the emitting of light from diode ?
- Q.11** How can you increase the emitting light ?
- Q.12** What is the output voltage of an inverting amplifier ?
- Q.13** What is the output voltage of a non-inverting voltage ?
- Q.14** What is the input offset current ?

### **11.15 Answers to Viva Questions**

- Ans.1:** It is a small integrated circuit which make on a chip where all components of circuit are present.
- Ans.2:** When depletion layer is form in a p-n diode present on chip then the boundaries of layers is used as a capacitor.
- Ans.3:** A RC is circuit made by the coupling of resistance and capacitance by which we find a fine amplify voltage. The coupling of capacitor is known as cascade and i.e. it is known as RC coupled amplifier.
- Ans.4:** In an IC circuit, we measure the power as an output at different resistance.
- Ans.5:** Those substances which on dissolving in a solvent convert into ions are known as electrolytes.
- Ans.6:** When LED discovered it emits blue light. After some time by invention Scientists are invented that it emits white light having much luminescence by mixing of Gallium Nitride in Gallium Phosphate solution.
- Ans.7:** It is a light emitting diode which having much of luminescence. When we connect a p-n junction diode in a forward bias, we get LED.
- Ans.8:** A process which converting binary form into its corresponding decimal number is known as Decoder.
- Ans.9:** The main disadvantage of IC is that it destroys easily because of only one component. If any one component is going to be destroying, the whole IC is destroying.
- Ans.10:** LED emits light in the form of energy. Here energy produces by the recombination of the charge carriers.



**Ans.11:** We can increase the amount of light emitting by the increase in the forward current.

*Amount of Light  $\propto$  Forward current.*

**Ans.12:** The output voltage of an inverting amplifier is given below:

$$V_0 = -\frac{R_f}{R_i} V_i.$$

Here negative sign represents the  $180^\circ$  phase shift which occurs between the input and output.

**Ans.13:** The output-inverting amplifier is

$$V_0 = \left(1 + \frac{R_f}{R_i}\right) V_i.$$

**Ans.14:** The input offset current is the difference between the two input currents driven from a common source. The input offset current for IC741 is 20nA.

## References and Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook
2. B.Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires.

# **UNIT-12**

## **Digital Timing Circuit using IC 555**

### **Structure of the Unit**

- 12.1 Aim
  - 12.2 Apparatus
  - 12.3 Diagram
  - 12.4 Formula
  - 12.5 Theory And Description
    - 12.5.1 Operation as a Free Running Multivibrator
    - 12.5.2 Operation as a Monostable Multivibrator
    - 12.5.3 Operation as a Long Duration Timer
  - 12.6 Self Learning Exercise -I
  - 12.7 Procedure
  - 12.8 Observation
  - 12.9 Calculations
  - 12.10 Result
  - 12.11 Discussion
  - 12.12 Precautions And Sources Of Error
  - 12.13 Self Learning Exercise -II
  - 12.14 Glossary
  - 12.15 Answers to Self Learning Exercises
  - 12.16 Viva Questions
  - 12.17 Answers to Viva Questions
- References and Suggested Readings

## 12.1 Aim

To study the digital timing circuit with the use of IC 555.

## 12.2 Apparatus

An IC (555) which is used, resistance, power supply, capacitance and input source and connection wires.

## 12.3 Diagram

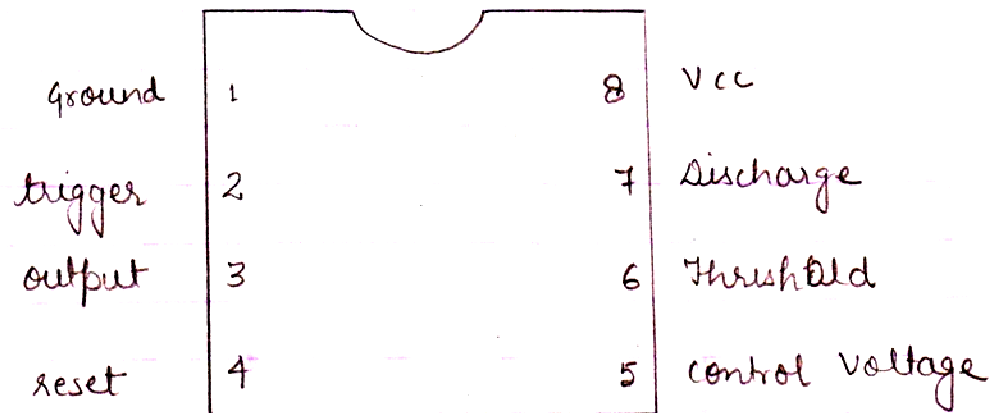


Figure 12.1: Pin connection of 555 Timer.

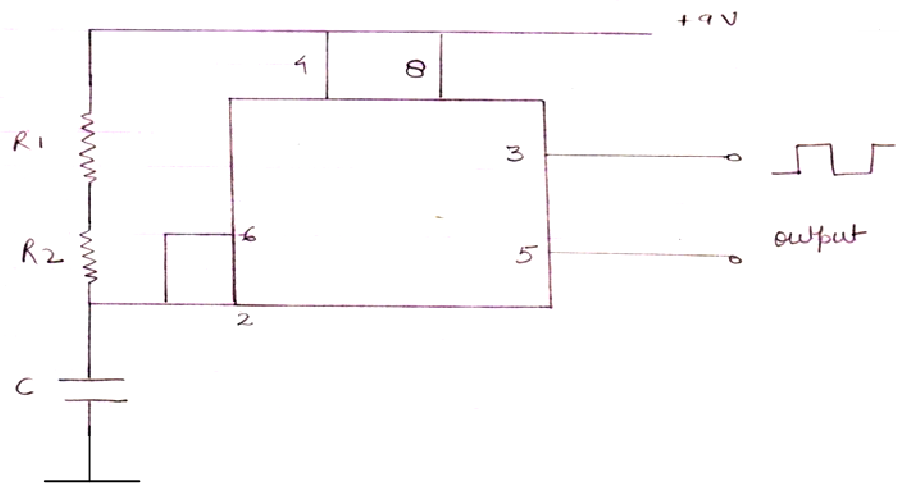


Figure 12.2: Free running Multivibrator.

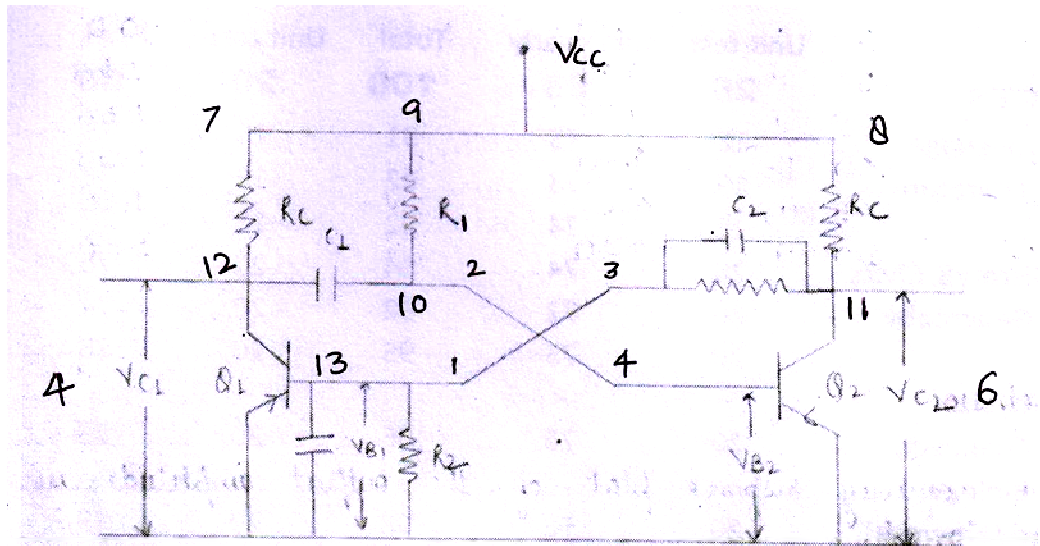


Figure 12.3: Monostable Multivibrator.

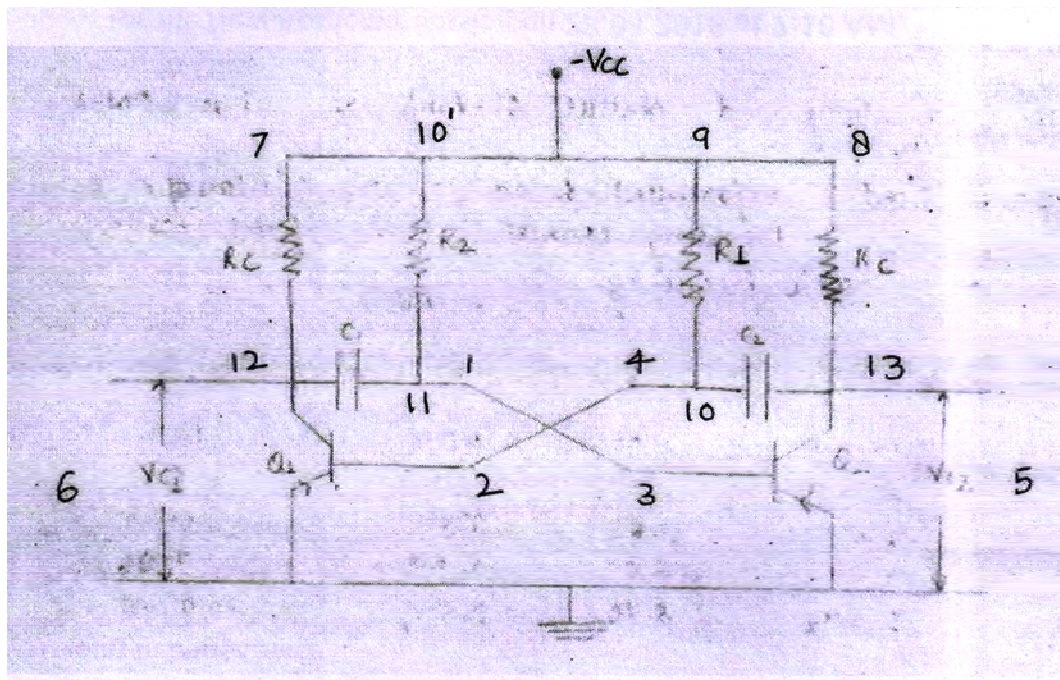


Figure 12.4: For Long duration Timer.

## 12.4 Formula

There are mainly three types of use of IC 555 timer which is:

**As free running multivibrator:**

Then frequency is defined by the formula which is:

$$f = \frac{1.44}{(R_1 + 2R_2)C}$$

**As Monostable multivibrator:**

The delay of monostable is given by expression

$$T = 1.1R_1C$$

**As a long duration timer:**

The time delay generated by circuit is given by the expression

$$T = 1.1 RC$$

## 12.5 Theory and Description

The IC 555 timer is used in three ways which is of following type:

1. A free running multivibrator
2. A monostable multivibrator
3. A long duration timer

IC timer 555 is a very accurate reliable and simple device which may be used in a variety of applications like precision timing, pulse generation, sequential timing, time delay generation pulse width modulation, pulse position modulation and linear ramp generation. Some of the features which make this device very useful are timing from  $\mu$  sec through hours, output and supply TTL compatible and temperature stability better than  $0.00590 \text{ per}^\circ\text{C}$ . The Pin diagram of 555 timer is given in fig. 12.1.

Internally the timer 555 timer consists of two comparators one flip flop and an output stage. All the timings are however set by external resistances and capacitances. Besides these a control voltage applied at pin 5 may also vary the timing of resulting waveform.

### **12.5.1 Operation As A Free Running Multivibrator:**

The circuit diagram for IC 555 for a free running operation is as shown in fig. 12.2. Referring to the layout of the experimental set up (fig 12.2), the following connections are required to be made:

1. Connect power supply by joining 2 to 4 & 1 to 3.
2. Connect  $R_1$ ,  $R_2$ , and  $C$  in series arrangement and then connect to 4.
3. Join 7 to 8
4. Connect CRO to 6 & 4.
5. All accept wire are in ground mode.

The frequency of oscillation is a function of  $R_1$ ,  $R_2$  and  $C$  is given by:

$$f = \frac{1.44}{(R_1 + 2R_2)C}$$

It may further be seen that frequency of oscillation can be varied by adjusting the potentiometer, for using this feature also join 13 & 14.

### **12.5.2 Operation as A Monostable Multivibrator:**

Figure 12.3 shows the schematic diagram for monostable operation using 555. Here an external trigger source is required. This is however, built in present experiment set up referring to lay out diagram in fig. 12.1. The following connections should be made.

1. Connect power supply by joining 1 to 3 and 2 to 4.
2. Connect  $R_1$  (between 9 and 10),  $R_c$  (between 1 and 12 and 8 to 11), and  $C_1$  and  $C_2$  as indicate in the figure 12.3 .
3. Now connect CRO to 6 & 4 and by the two voltage overlapping we find the pattern in CRO.

The delay of monostable is given by the expression,

$$T = 1.1 R_1 C$$

### **12.5.3 Operation as A Long Duration Timer:**

The basic circuit for this case is shown in figure 12.4. Following connections are necessary in this circuit or lay out diagram of experiment set up as shown in figure 12.1.

1. Connect power supply by joining 1 to 3 and 2 to 4.
2. Connect  $R_1$  (between 9 and 10),  $R_2$  (between 10' and 11), and  $C_1$  and  $C_2$  (between 11 and 12 and between 4 and 13 respectively).

3. Connect 7 to 8 for manual triggering.
4. Connect 6 to 5 for LED indication.

The time delay generated by circuit is given by the expression:

$$T = 1.1 RC$$

Where R and C are the total of resistance and capacitor.

## 12.6 Self Learning Exercise -I

- Q.1 Give the theory of Operational amplifier
- Q.2 Give the working procedure of the Astable multivibrator.

## 12.7 Procedure

### 12.7.1 Study of a Free Running Multivibrator

For the study of the free running multivibrator, we follow the below given points:

- (i) Make the connections as explained in theory, set  $R_1 = 10\text{ k}\Omega$ ,  $R_2 = 4.7\text{ k}\Omega$ ,  $C = 0.047\text{ }\mu\text{F}$ , voltage control potentiometer should be minimum.
- (ii) See the waveform on a CRO and measure its frequency.
- (iii) Compare the experimentally obtained frequency with the theoretical results computed from values of  $R_1$ ,  $R_2$  and  $C$ .
- (iv) Change the values of  $R_1$ ,  $R_2$  and set  $R_1 = 4.7\text{ k}\Omega$ ,  $R_2 = 10\text{ k}\Omega$ .
- (v) Repeat (ii) and (iii).
- (vi) For the setting as in (iv), measure the frequency for different values of voltage control potentiometer setting and note down the voltage in each value.
- (vii) Plot a graph between the voltage and frequency obtained in (vi). Comment on linearity of this plot.

### 12.7.2 Study of Monostable Multivibrator

- (i) Make the connections as explained in theory, set  $R_2 = 4.7\text{ k}\Omega$ ,  $C = 0.047\text{ }\mu\text{F}$ .

(ii) See the waveform on a CRO, complete the pulse width of this waveform and compare with theoretically obtained results.

(iii) Vary the trigger frequency control potentiometer and observe the waveform on CRO.

### 12.7.3 Study of a Long Duration Timer

(i) Make the connections as explained in theory, set  $R_1 = 4.7 M\Omega$ ,  $C = 10 \mu F$ .

(ii) Momentarily press the 'Manual Control' and observe that the LED indicator glows for a fixed period.

(iii) Measure this period with a stop watch and compare with theoretical result.

(iv) Repeat above  $R_1 = 1 M\Omega$ ,  $C = 10 \mu F$ .

## 12.8 Observation

### 12.8.1 For Astable Multivibrator

S. No.	$R_1$	$R_2$	$C$	Amplitudes	Pulse width	Time period observed	Time period theoretical

### 12.8.2 For Monostable Multivibrator

S. NO.	R	C	Amplitude	Pulse width	Time period observed	Time period theoretical




## 12.9 Calculation

.....

.....

.....

## 12.10 Result

Observed time period:

Theoretical time period:

**Percentage Error:**

$$\%error = \frac{\textit{Experimental value} - \textit{Standard value}}{\textit{standard value}} \times 100$$

.....

## 12.11 Discussion

.....

.....

.....

.....

## 12.12 Precautions and Sources of Errors

1. The output of power supply should not exceed 9V.
2. In case of monostable frequency of oscillator should not be change it should have fixed value.
3. It should be observed that theoretical value and experimental value should be almost same.

## 12.13 Self Learning Exercise -II

**Q.1** Give the Characteristics of an ideal OP-Amp.

**Q.2** Give the working procedure of Monostable multivibrator.

## 12.14 Glossary

**CMRR:** The ability of an OP-AMP to reject a common mode signal is expressed by its common mode rejection ratio. It is the ratio of differential voltage gain to the common-mode voltage gain.

**Output offset voltage:** It is the *d. c.* voltage appearing at the output terminal when the two input terminals are grounded.

**Input offset current:** The input offset current is the difference between the two input currents driven from a common source.

**Multivibrator:** An electronic circuit that generates square waves is known as a multivibrator.

## 12.15 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** An important setup is the introduction of operational amplifiers better known as OP-Amp which is the fundamental building block. It is high gain direct amplifier whose response characteristics are extremely controlled by negative feedback. It is used for accomplishing many linear as well as nonlinear functions and that is why it is called operational amplifier.

The use of diodes, transistors, tubes, etc. in various electronic devices has been made quite efficiently. In spite of their efficient use, the circuits using these devices have become quite cumbersome and costly. With the introduction of space technology and computers the need for miniaturization has been strongly felt. These days it has been possible to have a package of large number of transistors, diodes, resistors etc. on a very small chip. For this a monolithic integrated circuitry has been introduced with which a complete circuit without the use of external connecting wires can be manufactured on a silicon chip. In addition to reduction in

size and weight, it has been possible to have high accuracy and reduction in cost price also.

The basic use of the operational amplifier is Differential Amplifier.

### **Differential Amplifier:**

It is a basic unit of operational amplifier having differential input. In a broad way it amplifies the difference of two input signals. If the two input signals are  $v_1$  and  $v_2$  respectively then an ideal differential amplifier will give an output voltage

$$v_0 = A_d(v_1 - v_2)$$

where  $A_d$  is the gain of the amplifier.

The efficiency of a differential amplifier is measured by the ratio of  $A_d$  and  $A_c$ . This ratio is called common mode rejection ratio (CMRR).

We can write as

$$CMRR = \frac{\text{voltage gain for difference signal}}{\text{voltage gain for common mode signal}}$$

$$\rho = \left| \frac{A_d}{A_c} \right|$$

In Decibel we can written as

$$CMRR (\rho) = 20 \log_{10} \left[ \left| \frac{A_d}{A_c} \right| \right]$$

An OP-Amp is a high gain, direct coupled differential linear amplifier whose response characteristics are externally controlled by negative feedback. It is widely used in Computers, can perform mathematical operations such as summing, integration, and differentiation. Its input impedance is very high and output impedance is extremely low and its gain depends on the feedback ratio only.

**Ans.2:** For the working of astable multivibrator we make following connections and follow the method:

(1) Connect  $T_2$  to  $T_3$  &  $T_3$  to  $T_6$ . This connects  $V_{cc}$  to astable multivibrator check the supply voltage by a multimeter. It should be nearly 9V. Keep band switch in anyone position and note the value of capacitor associated.

- (2) Connect a CRO across  $T_4$  &  $T_6$ . A waveform similar to  $V_{c_2}$ ,  $V_{c_1}$  in figure will be seen on CRO screen. Trace this waveform.
- (3) Measure the time period and amplitude of this waveform with the help of CRO scales.
- (4) Connect the CRO across  $T_5$  &  $T_6$  and again trace the waveform seen at the base terminal. Measure its amplitude on CRO screen. Compare its shape with those given figure.
- (5) Repeat steps 2, 3 and 4 for other two positions of the band switch i.e. for the other capacitor value.
- (6) Next connect CRO across  $T_{15}$  &  $T_{14}$ . Measures the time-period of T of this input waveform from CRO scale.
- (7) Trace the waveform available at terminal  $T_{13}$ . Compare these waveform to those in figure.
- (8) Repeat steps 5, 6, 7 for all the three settings of the bandwidth in astable multivibrator.

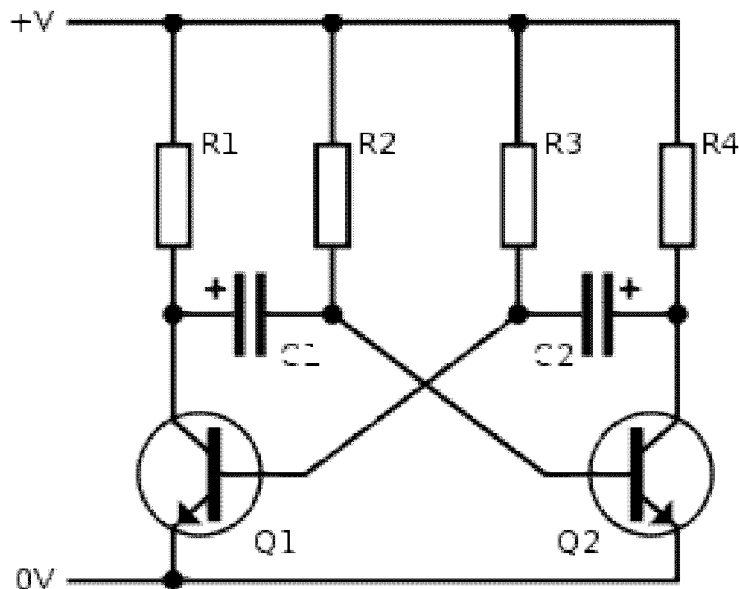


Figure: Astable Multivibrator

## Answers to Self Learning Exercise-II

**Ans.1:** The characteristics of an ideal OP-Amp is given below:

1. Infinite Voltage Gain,
2. Infinite Bandwidth,
3. Infinite Input Impedance,
4. Zero Output Impedance,
5. Perfectly Balanced Circuit,
6. Perfectly Stable Circuit.

Symbolically it has three leads other than power supply and ground leads as shown in given figure1. Terminal leads 1 and 2 function as two input ports and terminal 3 acts as output.

Infinite input impedance means that none of the input terminals draw any current i.e.  $I_1 = I_2 = 0$ . Zero output impedance means that any amount of current can be drawn without disturbing input voltage.

Terminal 1 is labelled (-)ve and terminal 2 is labelled as (+)ive. It means that if a signal is applied at terminal 1, it will be inverted at output i.e., appear as  $180^\circ$  out of phase with respect to input whereas if a signal is applied at terminal 2, it will not be inverted at output. On account of this function terminal 1 is called *Inverting Terminal* and terminal 2 is called *Non – Inverting Terminal*. Its symbol is shown in below given figure 2.

Its equivalent circuit is shown in figure 1.

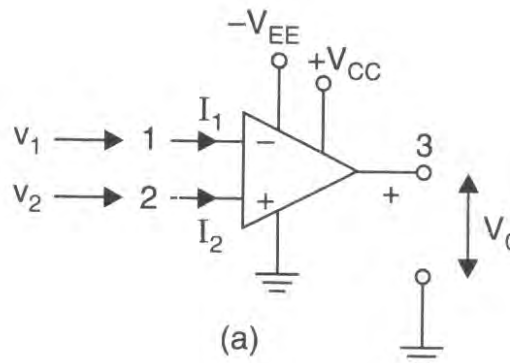
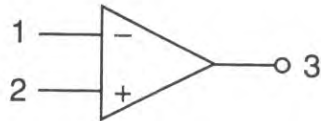


Figure : OP – Amp (a) Symbol OP – Amp



(b)

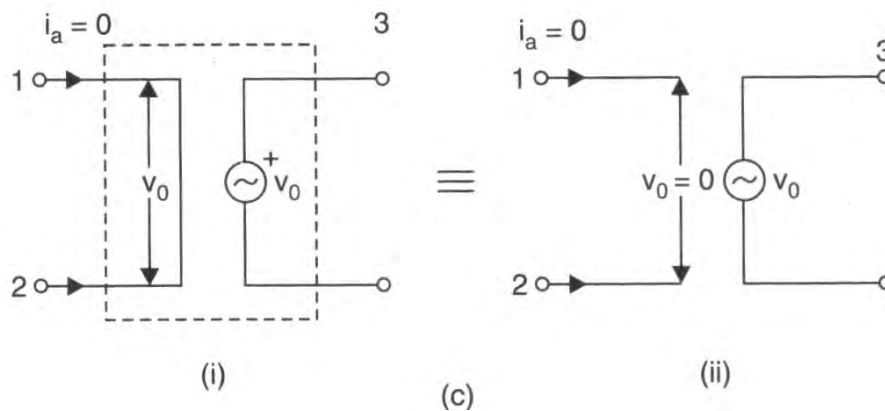


Figure : (b) (c) (i) (ii) Equivalent circuit

The magnitude of the output signal at terminal 3 is proportional to the difference of the two-input signals which is also called error voltage applied between terminals 1 and 2 respectively.

$$v_o \propto (v_1 - v_2)$$

$$v_o = A(v_1 - v_2) = A(v_i) \text{ where } v_i = (v_1 - v_2)$$

Here constant 'A' is called the **voltage gain** of the OP-Amp and  $v_i = (v_1 - v_2)$  is called the **error voltage**.

Now another figure of OP-Amp is given below (fig 1 d) which is known as Block Diagram. The input stage is a differential amplifier followed by more stages of gain and a class B pushpull Emitter Follower because a differential amplifier is the first stage.

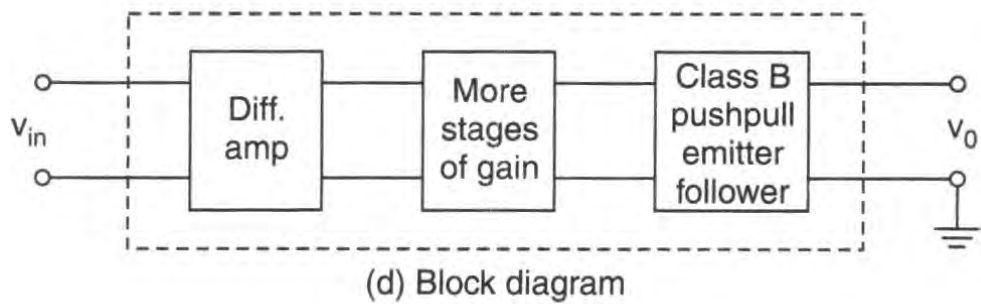
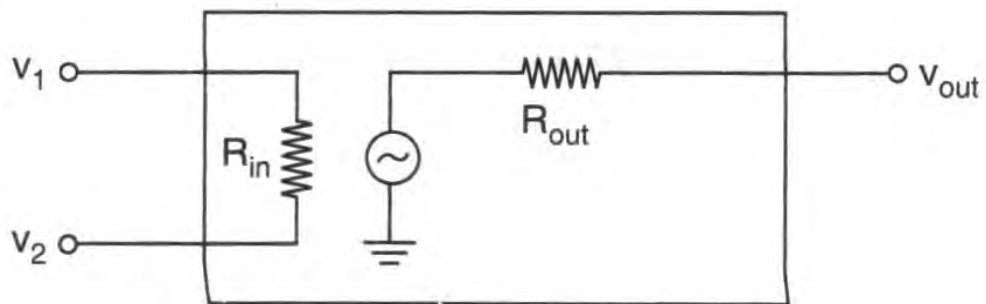


Figure 1 (d) Block diagram



It determines the input characteristics of the OP-Amp. In most OP-Amps the output is single ended as shown in figure 1(e). With positive and negative supplies, the single ended output is designed to have a quiescent value of zero. This way a zero input results into a zero output.

**Ans.2:** This generates voltage pulses and extended waveforms but require one driving pulse for generation of each cycle of waveform. First of all connect set up to a.c. mains and then connect the power supply terminals to the univibrator and free running multivibrator circuits with proper polarities.

Now measure the dc voltage with the help of a multimeter at the collector of both the transistor in monostable circuit, it will almost be equal to  $V_{cc}$ . Record these voltages in the observation table. Connection the output I of free running multivibrator to the input of univibrator (for this we connect ground terminal of free running to the ground terminal of univibrator). For the wave form of apply signal connect an oscilloscope at univibrator output.

## 12.16 Viva Questions

- Q.1 What is Chip ?
- Q.2 What is a photo mask ?
- Q.3 On which factor, multivibrator depends ?
- Q.4 What is astable multivibrator ?
- Q.5 What is a monostable multivibrator ?
- Q.6 What is bistable multivibrator ?
- Q.7 How many stages are present in multivibrator ?
- Q.8 What is the output voltage of an inverting amplifier ?
- Q.9 What is the output voltage of a non-inverting voltage ?
- Q.10 What is differentiating Circuit ?
- Q.11 What is integrating circuit ?
- Q.12 What is clipping circuit ?
- Q.13 What are the threshold and control voltages in a 555 timer ?

## 12.17 Answers to Viva Questions

- Ans.1:** Several hundred identical ICs are made at a time on a thin wafer several hundred centimeters wide, and the wafer is subsequently sliced into individual ICs called chips. The terms chip, silicon chip, microchip and IC are synonymous.
- Ans.2:** A photo mask is the actual size of the chip, replicated many times to fit on a round silicon wafer up to 12 inch in diameter.
- Ans.3:** A multivibrator is a switching circuit which depends for operation on positive feedback.
- Ans.4:** A multivibrator which generates square wave of its own without any external triggering pulse is known as an astable or free running multivibrator.
- Ans.5:** A multivibrator in which one transistor is always conducting and the other is non-conducting is called a monostable multivibrator.



**Ans.6:** A multivibrator which has both states stable is called a bistable multivibrator.

**Ans.7:** It is basically a two stage amplifier with output of one feedback to the input of the other.

**Ans.8:** The output voltage of an inverting amplifier is given below:

$$V_0 = -\frac{R_f}{R_i} V_i.$$

Here negative sign represents the  $180^\circ$  phase shift which occurs between the input and output.

**Ans.9:** The output-inverting amplifier is

$$V_0 = \left(1 + \frac{R_f}{R_i}\right) V_i.$$

**Ans.10:** A circuit in which output voltage is directly proportional to the derivative of the input is known as a differentiating circuit.

$$\text{Output} \propto \frac{d}{dt} (\text{Input})$$

**Ans.11:** A circuit in which output voltage is directly proportional to the integral of the input is known as an integrating circuit.

$$\text{Output} \propto \int \text{Input}$$

**Ans.12:** The circuit with which the waveform is shaped by removing a portion of the applied wave is known as a clipping circuit.

**Ans.13:** In a 555 timer, the non-inverting input of the upper op amp is called the threshold voltage whereas its inverting input is the control voltage. The former must be greater than the latter to set the RS flip-flop.

## References and Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook,
2. B. Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires

# UNIT-13

## 2Bit & 8 Bit Magnitude Comparators

### Structure of the Unit

- 13.1 Aim
  - 13.2 Apparatus
  - 13.3 Diagram
  - 13.4 Formula
  - 13.5 Theory And Description
    - 13.5.1 For 2-Bit Magnitude Comparator
    - 13.5.2 For 8-Bit Magnitude Comparator
  - 13.6 Self Learning Exercise -I
  - 13.7 Procedure
    - 13.7.1 For 2-Bit Magnitude Comparator
    - 13.7.2 For 8-Bit Magnitude Comparator
  - 13.8 Observation
  - 13.9 Calculations
  - 13.10 Result
  - 13.11 Discussion
  - 13.12 Precautions and Sources of Error
  - 13.13 Self Learning Exercise -II
  - 13.14 Glossary
  - 13.15 Answers to Self Learning Exercises
  - 13.16 Viva Questions
  - 13.17 Answers to Viva Questions
- References and Suggested Readings

## 13.1 Aim

To design and implement of:

1. 2-bit magnitude comparator using logic gates
2. 8-bit magnitude comparator using IC 7485.

## 13.2 Apparatus

Logic gates, TTL IC 7485 2-bit comparator, NPN transistor, volt-ohm-milliammeter, LED, Resistor 150 ohm, 470 ohm, 1 k ohm.

## 13.3 Diagram

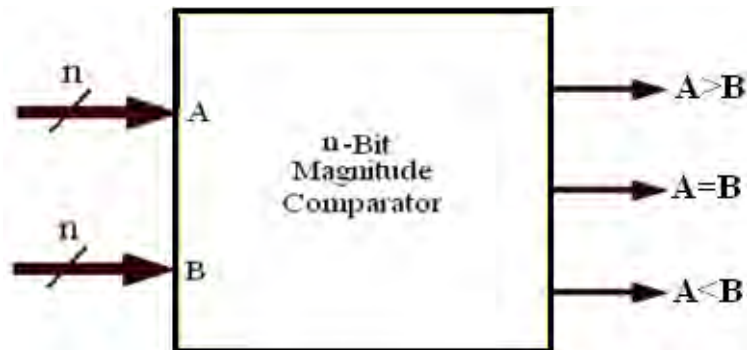


Figure 13.1: Basic Block Diagram of n-Bit Magnitude Comparator.

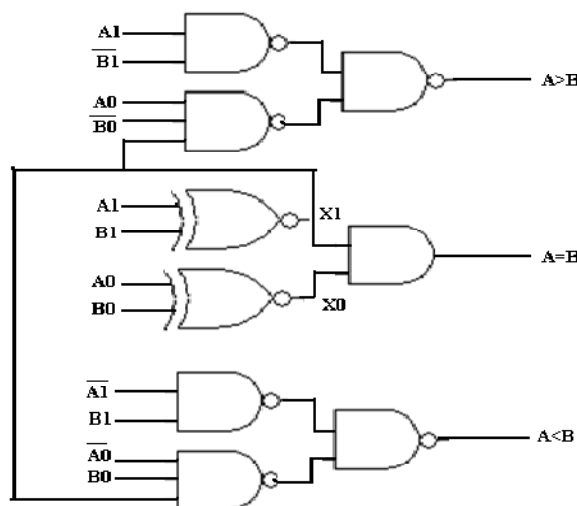


Figure 13.2: Logic gate diagram of 2-bit magnitude comparator.

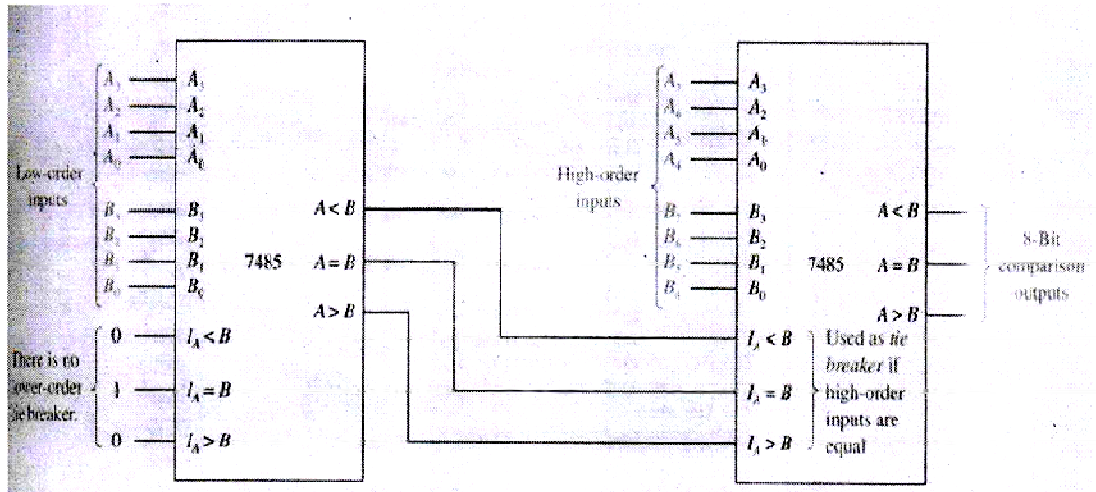


Figure 13.3: Design of 8-bit magnitude comparator using IC 7485.

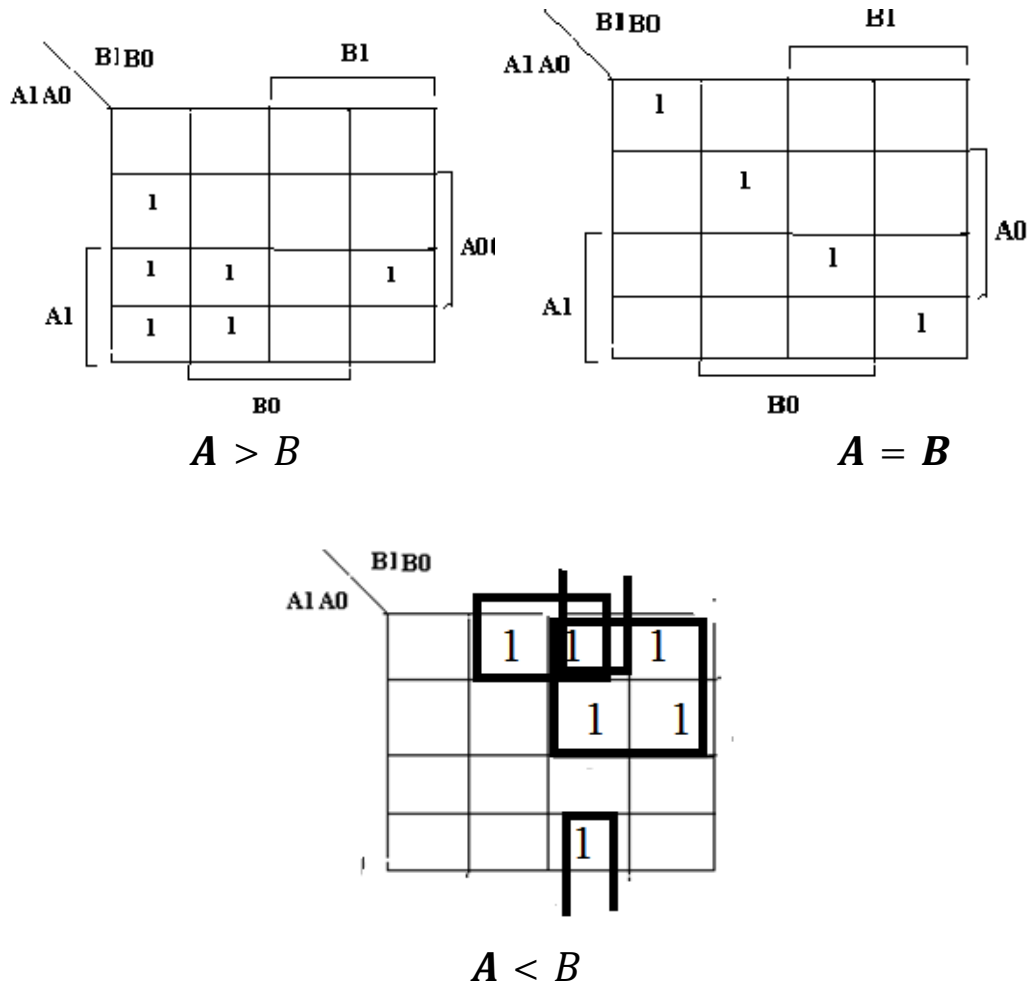


Figure 13.4: Three types of 8-bit magnitude given in above diagram.

## 13.4 Formula

We choose for the comparator in design AND, NAND and OR gate, so we use the Boolean equation of the all the three gate which are:

$$Y = A.B \text{ for AND gate}$$

$$Y = \overline{AB} \text{ for NAND gate}$$

$$Y = A + B \text{ for OR gate.}$$

## 13.5 Theory and Description

### 13.5.1 Theory for 2-Bit Comparator

Comparison is most basic arithmetic operation that determines if one number is greater than, equal to, or less than the other number. Comparator is most fundamental component that performs comparison operation. This brief present comparison between different logic styles used to design 2-Bit magnitude comparator. Comparison between different designs is calculated by simulation that is performed at 90nm technology.

The circuit, for comparing two n-Bit numbers, has 2n inputs &  $2^{2n}$  entries in the truth table, for 2-Bit numbers, 4-inputs & 16-rows in the truth table, similarly, for 3-Bit numbers 6-inputs & 64-rows in the truth table. The logic style used in logic gates basically influences the speed, size, power dissipation, and the wiring complexity of a circuit. Circuit size depends on the number of transistors and their sizes and on the wiring complexity. The wiring complexity is determined by the number of connections and their lengths. All these characteristics may vary considerably from one logic style to another and thus proper choice of logic style is very important for circuit performance. In order to differentiate all four designs, simulations are carried out for power, Delay, Power Delay Product at varying supply voltages from 0.6v to 1.4v in step of 0.2v. Simulations are performed at 90nm technology in Tanner EDA Tool.

### 13.5.2 Theory for 8-Bit Comparator

These advanced Schottky devices are capable of performing high-speed arithmetic or logic comparisons on two 8-bit binary or two's complement words. Two fully decoded decisions about words P and Q are externally available at two outputs. These devices are fully expandable to any number of bits without external

gates. To compare words of longer lengths, the  $P > QOUT$  and  $P < QOUT$  outputs of a stage handling less significant bits can be connected to the  $P > QIN$  and  $P < QIN$  inputs of the next stage handling more significant bits. The cascading paths are implemented with only a two-gate-level delay to reduce overall comparison times for long words. Two alternative methods of cascading are shown in application information.

The latch is transparent when P latch-enable (PLE) input is high; the P-input port is latched when PLE is low. This provides the designer with temporary storage for the P-data word. The enable circuitry is implemented with minimal delay times to enhance performance when cascaded for longer words. The PLE, P, and Q data inputs utilize pnp input transistors to reduce the low-level current input requirement to typically  $-0.25$  mA, which minimizes dc loading effects.

## 13.6 Self Learning Exercise-I

- Q.1 What are logic gates ?
- Q.2 What are Decoder ?
- Q.3 What are Encoders and Decoders ?

## 13.7 Procedure

### 13.7.1 For 2-Bit Magnitude Comparator

For the procedure of the 2-bit magnitude comparator, we should follow the below given process:

1. Using figure 13.2 first we should complete NAND gate and then after getting the output we send it into NAND gate, then we should get an final output which is gives us  $A > B$ .
2. And after that process we form a combination of OR gate and then it connect to AND gate. It should be noted that this combination is also connect with the above logic gate, then we should get a final value which is  $A = B$ .
3. Now at last we form a combination should be form same to the first combination and it should be noted that this is also connect with the above two combination and then we should get a final value which is  $A < B$ .

### 13.7.2: For 8-Bit Magnitude Comparator:

VHDL program for an 8-bit comparator with **IF\_THEN\_ELSE** statement.

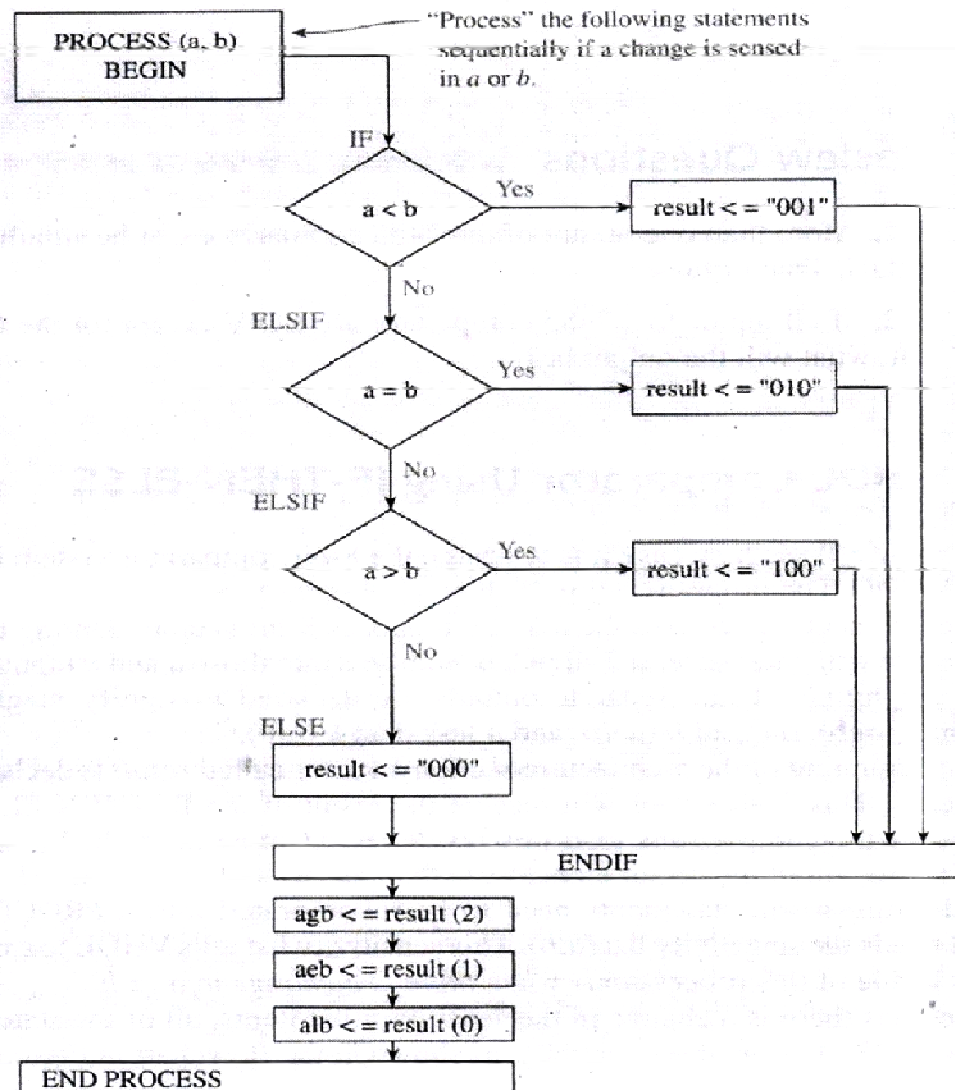
#### VHDL Design Part:

The entity section declares **A** and **B** as 8-bit vector. Three separate outputs are declared:

$A < B = a$  less than  $b$ ;

$A = B = a$  equal  $b$ ;

$A > B = a$  greater than  $b$ .



Go to according the above given flowchart, we can form the 8-bit magnitude comparator.

## 13.8 Observation

### 13.8.1 Table for the 2-bit magnitude comparator:

2-Bit Magnitude Comparator Compares two numbers each having two bits  $A_1 A_0$  and  $B_1 B_0$ . For this arrangement truth table should have 4 inputs & 16 entries:

INPUT				OUTPUT		
$A_1$	$A_0$	$B_1$	$B_0$	$A > B$	$A=B$	$A < B$

### 13.8.2 Table for the 8-bit magnitude comparator:

Table is same as the table of the 2-bit magnitude comparator but only difference is that there is only change in the component of the applying signals:

INPUT				OUTPUT		
$A_1$	$A_0$	$B_1$	$B_0$	$A > B$	$A=B$	$A < B$




### 13.9 Calculation

$$\begin{aligned}
 A > B &= A_1 B_1' + A_0 B_0' A_1' B_1' + A_0 B_0' A_1 B_1 \\
 &= A_1 B_1' + A_0 B_0' (A_1' B_1' + A_1 B_1) \\
 &= A_1 B_1' + A_0 B_0' X_1.
 \end{aligned}$$

$$\begin{aligned}
 A = B &= A_1' A_0' B_1' B_0' + A_1' A_0 B_1' B_0 + A_1 A_0' B_1 B_0' + A_1 A_0 B_1 B_0 \\
 &= (A_1' B_1' + A_1 B_1) (A_0' B_0' + A_0 B_0) \\
 &= X_1 X_0.
 \end{aligned}$$

$$\begin{aligned}
 A < B &= A_1' B_1 + A_0' B_0 A_1' B_1' + A_0' B_0 A_1 B_1 \\
 &= A_1' B_1 + A_0' B_0 (A_1' B_1' + A_1 B_1) \\
 &= A_1' B_1 + A_0' B_0 X_1.
 \end{aligned}$$

### 13.10 Result

### 13.11 Discussion

## 13.12 Precautions and Sources of Errors

1. It should be noted that every Boolean equation is verified.
2. Every connection should be checked by teacher.
3. Every reading is taken by very precise manner.
4. It should be clear between the forward bias and the reverse bias.
5. Connection of gates should be make vary precisely because it is so sensitive.

## 13.13 Self Learning Exercise-II

- Q.1** Explain about setup time and hold time, what will happen if there is setup time and hold time violation, how to overcome this?
- Q.2** Difference between one hot and binary encoding?
- Q.3** What is a p-n junction diode? State and explain some of its applications.
- Q.4** Identify each of these logic gates by name, and complete their respective truth tables



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	Output
0	
1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	



A	B	Output
0	0	
0	1	
1	0	
1	1	

Q.5 Give the Properties of the Digital electronics.

### 13.14 Glossary

**Bit:** A binary digit is known as bit.

**Logic Gates:** Logic gates are the basic elements that make up a digital system. The electronic gate is a circuit that is able to operate on a number of binary inputs in order to perform a particular logical function.

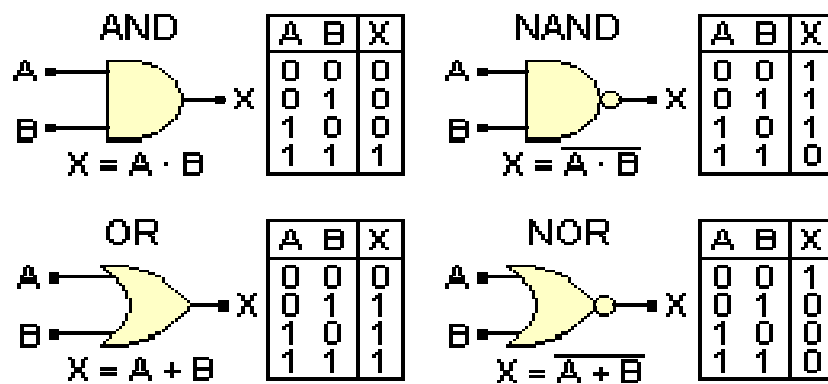
**Digital Circuit:** Digital circuits are electronics that handle digital signals – discrete bands of analog levels – rather than by continuous ranges as used in analog electronics.

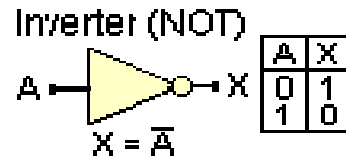
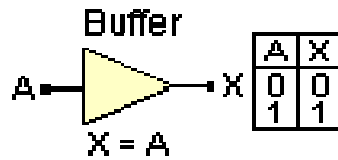
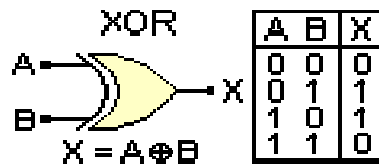
**IC:** Integrated Circuits

### 13.15 Answer to Self Learning Exercises

#### *Answer to Self Learning Exercise-I*

**Ans.1:** Logic gates (or simply gates) are the fundamental building blocks of digital circuitry. As their name implies, they function by "opening" or "closing" to admit or reject the flow of digital information. Gates implement electronically simple logical operations on Boolean (Bool's algebra) variables, i.e. variables that can have only one of two states (0/1, low/high, false/true). From an electrical point of view and for the TTL (transistor-transistor-logic) family of digital electronics, any voltage in the range 0-0.7 V and in the range 2.5-5 V, represent logic states 0 and 1, respectively. In the following figure the accepted electronic symbols for different gates are shown, along with their corresponding "truth tables" and their symbolic logical expressions. All variables (X, A, B, ...) are Booleans.



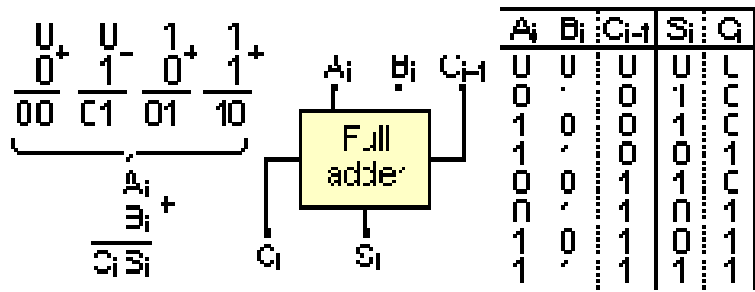


The most typical logical operations are implemented by AND and OR gates. The logical expression for the AND operation is “if A is true AND B is true then X is true”, and for the OR operation is “if A is true OR B is true then X is true”. The inverted logic AND and OR gates are commonly known as NAND (Not AND) and NOR (Not OR) gates. A XOR (Exclusive-OR) gate implements the logical expression “if A is different than B then X is true”, hence sometimes this gate is called “inequality comparator”.

The buffer and the inverter are not gates but their use is closely associated with them. A buffer doesn't change the logic state of its input. It is only occasionally used for increasing the fan-out, i.e. the capability of the output of one gate to drive a number of other gates. The inverter is much more important and it is used for inverting a logic state, i.e. for performing the logical operation of negation (NOT). The logical expressions for a buffer and an inverter are “X is A” and “X is NOT A”, respectively. AND, OR, NAND and NOR gates can have more than 2 inputs. In this case their truth tables are extended to all inputs combinations and their corresponding expressions as well. For example, the logical expression for a 4-input AND is “if A is true AND B is true AND C is true AND D is true then X is true”. The corresponding expression for a 3-input NOR gate is “if A is true OR B is true OR C is true then X is false”

**Ans.2:** Decoders are circuits with two or more inputs and one or more outputs, resulting by combining various types of gates. Their basic function is to accept a binary word (code) as an input and create a different binary word as an output. A typical decoder is the so-called full adder (3 inputs-2 outputs) implementing the addition of two one-digit numbers ( $A_i$ ,  $B_i$ ) taking into consideration the status of any previous carry ( $C_{i-1}$ ), resulting into the sum ( $S_i$ ), and generating a new carry

(C<sub>i</sub>). The addition of two 1-digits numbers and the corresponding truth table of full adder are shown below:



N full adders can be cascaded to form a unit for the addition of two N-digits binary numbers. Decoders with any type of truth table can be constructed by using simple or complicated combinations of gates. Implementation of Bool’s algebra rules generally simplifies the overall design. Simple and useful decoders are the so-called “2-to-4” and “3-to-8” decoders.

**Ans.3:** Encoders are logic circuits which change a set of signals into a code using certain logic. It has a set of input signals of which only one line is active at a certain instant of time.

Decoders are logic circuits which accept a set of inputs that represent a binary number and activate only the output that corresponds to the input number. It produces only one output as high for an input set.

Encoding and Decoding is done for the safe transmission of data. Now a days, data is encrypted using 128-bit encryption. It may have 2 to the power of 128 possible combinations. This is widely regarded as immune to brutal force attacks from hackers simply because it would take too long to crack.

### ***Answer to Self Learning Exercise-II***

**Ans.1:** For Synchronous flip-flops, we have special requirements for the inputs with respect to clock signal input there are Setup Time: Minimum time Period during which data must be stable before the clock makes a valid transition. E.g. for a positive edge triggered flip-flop having a setup time of 2ns so input data should be Stable for 2ns before the clock makes a valid transaction from zero to one.

**Hold Time:** Minimum time period during which data must be stable after the clock has made a valid transition. E.g. for a posedge triggered flip-flop, with a hold time of 1 ns. Input Data (i.e. R and S in the case of RS flip-flop) should be stable for at least 1 ns after clock has made transition from 0 to 1. Hold time is the amount of time after the clock edge that same input signal has to be held before changing it to make sure it is sensed properly at the clock edge. Whenever there are setup and hold time violations in any flip-flop, it enters a state where its output is unpredictable: this state is known as metastable state (quasi stable state); at the end of metastable state, the flip-flop settles down to either '1' or '0'. This whole process is known as metastability.

**Ans.2:** Common classifications used to describe the state encoding of an FSM are Binary (or highly encoded) and One hot. A binary-encoded FSM design only requires as many flip-flops as are needed to uniquely encode the number of states in the state machine. The actual number of flip-flops required is equal to the ceiling of the log-base-2 of the number of states in the FSM. A one hot FSM design requires a flip-flop for each state in the design and only one flip-flop (the flip-flop representing the current or "hot" state) is set at a time in a one hot FSM design. For a state machine with 9- 16 states, a binary FSM only requires 4 flip-flops while a one hot FSM requires a flip-flop for each state in the design. FPGA vendors frequently recommend using a one hot state encoding style because flip-flops are plentiful in an FPGA and the combinational logic required to implement a one hot FSM design is typically smaller than most binary encoding styles.

Since FPGA performance is typically related to the combinational logic size of the FPGA design, one hot FSMs typically run faster than a binary encoded FSM with larger combinational logic blocks.

**Ans.3:** A p-n junction is a metallurgical junction formed at the boundary between a p-type and n-type semiconductor created in a single crystal of semiconductor by doping. It acts as a diode as is a two-terminal electronic component with asymmetric transfer characteristics. The following are some of its applications:-

**Rectifiers:** A rectifier is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction.

The types of rectifiers are-

- i. Half wave rectifier- Uses one p-n junction,
- ii. Full wave rectifier- Uses two p-n junction.

Signal diodes: These diodes are used in circuits that mix signals together (mixers), detect the presence of a signal (detector), and act as a switch “to open or close a circuit”.

Tunnel diodes: Very high frequency applications using the tunnel diode are possible because the tunnelling action occurs so rapidly that there is no transit time effect and therefore no signal distortion. Tunnel diodes are also used extensively in high-speed switching circuits because of the speed of the tunnelling action.

Zener diodes: It is very highly doped acts as a voltage regulator and operates under reverse bias in the breakdown region.

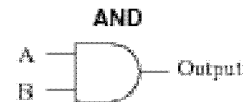
**Ans.4:** Answer is given by the figure:



A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



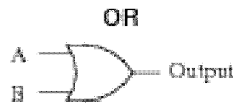
A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0



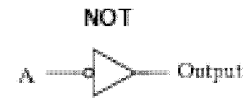
A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1



A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1



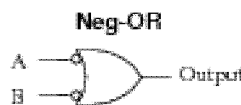
A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1



A	Output
0	1
1	0



A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0



A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

**Ans.5:** An advantage of digital circuits when compared to analog circuits is that signals represented digitally can be transmitted without degradation due to noise.<sup>[8]</sup> For example, a continuous audio signal transmitted as a sequence of 1s and 0s, can be reconstructed without error, provided the noise picked up in transmission is not enough to prevent identification of the 1s and 0s. An hour of music can be stored on a compact disc using about 6 billion binary digits.

In a digital system, a more precise representation of a signal can be obtained by using more binary digits to represent it. While this requires more digital circuits to process the signals, each digit is handled by the same kind of hardware, resulting in an easily scalable system. In an analog system, additional resolution requires fundamental improvements in the linearity and noise characteristics of each step of the signal chain.

Computer-controlled digital systems can be controlled by software, allowing new functions to be added without changing hardware. Often this can be done outside of the factory by updating the product's software. So, the product's design errors can be corrected after the product is in a customer's hands.

Information storage can be easier in digital systems than in analog ones. The noise-immunity of digital systems permits data to be stored and retrieved without degradation. In an analog system, noise from aging and wear degrade the information stored. In a digital system, as long as the total noise is below a certain level, the information can be recovered perfectly.

Even when more significant noise is present, the use of redundancy permits the recovery of the original data provided too many errors do not occur.

In some cases, digital circuits use more energy than analog circuits to accomplish the same tasks, thus producing more heat which increases the complexity of the circuits such as the inclusion of heat sinks. In portable or battery-powered systems this can limit use of digital systems.

For example, battery-powered cellular telephones often use a low-power analog front-end to amplify and tune in the radio signals from the base station. However, a base station has grid power and can use power-hungry, but very flexible software radios. Such base stations can be easily reprogrammed to process the signals used in new cellular standards.



Digital circuits are sometimes more expensive, especially in small quantities.

Most useful digital systems must translate from continuous analog signals to discrete digital signals. This causes quantization errors. Quantization error can be reduced if the system stores enough digital data to represent the signal to the desired degree of fidelity. The Nyquist-Shannon sampling theorem provides an important guideline as to how much digital data is needed to accurately portray a given analog signal.

In some systems, if a single piece of digital data is lost or misinterpreted, the meaning of large blocks of related data can completely change. Because of the cliff effect, it can be difficult for users to tell if a particular system is right on the edge of failure, or if it can tolerate much more noise before failing.

Digital fragility can be reduced by designing a digital system for robustness. For example, a parity bit or other error management method can be inserted into the signal path. These schemes help the system detect errors, and then either correct the errors, or at least ask for a new copy of the data. In a state-machine, the state transition logic can be designed to catch unused states and trigger a reset sequence or other error recovery routine.

Digital memory and transmission systems can use techniques such as error detection and correction to use additional data to correct any errors in transmission and storage.

On the other hand, some techniques used in digital systems make those systems more vulnerable to single-bit errors. These techniques are acceptable when the underlying bits are reliable enough that such errors are highly unlikely.

A single-bit error in audio data stored directly as linear pulse code modulation (such as on a CD-ROM) causes, at worst, a single click. Instead, many people use audio compression to save storage space and download time, even though a single-bit error may corrupt the entire song.

### **13.16 Viva Questions**

- Q.1** State some of applications of buffer?
- Q.2** Define Digital Systems.
- Q.3** What is meant by bit?

- Q.4** What is the best example of digital system?
- Q.5** Define Radix.
- Q.6** Define Nibble and Byte.
- Q.7** List the number systems?
- Q.8** Define binary logic.
- Q.9** What are Logic gates?
- Q.10** What are the basic digital logic gates?

### 13.17 Answers to Viva Questions

**Ans.1:** (a) They are used to introduce small delays.

b) They are used to eliminate cross talk caused due to inter electrode capacitance due to close routing.

c) They are used to support high fan-out,

**Ans.2:** A system which processing discrete or digital signal is called as Digital System.

**Ans.3:** A binary digit is called bit.

**Ans.4:** Digital computer is the best example of a digital system.

**Ans.5:** It specifies the number of symbols used for corresponding number system. .

**Ans.6:** i). In binary number a group of four bits.

ii). A group of 8 bits are called Byte.

**Ans.7:** i) Decimal Number system

ii) Binary Number system

iii) Octal Number system

iv) Hexadecimal Number system

**Ans.8:** Binary logic consists of binary variables and logical operations. The variables are designated by the alphabets such as A, B, C, x, y, z, etc., with each variable having only two distinct values: 1 and 0. There are three basic logic operations: AND, OR, and NOT.

**Ans.9:** Logic gates are the basic elements that make up a digital system. The

electronic gate is a circuit that is able to operate on a number of binary inputs in order to perform a particular logical function.

**Ans.10:** The three basic logic gates are:

1. AND gate
2. OR gate
3. NOT gate.

## References and Suggested Readings

1. Tokjeim R.L; Digital Principles; McGraw Hill Book.
2. TTL Logic Data Book.
3. Digital Technology by Nashelsky L.
4. Digital Circuits and Systems by Douglas V. Hall.

# UNIT-14

## Colpitt Oscillator

### Structure of the Unit

- 14.1 Aim
  - 14.2 Apparatus
  - 14.3 Diagram
  - 14.4 Formula
  - 14.5 Theory and Description
  - 14.6 Self Learning Exercise -I
  - 14.7 Procedure
  - 14.8 Observation
  - 14.9 Calculations
  - 14.10 Result
  - 14.11 Discussion
  - 14.12 Precautions and Sources of Error
  - 14.13 Self Learning Exercise -II
  - 14.14 Glossary
  - 14.15 Answers to Self Learning Exercises
  - 14.16 Viva Questions
  - 14.17 Answers to Viva Questions
- References and Suggested Readings

### 14.1 Aim

To study the characteristics of the Colpitt's Oscillator.

### 14.2 Apparatus

CRO, VTVM, an A. C. milli or microvoltmeter, a transistor, Resistance.

## 14.3 Diagram

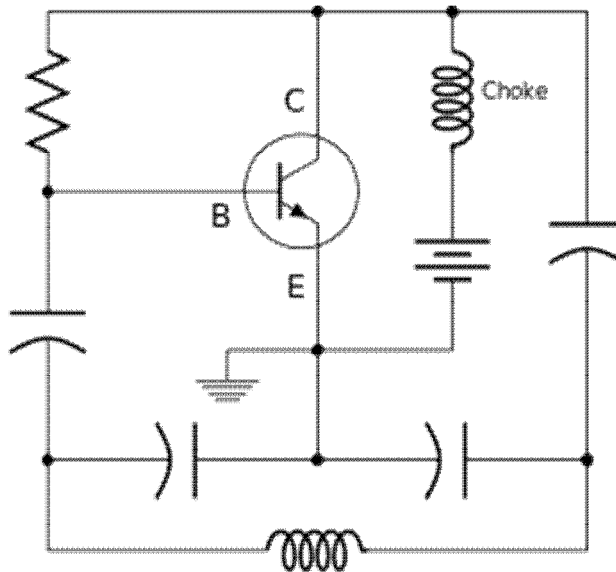


Figure 14.1: Colpitt Oscillator.

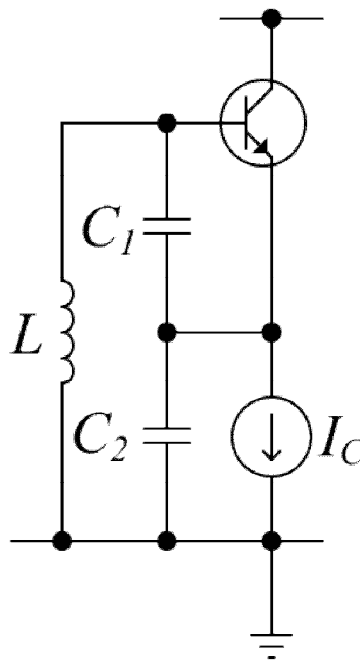


Figure 14.2: Common collector colpitt oscillator

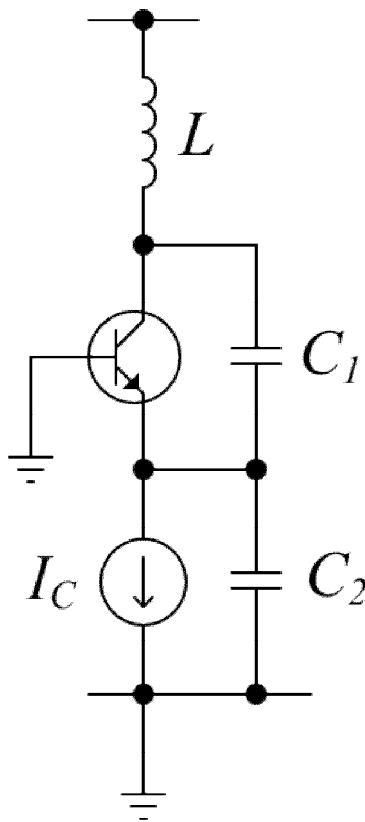


figure 14.3: Common base.

### 14.4 Formula

We know that for the oscillator, we have a Barkhausen condition, which is

$$Z + \frac{R_a}{1 + \alpha\mu} = 0$$

Here

$Z$  is the impedance of coupling output network,

$R_a$  anode slope resistance and

$\mu$  amplification factor of the valve and

$\alpha$  is the feedback factor i.e.

$$\alpha = \frac{\text{feedback voltage}}{\text{output voltage}}$$

## 14.5 Theory and Description

It is basically a CE amplifier with a feedback path through  $C_e$ . It is similar to Hartley oscillator except that it employs tapped capacitor  $C_1$  and  $C_2$  in place of tapped inductor. The two capacitors  $C_1$  and  $C_2$  in series form the voltage divider. The voltage developed across  $C_2$  serves as feedback voltage. The feedback factor (feedback fraction)  $m$  is given by  $C_1/C_2$ .

At lower frequencies the output voltage  $V_o$  is  $180^\circ$  out of phase with the input, but at higher frequencies a phase shift takes place at the LC-network and at a certain higher frequency  $V_o$  is in phase with the input. This in-phase output voltage  $V_o$  is fed back to the input through  $C_c$ . The R.F. Choke prevents the R.F. currents from flowing to the battery. The resistance  $R_1$  and  $R_2$  are for maintaining the bias.

The output voltage of the oscillator is sinusoidal because the resonant tank circuit offers high impedance only to the component of current with fundamental frequency.

The frequency of oscillation is given by,

$$F = 1/2\pi\sqrt{LC}$$

$$\text{Where, } C = C_1C_2/(C_1+C_2)$$

As the source of D.C. supply is switched on, there is a sudden rise in collector current which initiates oscillations in the tank circuit. These oscillations are sustained by the positive feedback and the amplifying action of the transistor. The output may be taken by inductive coupling to L.

## 14.6 Self Learning Exercise -I

**Q.1** Determine the

- (i) Operating frequency and
- (ii) Feedback fraction for Colpitt's oscillator having

$$C_1 = 0.001 \text{ and } C_2 = 0.01 \text{ and } L = 15 \mu H$$

**Q.2** In a Colpitt oscillator

$$C_1 = 30 \text{ pF and } C_2 = 60 \text{ pF and } L = 10 \mu H,$$

Find the operating frequency of the oscillator.

## 14.7 Procedure

1. Set an oscillator and note d.c. operating conditions. Also note conditions of sustained oscillations and expected frequency range.
2. In order to measure the output voltage, either use a calibrated CRO or calibrate its y-direction in terms of known a.c. voltage and plot a calibration curve for a definite gain of Y-amplifier provided CRO. However, voltage output of audio frequency oscillations is of the order of volts while that of radio frequency oscillations is usually quite low of order milli or micro volt. Hence the sensitivity of CRO is also to be kept in mind before using it.
3. Alternately low voltage output of oscillator may be connected at the input of FET and the output current may be measured by a sensitive  $\mu A$  having a bridge rectifier, say made by point contact rectifiers. If the  $\mu A$  has been precalibrated in terms of known input, the unknown input voltage may be found. Measure the Y trace length on scope screen and reduce it to volts.
4. In order to find the power output of oscillator, directly measure its value with the help of calibrated output meter. The open circuit output current of the oscillator may also be measured by connecting a suitable ammeter with a bridge rectifier. Similarly, V may be measured and power  $P = VI$  and  $Z = V/I$  may be computed.
5. In order to determine the internal impedance of the oscillator, first measure its open circuit output voltage, say V, by CRO. Next connect a known non inductive variable resistance at its output and also connect the measuring voltage instrument CRO across it. Adjust resistance so that the reading becomes half of the open circuit value, say  $V/2$ . Read the resistance R, this is equal to the internal impedance of the oscillator.
6. In order to note the wave shape, frequency and voltage amplitude of the wave of the oscillator, connect its output at y-deflection of CRO and give it a suitable gain. Work the internal time base which gives X-deflection. Adjust time base frequency and gain so that one complete wave is displayed on screen. Read the time base frequency, this is equal to the frequency of the unknown wave. Measure peak to peak voltage corresponding to this wave and note it. Trace the



wave on graph paper. If the number of waves on screen are an integer, say  $n$ , where  $n=2, 3, 4, 5\dots$ . Then unknown frequency is the product of time base frequency and  $n$ .

- Repeat experiment for different oscillator frequencies and plot the variations graphically. The dial attached with the gang capacitor of the resonant circuit is calibrated in terms of frequency.

## 14.8 Observation

**Table 1: Circuit parameters for optimum operating conditions.**

Valve specifications				Load $R_1$ $K\Omega$	Gain $A$	Feedback factor $\alpha$	Coupling and frequency control				
Valve No.	$R_a$ $K\Omega$	$\mu$	$g_m$ m/o hm				$L$ $\mu H$	$L'$ $\mu H$	$M$ $= \sqrt{LL'}$ $\mu H$	Range of C $pF$	R $K\Omega$

**Table 2: Experimentally observed values.**

S. No..	Output values				Peak to peak voltage	Frequency of wave	Wave shape
	Voltage V	Current $I$	Power $P = VI$	Impedance $Z = V/I$			
	Volt	Amp.	Watt	$K\Omega$	Volt	Hz	....

## 14.9 Calculations

Calculation is based on the formula which is given by

$$Z + \frac{R_a}{1 + \alpha\mu} = 0$$

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## 14.10 Result

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**Percentage Error:**

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

## 14.11 Discussion

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## 14.12 Precautions and Sources of Error

1. It should be noted that the output and the input should be in same phase.
2. Circuit should be checked first checked by teacher.
3. All the connections should be well connected.

## 14.13 Self Learning Exercise -II

- Q.1** Give the Limitations of LC and RC Oscillators.
- Q.2** What are transistors? Give the theory of Field effect transistor.
- Q.3** Give the theory of one Power supply filter.

## 14.14 Glossary

**Impedance:** In LCR circuit the ratio of total voltage with the total current, is known as Impedance.

**Inductance:** The property of an electric conductor or circuit that causes an electromotive force to be generated by a change in the current flowing.

**Capacitance:** The ability of a system to store an electric charge.

**Resistance:** A measure of the degree to which conductor opposes an electric current through that conductor.

**Feedback Circuit:** The feedback circuit supplies a part of collector energy to the tank circuit in correct phase to aid the oscillations i.e. it provides positive feedback.

**Frequency of oscillations:** The frequency of oscillations in the tank circuit is determined by the constants of the circuit L and C. The actual frequency of oscillations is the resonant frequency of the tank circuit given by:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

## 14.15 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:(i) Operating Frequency:**

The operating frequency of the circuit is always equal to the resonant frequency of the feedback network. As noted previously, the capacitors  $C_1$  and  $C_2$  are in series

$$\begin{aligned} C_T &= \frac{C_1 C_2}{C_1 + C_2} = \frac{0.001 \times 0.01}{0.001 + 0.01} = 9.09 \times 10^{-4} \mu F \\ &= 909 \times 10^{-12} F \end{aligned}$$

Operating frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC_T}} = \frac{1}{2\pi\sqrt{15 \times 909 \times 10^{-18}}}$$

$$f = 1361 \text{ kHz}$$

(ii) Feedback fraction:

$$m_v = \frac{C_1}{C_2} = \frac{0.001}{0.01} = 0.1$$

**Answer 2: Operating Frequency:** The operating frequency of the circuit is always equal to the resonant frequency of the feedback network.

The capacitors  $C_1$  and  $C_2$  are in series

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{30 \times 60}{30 + 60} = 20 \times 10^{-12} \text{ F}$$

Operating frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 20 \times 10^{-18}}}$$

$$f = 11.25 \text{ MHz}$$

### **Answers to Self Learning Exercise-II**

**Ans.1:** The LC and RC oscillators discussed so far have their own limitations. The major problem in such circuits is that their operating frequency does not remain strictly constant. There are two principal reasons for it

(i) As the circuit operates, it will warm up. Consequently, the values of resistors and inductors, which are the frequency determining factors in these circuits, will change with temperature. This causes the change in frequency of the oscillator.

(ii) If any component in the feedback network is changed, it will shift the operating frequency of the oscillator.

However, in many applications, it is desirable and necessary to maintain the frequency constant with extreme low tolerances. For example, the frequency tolerance for broadcasting station should not exceed 0.002% i.e. change in frequency due to any reason should not be more than 0.002% of the specified frequency. The broadcasting stations have frequencies which are quite close to each other. In fact, the frequency difference between two broadcasting stations is less than 1%. It is apparent that if we employ LC and RC circuits, a change of temperature may cause the frequencies of adjacent broadcasting stations to overlap.

**Ans.2:** Transistor is a semiconductor device consisting of two  $p - n$  junctions placed back to back, one under a forward bias and the other under a reverse bias. A forward-biased  $p - n$  junction offers a low resistance, while a reverse-biased one offers high resistance. Therefore, a small signal applied across the forward-biased junction would appear with a much higher voltage across the reverse-biased junction. The device can thus act as an amplifier.

Transistor classified in two types:

- (1) Unipolar transistor,
- (2) Bipolar transistor.

A unipolar transistor is a device in which the current flows due to the majority carriers.

A bipolar transistor is a device in which both majority and minority carriers play significant roles.

**Field Effect Transistor:** A field effect transistor is a semiconductor electronic device in which the current is controlled by the variation of an electric field and is carried by majority carriers only. The FET is an unipolar device because current is flow from only one type of carrier.

There are two types of FET: (1) Junction field-effect transistor (JFET), (2) Metal-oxide-semiconductor field-effect transistor (MOSFET).

There are two type of classes of JFET

:  $n$  -channel JFET and

$p$  -channel JFET.

An n-channel JFET consists of lightly doped n-type semiconductor bar, with heavily-doped p-type regions diffused on both sides (fig.1 a). These regions are internally connected and are called 'Gate' G. The n-type region between the two gate-regions is called the 'channel'. Terminals are attached through ohmic contacts to both the ends of the channel. One of them is called the 'Source' S and the other is called the 'Drain' D. During operation, majority carriers enter the channel through the source S and leave it through the drain D. The current is controlled by the gate which is always *reverse - biased* and hence draws very small current.

The Fig. 1(b) shows the symbol of the n-channel JFET. The arrow at the gate terminal points in the direction in which the conventional current would flow if the gate junction were forward-biased.

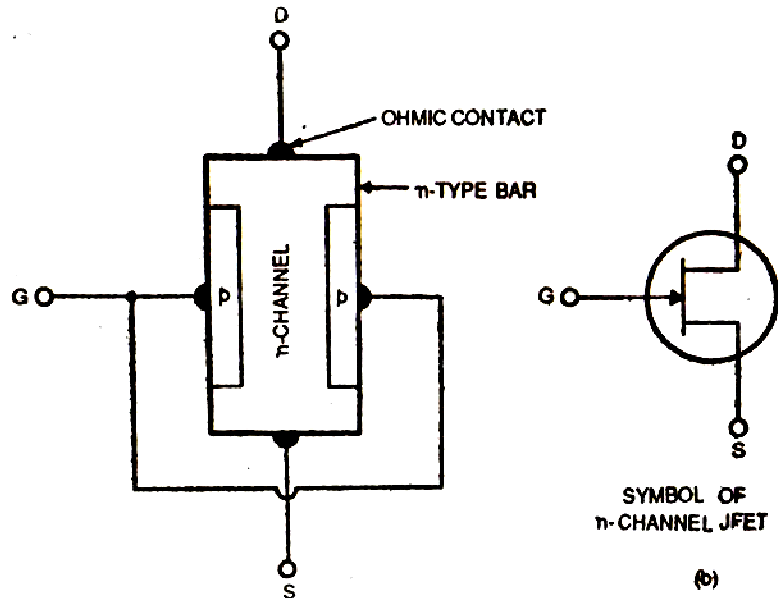


Figure 1(a),(b)

The p-channel JFET is similar in construction except that it uses a p-type semiconductor bar with n-type regions diffused on both sides Fig. 2(a).

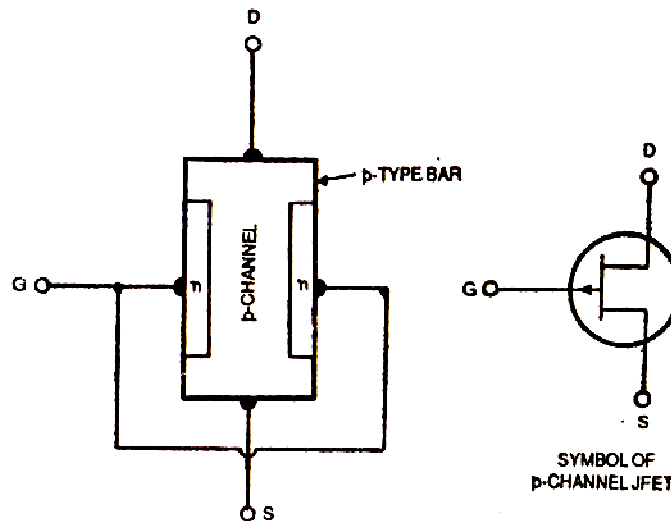


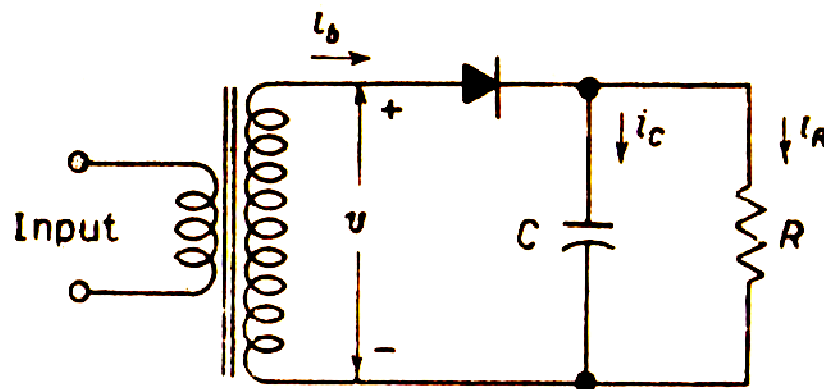
Figure 2(a),(b)

In this case, the majority charge carriers which flow through the p-channel are “holes”. The symbol of p-channel JFET is shown in figure 2(b), the arrow points out, like that in an n-p-n transistor.

**Ans.3:** Filter circuits are employed to reduce the rectifier output ripple. This is achieved by either by passing the ac output components around the load by a shunt capacitance, or limiting their magnitude to a low value in the load by a series inductance or a combination of these two for more efficient circuits.

The circuit arrangement of the capacitor filter is shown in below given figure 1(a). The capacitance is so chosen that  $X_C \ll R$ , and alternating currents find a low-reactance shunt in C.

Only a small alternating current component passes in C. Only a small alternating current component passes in R, producing a small ripple voltage. The capacitor alters the conditions under which the diode operates.



**Figure 1(a)**

When the diode output voltage is increasing, the capacitor stores energy, by charging to the peak to the input cycle as shown in Figure 1(b). With falling source-voltage, the diode disconnects the source from the load at the instant when the source voltage starts to fall faster than the capacitor voltage can fall, as determined by the time constant of C and the load. The capacitor continues to maintain the load voltage at a higher value and lower ripple than if the capacitor were not present. The diode delivers a charging pulse of current in each cycle and

then disconnects source from the load. The circuit is excited by repeated current transients as shown in figure 1 (c).

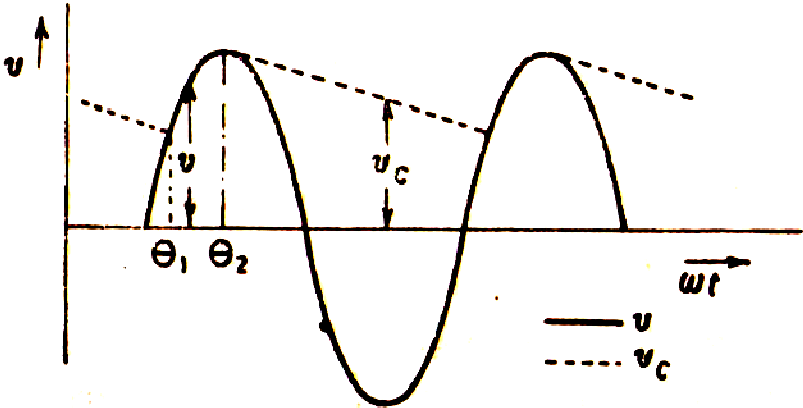


Figure 1(b)

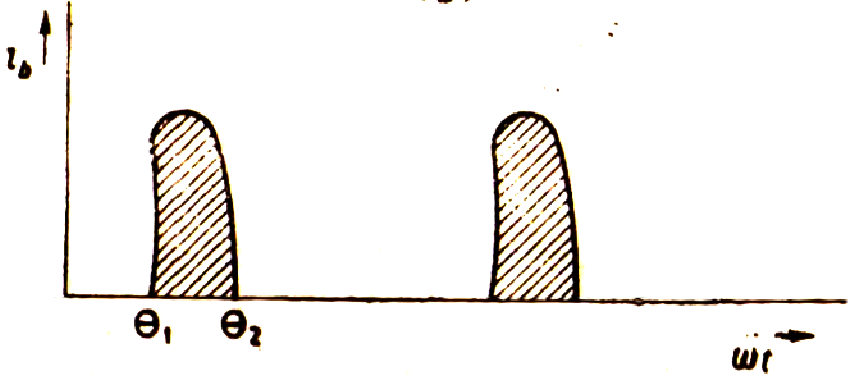


Figure 1(c)

Similarly we can define for full-wave circuit, whose circuit is given below in figure 2.

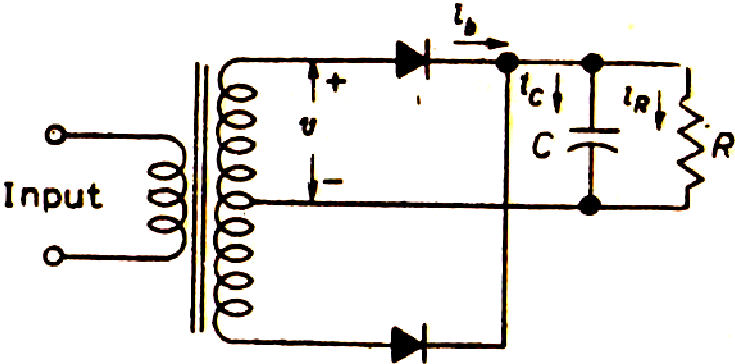


Figure 2(a)



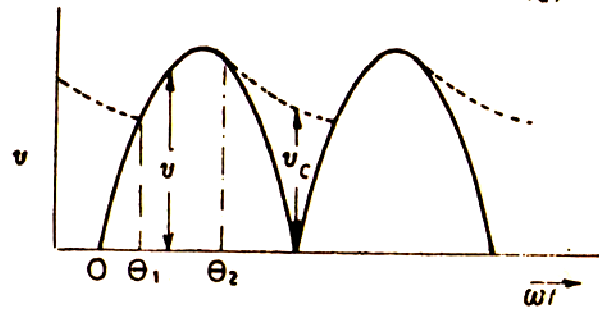


Figure 2(b)

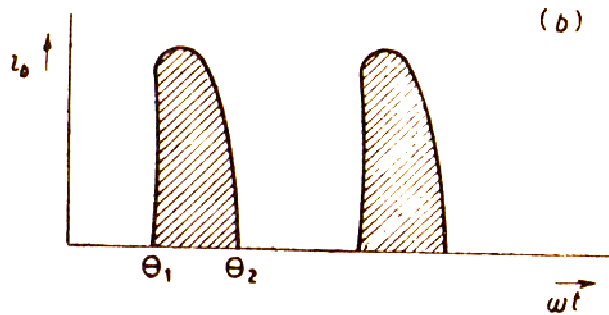


Figure 2(c)

## 14.16 Viva Questions

- Q.1 What is rectification?
- Q.2 What is rectifier?
- Q.3 What is tank circuit?
- Q.4 What is audio frequency generator?
- Q.5 Define Dissipation factor.
- Q.6 Define quality factor.
- Q.7 What should be the minimum voltage gain in a phase shift oscillator so as to maintain oscillations?
- Q.8 What are Filter circuits?
- Q.9 What are Power amplifiers?
- Q.10 What is feedback?

## 14.17 Answers to Viva Questions

**Ans.1:** A process by which the alternating current is changes into direct current known as Rectification.

**Ans.2:** An electronic circuit in which input is alternating current and output is direct current.

**Ans.3:** An electronic circuit which produces electrical oscillations of any desired frequency is known as an oscillatory circuit or tank circuit.

**Ans.4:** An electronic device which supplies the frequency ranging from 20 Hz to 20 kHz.

**Ans.5:** It is the reciprocal of the 'Quality'  $Q$ , i.e. the dissipation factor is the ratio of the resistance to reactance. Hence

$$D = \frac{1}{Q} = \frac{R}{X}$$

Here  $X$  is the reactance.

**Ans.6:** The quality factor is a dimensionless parameter that describes how under damped an oscillator or resonator is and it is also defines as well as a resonator's bandwidth relative to higher frequency of resonator.

For the loss of low energy, we have the quality factor large.

$$Q = \frac{X}{R}$$

**Ans.7:** The minimum voltage gain in a phase shift oscillator be 29 so that the condition  $\beta A = 1$  is satisfied and self-sustained oscillations start.

**Ans.8:** A filter circuit is a device which removes the a.c. component of rectifier output but allows the d.c. component to reach the load.

**Ans.9:** A transistor amplifier which raises the power level of the signals that have audio frequency range is known as transistor audio power amplifier.

**Ans.10:** The process of injecting a fraction of output energy of some device back to the input is known as feedback.

## References and Suggested Readings

1. "Electronic Devices Applications And Integrated Circuits" by "Satnam P. Mathur" "Durgesh C. Kulshreshtha" and "Prem R. Chadha".
2. Electronics Theory And Applications by "S.L. Kakani" and "K.C. Bhandari"

# UNIT-15

## Applications of Operational Amplifier

### Structure of the Unit

- 15.1 Aim
  - 15.2 Apparatus
  - 15.3 Diagram
  - 15.4 Formula
  - 15.5 Theory and Description
  - 15.6 Self Learning Exercise-I
  - 15.7 Procedure
  - 15.8 Observation
  - 15.9 Calculations
  - 15.10 Result
  - 15.11 Discussion
  - 15.12 Precautions and Sources of Error
  - 15.13 Self Learning Exercise-II
  - 15.14 Glossary
  - 15.15 Answers to Self Learning Exercises
  - 15.16 Viva Questions
  - 15.17 Answers to Viva Questions
- References and Suggested Readings

### 15.1 Aim

- (i) To study the inverting, non inverting, addition, integration and differentiation properties of an OP-amp.
- (ii) To study the operational amplifier in (a) Inverting mode, (b) non inverting mode (c) addition, (d) Integration and (e) Differentiation amplifier.

## 15.2 Apparatus

Operational amplifier (741 or 741C), audio frequency oscillator, oscilloscope, DC voltage supply, non sinusoidal (alternating) input signal, CRO, VTVM, different resistances and cords for connection etc.

## 15.3 Diagram

### (A) Inverting Amplifier

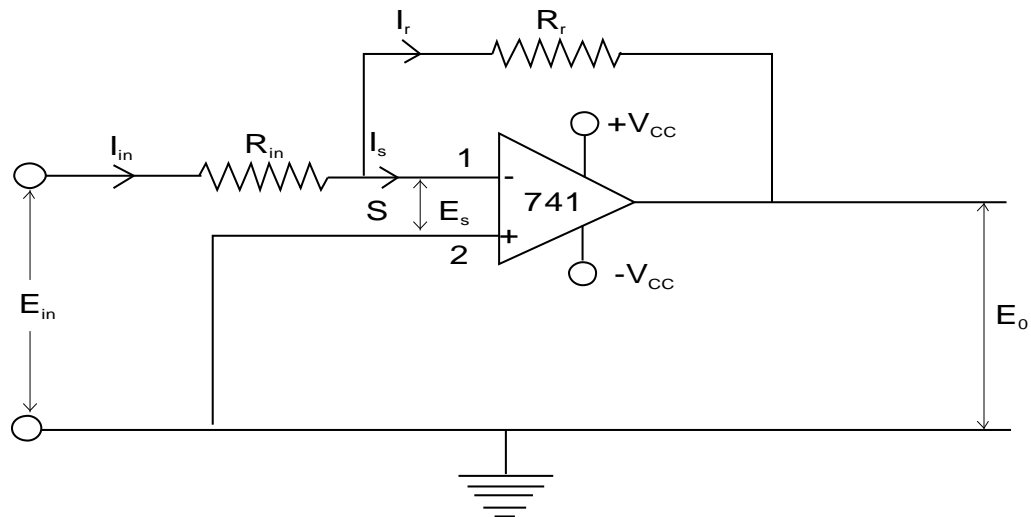


Fig. 15.1: Inverting Amplifier

### (b) Non-Inverting amplifier:-

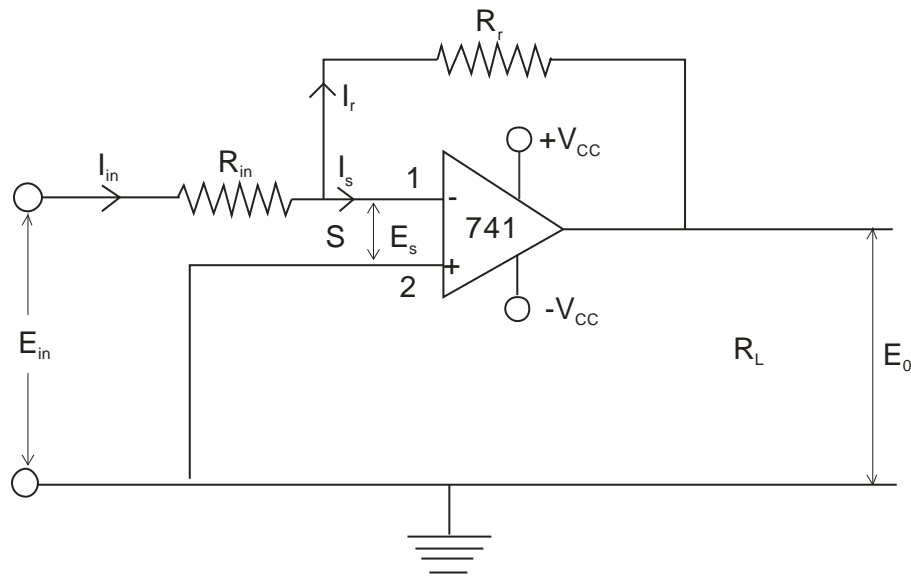
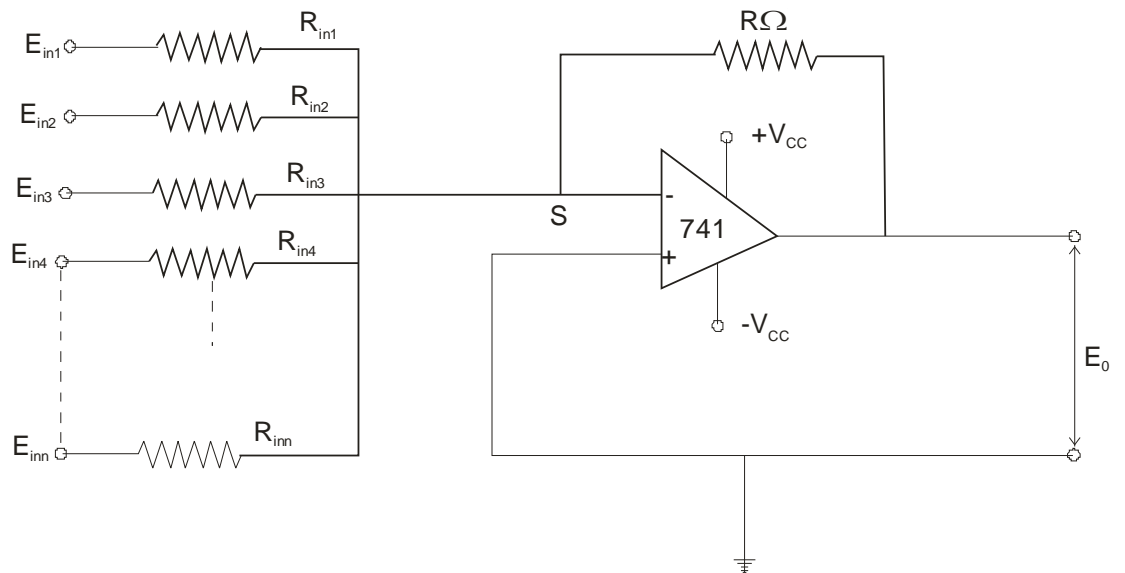


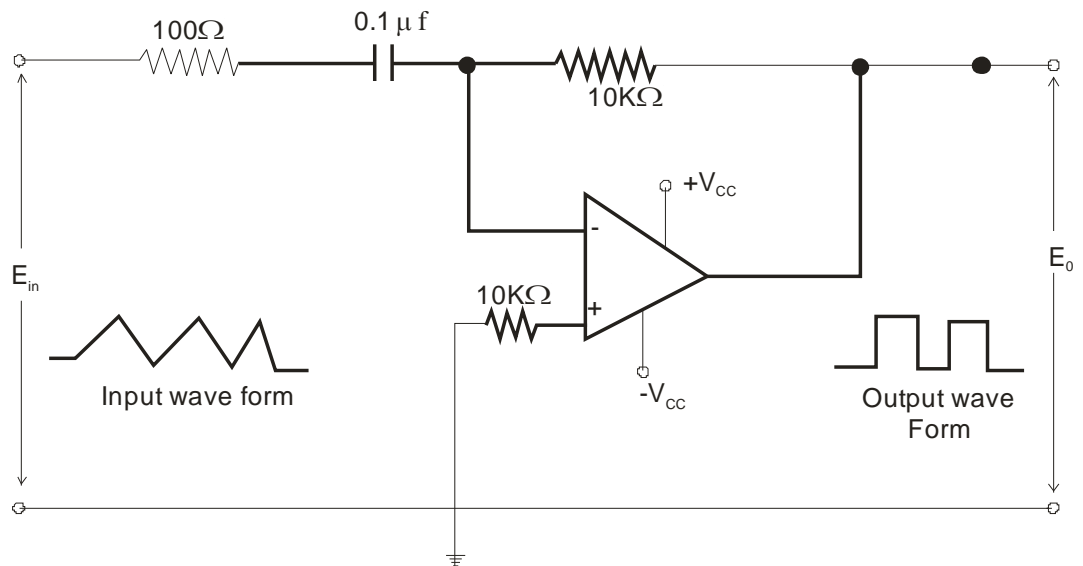
Fig. 15.2: Inverting Amplifier

**(c) Addition:-**



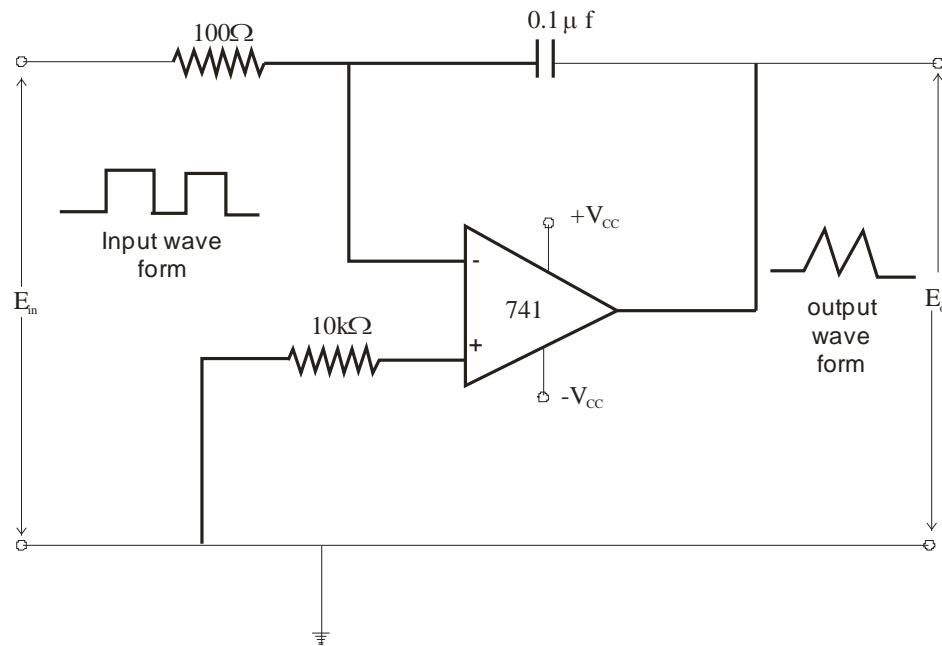
**Fig. 15.3: Operational Amplifier As An Adder.**

**(d) Differentiation:-**



**Fig. 15.4: OP-amp Differentiation**

(e) **Integration:-**



**Fig. 15.5: OP-amp Integrator**

## 15.4 Formula

The voltage gain of Inverting OP-amp is

$$Av_o = \frac{E_o}{E_{in}} = -\frac{R_f}{R_{in}} \quad (1)$$

The voltage gain of Non-inverting is

$$Av_o = \frac{E_o}{E_{in}} = 1 + \frac{R_f}{R_{in}} \quad (2)$$

The output voltage is equal to the sum of the individual input voltages; this process is called scaling of variables.

$$E_o = -\frac{R_f}{R_{in}} [E_{in1} + E_{in2} + \dots + E_{inn}]$$

If  $R_{in} = R_f$

$$E_o = - [E_{in1} + E_{in2} + \dots + E_{inn}] \quad (3)$$

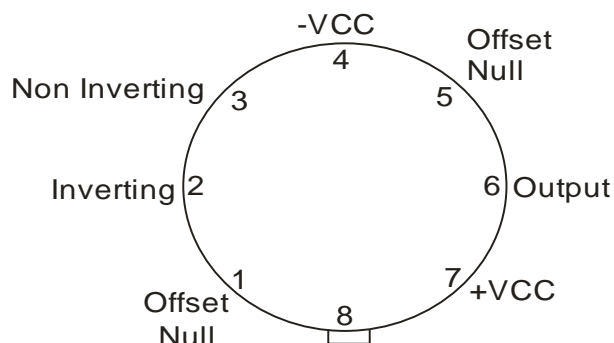
## 15.5 Theory And Description

Operational amplifier (in brief Op. Amp.) is a direct coupled amplifier whose gain is very high ( $A=75000$ ). This amplifier can be used to solve many linear and non

linear mathematical equations. Operational amplifier can also be used to perform mathematical operations like summation, subtraction, differentiation, integration etc. It is also used extensively in design of many electronic circuits e.g. voltage regulation phase change, wave generation, time base circuits and filter circuits and therefore, it is called operational amplifier. Its actual circuit configuration is very complex, it consists of many electronic devices and elements but because of integrated chip circuit e.g. I.C. 741 operational amplifiers are available in very small size. Only several terminals are accessible through which connections are made for application circuits. These terminals called pins are identified through numbers say 1 to 8 and in manufacturers manual it is specified that which pin connects the outside circuit at which point (See Fig. 15.6) e.g., pin 2 is inverting terminal. The connections are made through the connecting leads. I.C. Operational amplifier is easily available, versatile and cheap.

Operational amplifier consists of two input terminals and one output terminal. This amplifier amplifies the difference of voltages ( $V_0 = V_1 - V_2$ ) between input terminals. In Fig. 15.7 block diagram of operational amplifier circuit is shown.

Two inputs 1 and 2 are represented by - or +, which does not represent the negative or positive signals but these are representing the inverting (-) and non-inverting (+) terminals, respectively. Meaning of Negative sign at terminal is that the output would be with opposite polarity as compared to the polarity of input signal. For this input, the phase change produce by the Op. Amp is  $180^\circ$ . So this terminal or end is known as inverting input terminal. Positive sign at the terminal 2 indicates that the polarity of signal voltage at input does not change at output terminal i.e. input voltage at terminal 2 and output voltage at 0 are in same phase. Therefore, terminal 2 is known as non-inverting input terminal.



**Fig. 15.6 View of base plane of Op- Amp.**

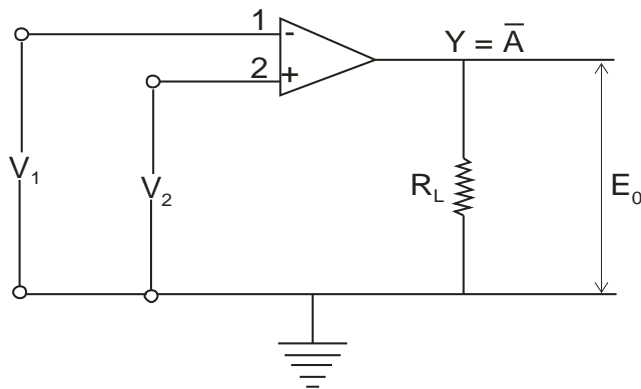
The value of output voltage  $V_0$  at terminal 1 and 2 is directly proportional to difference of input voltage ( $V_1 - V_2$ ) i.e.

$$V_0 \propto (V_1 - V_2)$$

or 
$$V_0 = A_d (V_1 - V_2) \quad \dots\dots(1)$$

Proportionality constant  $A_d$  is known as voltage gain of operational amplifier for difference signal input. For an ideal operational amplifier this gain is infinity for all signal frequencies.

In fig. 15.6 the pin diagram of operational amplifier is shown. According to this pin 2 and 3 are inverting and non-inverting inputs, respectively. Pin 4 and 7 are for supply voltage. Pin 6 is for output and pin 1 and 5 are for offset adjustments. Generally, pin 1, 5 and 8 are not used. Position of pin 1 is represented by specific mark like dot, Notch etc.



**Fig. 15.7: Block Diagram Of Operational Amplifier  
(1-inverting terminal, 2 - Non- inverting terminals)**

Operational amplifier is a high gain amplifier. The general use of this in its ideal condition is because of following characteristics -

1. Infinite voltage gain
2. Infinite band width
3. Infinite input impedance
4. Zero output impedance
5. Complete balance (at  $V_1 = V_2, V_0 = 0$ )
6. Stability in characteristics with change in temperature and voltage.



Practical operational amplifier is not an ideal one but the behavior is very close to above mentioned characteristics.

Operational amplifier has many applications but here we are studying the characteristics of an Operational amplifier in (a) Inverting mode (b) Non-inverting mode and their applications as (a) adder amplifier, (b) Integration and (c) differentiation.

**(a) Characteristics of Operational amplifier in Inverting mode operation:-**

A basic inverting operational amplifier circuit is shown in Fig. 15.1  $R_{in}$  is input resistance and  $R_f$  is feedback resistance. Non inverting terminal is grounded. The input signal (here d. c.) is applied at inverting terminal through a resistor  $R_{in}$ .  $R_f$  connected across the inverting terminal and output provides the feedback.

Since the terminal 1 is virtual ground, the current  $I_{in}$  flowing through the resistance  $R_{in}$  will be-

$$I_{in} = \frac{E_{in}}{R_{in}}$$

$$\therefore E_{in} = I_{in} R_{in} \quad (2)$$

Besides this, because of infinite input impedance, the value of  $I_s$  becomes zero, applying Kirchhoff's Current Law (KCL) at node A, we get

$$I_F + I_{in} + I_s = 0 \quad (\because I_s = 0)$$

$$\therefore I_F = -I_{in} \quad (3)$$

The equivalent circuit of Operational amplifier is shown in Fig. (15.1). It is quite obvious from fig. (15.1), that

$$E_o = I_F R_f = -I_{in} R_f \quad (4)$$

From eqn. (2) and (4) we get the gain of Operational amplifier in inverting mode  $A_{v_{inv}}$  as.

$$\text{or } A_{v_{inv}} = \frac{E_o}{E_{in}} = -\frac{R_f}{R_{in}} \quad (5)$$

So the voltage gain ( $A_{v_{inv}}$ ) of inverting amplifier depends upon ratio of feedback resistance  $R_f$  and input resistance  $R_{in}$ . It does not depend upon the open loop gain

(A) of the amplifier. Negative sign in eq. (5) represents that output voltage ( $E_o$ ) and input voltage ( $E_{in}$ ) are in opposite phase.

**(b) Non Inverting amplifier:**

A basic non-inverting operational amplifier circuit is shown in Fig. (15.2). In this circuit feedback resistance  $R_f$  is connected at inverting input (-) like inverting amplifier. From this terminal 1,  $R_{in}$  is connected with ground. Input signal  $E_{in}$  is connected with non-inverting input (+).

Some part of the output voltage ( $E_o$ ) is applied at the negative terminals of input ends with the help of voltage divider. As input current is not flowing at the input ends and because  $E_s = 0$ , applying KCL at terminal 1.

$$\frac{E_o - E_{in}}{E_f} = \frac{E_{in}}{R_{in}} \quad (6)$$

or 
$$E_{in} = \frac{R_{in}}{R_{in} + E_f} E_o$$

or 
$$A_{v_{inv}} = \frac{E_o}{E_{in}} = 1 + \frac{R_f}{R_{in}} \quad (7)$$

So, voltage gain of non-inverting amplifier is positive i. e. output voltage and input signal are in same phase. Clearly, without change in phase, sufficient gain can be obtained by this. Here, voltage gain depends only upon the ratio of  $R_f$  and  $R_{in}$ .

**(c) Summing Amplifier or Adder Amplifier:**

Operational amplifier can be used for the addition of input signals. An adder amplifier circuit is shown in Fig. 15.3.

When more than one input signals are taken at the inverting input end then addition of all input signals are obtained in opposite phase at the output end. This is also same like inverting amplifier but here more than one input signals are applied at the inverting input end.

At the junction point S of inverting end, total addition of current is zero.

$$I_{in1} + I_{in2} + I_{in3} + \dots + I_{in_n} + \frac{E_o}{R_f} = 0$$

$$\text{or } \frac{E_{in1}}{R_{in1}} + \frac{E_{in2}}{R_{in2}} + \frac{E_{in3}}{R_{in3}} + \dots + \frac{E_{inn}}{R_{inn}} = -\frac{E_0}{R_{if}}$$

$$\text{or } E_0 = -\left[ E_{in1} \left( \frac{R_f}{R_{in1}} \right) + E_{in2} \left( \frac{R_f}{R_{in2}} \right) + E_{in3} \left( \frac{R_f}{R_{in3}} \right) + \dots \right] \quad (8)$$

i.e., the value of output voltage is algebraic sum of input voltages, along with measured weight coefficient by  $R_f$ , Fig. (15.3) shows circuit for use of operational amplifier as an adder.

$$\text{If } R_{in1} = R_{in2} = R_{in3} = \dots = R_{inn} = R_f$$

$$E_0 = -\left[ E_{in1} + E_{in2} + E_{in3} + \dots + E_{inn} \right] \quad (9)$$

From eq. (8) and (9) it is quite clear that op. amp can be used as both simple adder and proportionate adder.

#### (d) Differentiator:

A differentiator in an Op-amp is a circuit that performs the output wave form which is derivative of input wave form.

In this circuit a capacitor is connected to the inverting terminal of Op-amp. Negative feedback is provided with the resistance  $R_f$  when the input voltage varies as the capacitor charges or discharges, because of virtual ground capacitor passes through the feedback resistor.

$\therefore$  By Kirchoff's current law equation's becomes

$$I_c = I_f + I_B$$

$$I_c = I_f \quad (\because I_B = 0)$$

$$C_1 \frac{d}{dt} (V_{in} - V_2) = \frac{V_2 - V_0}{R_f}$$

But A is very large then  $V_1 = V_2 = 0$

$$\text{Hence } C_1 \frac{d}{dt} (V_{in}) = \frac{-V_0}{R_f}$$

$$V_0 = -R_f C_1 \frac{d}{dt} V_{in}$$

Hence the output voltage is proportional to the differentiation of input voltage.

We have a negative sign which indicates the input is given at the inverting terminals.

**(e) Integrator:**

For an integrator OP-amp acts in inverting configuration. An integrator is a circuit that produces an output voltage wave form is the integration of the input voltage wave form.

A resistance  $R_1$  is connected to the inverting terminal of the OP - amp and feed back is provided by  $C_f$  capacitor.

By writing Kirchoff's current equation at node  $V_2$  we get

$$I_1 = I_f + I_B$$

$$I_1 = I_f (\because A \text{ is very large then } I_B \approx 0)$$

$$\frac{V_{in} - V_2}{R_1} = C_f \frac{d}{dt}(V_2 - V_0)$$

$\therefore$  A is very large then  $V_1 = V_2 = 0$

$$\text{Then } \frac{V_{in}}{R_1} = C_f \frac{d}{dt}(-V_0)$$

$$V_0 = -\frac{1}{R_1 C_f} \int_0^t V_{in} dt + C$$

So output is equal to the negative integration of input voltage.

## 15.6 Self Learning Exercise-I

- Q.1** Why this circuit is known as operational amplifier?
- Q.2** What are main features of operational amplifier circuit?
- Q.3** Mention different uses of operational amplifier?
- Q.4** If some e.m.f sources are connected in series then voltage between first and last end will be sum of voltages, then what is the utility of operational amplifier as an adder?
- Q.5** What are the main characteristics of an ideal operational amplifier?

## 15.7 Procedure

### For inverting amplifier:

1. Connect a circuit of an inverting operational amplifier according to Fig. 15.1.
2. Fix the theoretical voltage gain of an amplifier by choosing the value of  $R_f$  and  $R_{in}$ .
3. Apply voltage ( $E_{in}$ ) at input ends, order of this should be 200 mV. For every input voltage ( $E_{in}$ ) note down the corresponding value of output voltage ( $E_0$ ) in table 1.
4. Calculate the experimental voltage gain by knowing the ratio of  $E_0/E_{in}$  and compare this with theoretical voltage gain.
5. Display the input and output signals on the screen of CRO and see that input and output waves are having a phase difference of  $\pi$ .
6. By changing the value of  $R_f$  repeat steps from 2 to 5.

### (B) For Non Inverting Amplifier:

7. Connect a circuit of non-inverting amplifier according to Fig. 15.2
8. Fix the theoretical voltage gain by choosing the value of  $R_f$  and  $R_{in}$ .
9. By applying the input voltage ( $E_{in}$ ) observe the value of corresponding output voltage ( $E_0$ ).
10. Calculate the voltage gain by both theoretical and observed values.
11. Phase relationship (coherence) between output and input can be seen on the screen of CRO by applying input or output waves at two channels of CRO.

### (C) For an addition OP-Amplifier:

12. Make a circuit of an adder amplifier according to Fig. 15.3 In this circuit apply two or three different input signals.
13. At different values of  $R_{in1}$ ,  $R_{in2}$ , and  $R_{in3}$  observe the value of output voltage ( $E_0$ ) by VTVM or milli voltmeter.
14. With the help of eq. 9 calculate the theoretical value of  $E_0$  and compare it with observed value.

**(d) Differentiator:**

15. Make circuit connections as shown in Fig. 15.4. Give the input from oscillator as triangular wave may be fed. The output of the differentiator will be square wave of triangular wave .

**(e) Integrator:**

16. Make circuit connections as shown in Fig. 15.5. Give input of 1KHz observe the output. It will be integration of the input.

**15.8 Observation**

(a) Number of used operational amplifier = 741

(b) Value of used DC bias voltage = ..... Volt.

**(a) Table 1: gain for an Inverting amplifier:-**

When  $R_f = \dots K\Omega$ ,  $R_{in} = \dots K\Omega$

Then theoretical voltage gain

$$A = \frac{R_f}{R_{in}} = \dots\dots\dots$$

S.No.	Input Voltage ( $E_{in}$ ) In volts	Output Voltage ( $E_o$ ) in volts	Observed Voltage gain ( $E_o/E_{in}$ )
1			
2			
3			
4			
5			
6			
7			

**(b) Table 2: Gain for non inverting amplifier.**

When  $R_f = \dots K\Omega$ ,  $R_{in} = \dots K\Omega$

Then theoretical voltage gain

$$A = 1 + \frac{R_f}{R_{in}} = \dots\dots\dots$$

S.No.	Input Voltage (E <sub>in</sub> ) Volts	Output Voltage (E <sub>o</sub> ) Volts	Observed Voltage gain (E <sub>o</sub> /E <sub>in</sub> )
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

**(c) Table 3: For an adder amplifier**

When R<sub>1</sub> = R<sub>2</sub> = .....KΩ, R'<sub>1</sub>=.....KΩ and R<sub>f</sub> = .....KΩ.

Then we calculate amplification =  $\frac{R_f}{R_1} = \dots\dots\dots$

Now voltage gain for adder configuration is unity.

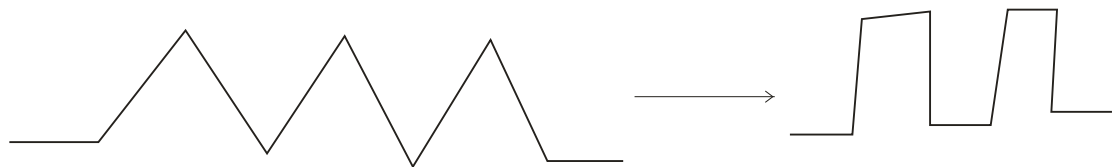
Output voltage will be the sum of two input voltage.

S.No.	Input Voltage 1 (E <sub>in1</sub> ) (volts)	Input Voltage 2 (E <sub>in2</sub> ) (volts)	Observed Voltage (E <sub>o</sub> =E <sub>in1</sub> + E <sub>in2</sub> ) (Volts)
1			
2			
3			

4			
5			
6			
7			
8			
9			
10			

**(d) Differentiator:**

Here if we give input wave from in the form of saw-tooth wave then we get differentiated output wave form in the form of square wave.



**(e) Integrator:**

Here if we give input pulse wave in the form of square wave then we get integrated output pulse wave in the form of saw tooth wave.



**15.9 Calculations**

**For an inverting operational amplifier**

Theoretical voltage gain  $A = - \frac{R_F}{R_{in}} = - \dots\dots\dots$

Practical voltage gain  $A = - \frac{E_o}{E_{in}} = - \dots\dots\dots$

Average observed voltage gain = - .....



Percentage error = - .....%

**For non- inverting operational amplifier**

Theoretical voltage gain  $A_V = 1 + \frac{R_F}{R_{in}} = \dots\dots\dots$

Practical Voltage gain  $A_V = \frac{E_o}{E_{in}} = \dots\dots\dots$

Average observed voltage gain = + .....%

Percentage error = .....%

**15.10 Result**

1. For inverting and non-inverting amplifier the observed value of voltage gain is obtained almost equal to theoretical value according to the table 1 and 2.
2. By increasing the value of  $R_f$  voltage gain of inverting amplifier is varying from unity to infinity.
3. It is clear from table 3 that the total sum of input signals is equal to output signal.
4. In this configuration saw tooth wave is differentiated to square wave form.
5. In this configuration square wave is integrated to saw tooth wave form.

**15.11 Discussion**

.....  
.....  
.....  
.....  
.....  
.....

**15.12 Precautions and Sources of Error**

1. Connect the wires carefully and correctly in different configurations.
2. Input and output voltages should be measured properly and least counts should also be taken in care.

3. Input voltage supply should be given in limits otherwise OP-amp reaches the saturation state.
4. Input offset current is measured with intense care.
5. Use the CRO very carefully.
6. Input resistance must be greater and output resistance must be lesser.

### 15.13 Self Learning Exercise - II

- Q.1 What is the formula for Inverting Amplifier?
- Q.2 What is the formula for non-inverting Amplifier?
- Q.3 Why OP AMP Call direct coupled high differential circuit?
- Q.4 What is the output Differentiator and Integrator?
- Q.5 What are the assumptions made from ideal OP Amp Characteristics?

### 15.14 Glossary

**Amplifier:** A device that increases the strength of an electrical signal by drawing energy from a separate source to that of the signal.

**Common Mode Rejection Ratio:** The ratio of difference mode gain  $A_d$  to common mode gain  $A_c$  is called

$$\text{CMRR (dB)} = 20 \log \frac{A_d}{A_c}$$

**Input Offset Current:** Difference between the currents into the two input terminals.

**Differential:** The result of Mathematical differentiation.

### 15.15 Answer To Self Learning Exercises

#### *Answer To Self Learning Exercise - I*

**Ans.1:** Operational amplifier is a direct coupled amplifier whose gain is very high. This amplifier can be used to solve many linear and non linear mathematical equations. OP-AMP can also be used to perform mathematical operations like adder, differentiation, integration etc. It is also used extensively in design of many electronic circuits, therefore it is called operational amplifier.

**Ans.2:** A simple operational amplifier is a direct coupled amplifier whose characteristics are controlled by feedback.

**Ans.3:** There are many uses of operational amplifier like-addition. Subtraction, differentiation, integration, buffer amplifier, impedance matching and analytical solutions of different mathematical equations, log amplifier.

**Ans.4:** In first situation sources are effectively not in open circuit but in close circuit. So the voltage between first and last ends is equal to sum of terminal voltages of sources. But when operational amplifier will work as an adder then sources are in open circuit and output voltage is equal to sum of all the input emfs.

**Ans.5:** For an ideal operational amplifier  $R_{in} = \infty$ ,  $R_{out} = 0$  voltage amplifier.  $A = -\infty$  frequency Band width  $BW = \infty$ , balance  $CMRR = \infty$  and no change on characteristics due to temperature.

### ***Answer To Self Learning Exercise-II***

**Ans.1:** The formula for inverting amplifier is given as  $-R_f/R_{in}$ .

**Ans.2:** The formulas for noninverting amplifier is given as  $1+R_f/R_{in}$ .

**Ans.3:** OP AMP is called direct coupled because the input of one OP AMP is inserted into the input of another OPAMP. It is called high gain differential circuit because the difference of the two inputs is amplified.

**Ans.4:** If we give the sinusoidal input in differentiator we will get the output of differentiator as a square output. If we give Sinusoidal input in integrator we will get the output of integrator as a ramp output.

**Ans.5:** (i) The current drawn by either of the input terminals (non inverting / inverting) is negligible.

(ii) The potential difference between the inverting and non-inverting input terminals is zero.

### **15.16 Viva Questions**

**Q.1** What are characteristics of ideal OP AMP?

**Q.2** What are the main features of IC741?

**Q.3** What do you mean by input biased current?

- Q.4** What do you mean by differential input resistance?
- Q.5** What do you mean common mode rejection ratio (CMMR)?
- Q.6** What do you mean by supply voltage rejection ratio Q (SVRR)?
- Q.7** What do you mean by output resistance of an OP-amp?
- Q.8** What do you mean by slew rate of an OP - amp?
- Q.9** What do you mean by gain band width product of an OP - amp?
- Q.10** What do you mean by an open loop configuration f an OP-amp?
- Q.11** How many configurations in close loop form op-amp can be used ?
- Q.12** What do you mean by voltage follower?
- Q.13** What are the areas of application of non-linear Op-amp circuits?
- Q.14** What happens when the common terminal of  $V_+$  and  $V_-$  sources is not grounded?
- Q.15** In practical OP-amps, what is the effect of high frequency on its performance?
- Q.16** What is the need for frequency compensation in practical OP-amps?
- Q.17** Define slew rate.
- Q.18** What is amplifier?
- Q.19** What is perfect balance in OP AMP?
- Q.20** Which OP AMP don't have feedback loop?

### 15.17 Answers to Viva Questions

**Ans.1:** An ideal OP-amp would have the following characteristics:-

- |                             |                                |
|-----------------------------|--------------------------------|
| (i) Infinite voltage gain   | (ii) Infinite input resistance |
| (iii) Zero output impedance | (iv) Infinite input impedance  |
| (v) Infinite slew rate      | (vi) Infinite CMRR             |

**Ans.2:** The main features of IC741 are

- (i) No external frequency compensation required.
- (ii) Short circuit protection.
- (iii) Offset null capability.

- (iv) Large common mode and differential voltage range.
- (v) Low power consumption.
- (vi) No latch up problem.

**Ans.3:** Input biased circuit is the average of the current that flow into the inverting and non-inverting input terminals of the OP-amps.

**Ans.4:** Differential input resistance is the equivalent resistance that can be measured at either the inverting and non- inverting input terminal with the other terminal connected to ground.

**Ans.5:** Common mode rejection ratio defines as it is the ratio of the differential voltage gain to the common mode gain of an OP-amp.

$$\text{CMRR} = \text{differential gain} / \text{common mode gain}$$

**Ans.6:** The change in an OP-amp input-offset voltage caused by variation in supply voltage is called supply voltage rejection ratio (SVRR).

**Ans.7:** The output resistance is the equivalent resistance that can be measured between the terminal of the OP-amp and ground.

**Ans.8:** Slew rate is the maximum rate of change of output voltage per unit of time.

**Ans.9:** The gain band width of an op-amp is the "band width when the voltage gain is unity".

**Ans.10:** The open loop configuration of an OP-amp indicate the No connection, exist between the output signal is not fed-back in any form into the input.

**Ans.11:** There are three open-loop op-amp configurations.

- (i) Differential amplifier
- (ii) Inverting amplifier
- (iii) Non-inverting amplifier.

**Ans.12:** The lowest gain that can be obtained from a non-inverting amplifier with feedback is unity when the non-inverting amplifier is configured for unity, it is called a voltage followers.

**Ans.13:** (i) Industrial Instrumentation

- (ii) Communication
- (iii) Signal processing.

**Ans.14:** If the common point of the two supplies is not grounded, twice the supply voltage will get applied and it may damage the OP-amp.

**Ans.15:** The open-loop gain of OP-amp decreases at higher frequencies due to the presence of parasitic capacitance. The closed loop gain increases at higher frequencies and leads to instability.

**Ans.16:** Frequency compensation is needed when large bandwidth and lower closed loop gain is desired. Compensating networks are used to control the phase shift and hence to improve the stability.

**Ans.17:** The slew rate is defined as the maximum rate of change of output voltage caused by a step input voltage. An ideal slew rate is infinite which means that OP-amp's output voltage should change instantaneously in response to input step voltage.

**Ans.18:** Amplifier is a device that makes sound louder and signal level greater.

**Ans.19:** Perfect balance is the characteristics of ideal OP AMP and if there is some input applied then we will get the output zero. In this condition it is known as perfect balance.

**Ans.20:** Comparator OP AMP don't have feedback loop.

## References and Suggested Readings

1. Ram Kanth Gayakwad, OP- AMP and Linear Integrated Circuits, PHI Learning private limited, 2011.
2. Gupta Kumar, Hand Book of electronics, Pragati Prakashan, 2014.
3. S.L. Kakani and K.C. Bhandari, A text book of electronics, 2e, New age International (P) Limited Publishers, 2010.

# UNIT-16

## Characteristics of Operational Amplifier

### Structure of the Unit

- 16.1 Aim
  - 16.2 Apparatus
  - 16.3 Diagram
  - 16.4 Formula
  - 16.5 Theory and Description
  - 16.6 Self Learning Exercise-I
  - 16.7 Procedure
  - 16.8 Observation
  - 16.9 Calculations
  - 16.10 Result
  - 16.11 Discussion
  - 16.12 Precautions and Source of Error
  - 16.13 Self Learning Exercise-II
  - 16.14 Glossary
  - 16.15 Answers to Self Learning Exercises
  - 16.16 Viva Questions
  - 16.17 Answers to Viva Questions
- References and Suggested Readings

### 16.1 Aim

- (i) To study of characteristics of an operational amplifier.
- (ii) To measure the input bias current, input offset current input offset voltage, input and output voltage range, the slew rate and band width of OP amp.

## 16.2 Apparatus

Operational amplifier box, battery eliminator, resistance box, voltmeter, ammeter, connecting wires etc.

## 16.3 Diagram

### 1. Input bias current and input offset current.

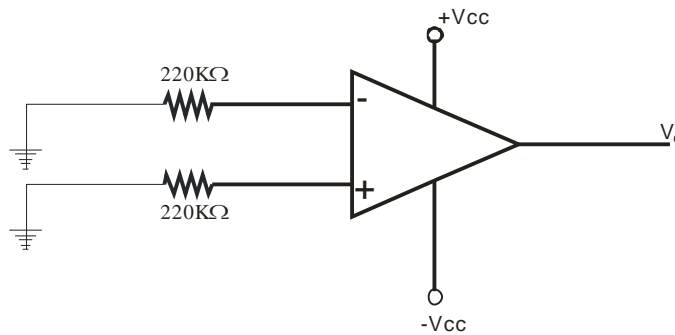


Fig. 16.1: Input bias and input offset current.

### 2. Input offset voltage

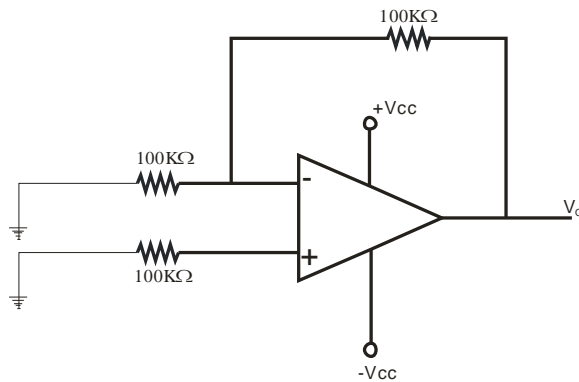


Fig. 16.2: Input offset voltage

### 3. Slew rate and band width:-

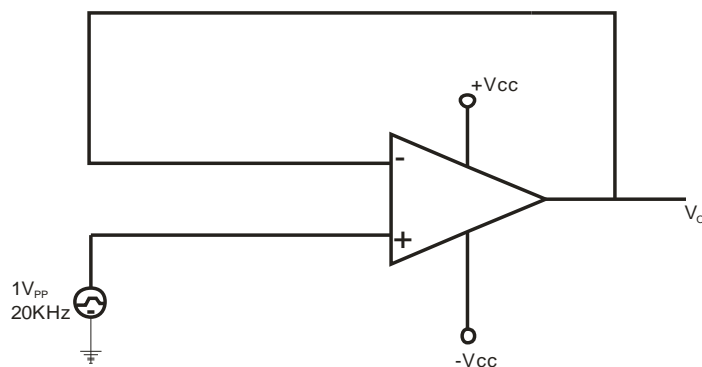
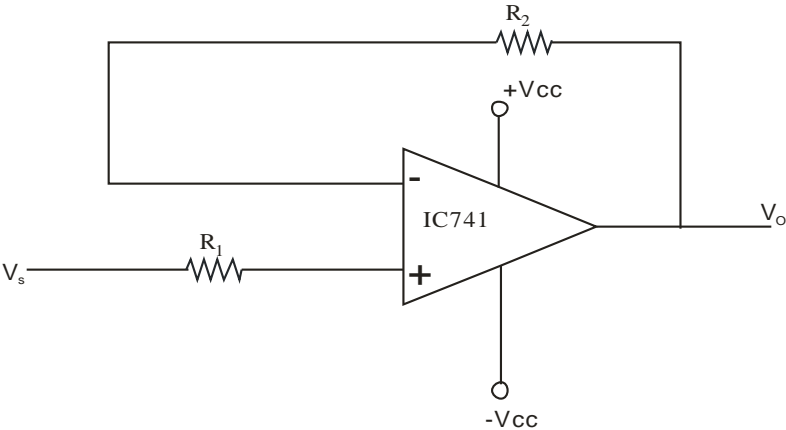


Fig. 16.3: Slew rate and band width

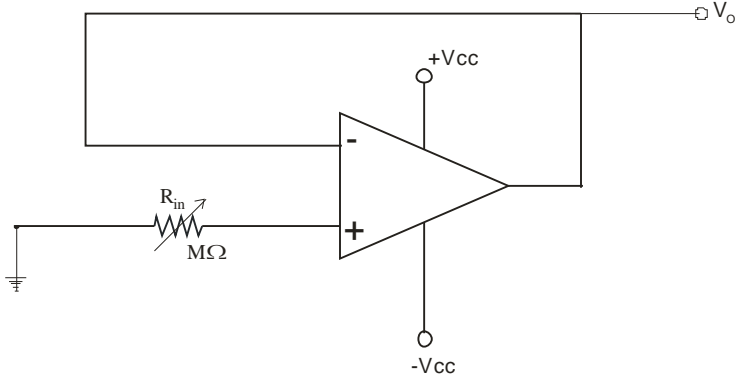


**4. Input and output voltage ranges:-**



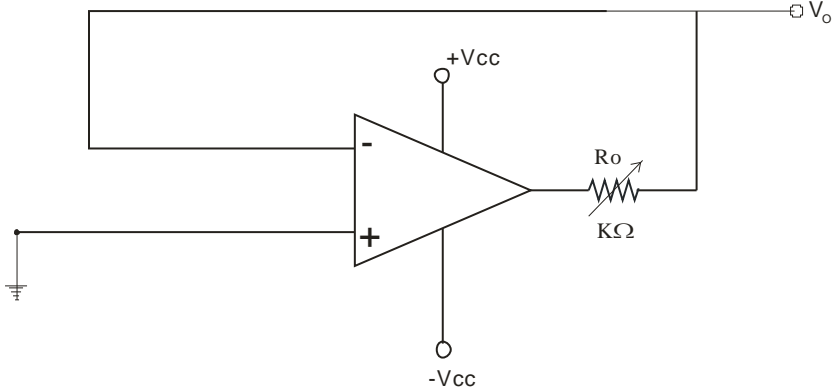
**Fig. 16.4: Input and output voltage range**

**5. Input Impedance:**



**Fig. 16.5: Input Impedance**

**6. Output Impedance:-**



**Fig.: 16.6: Output Impedance**

## 16.4 Formula

- (i) Input bias current  $I_B = \frac{I_B^+ + I_B^-}{2}$
- (ii) Input offset current  $I_{OS} = |I_B^+ - I_B^-|$
- (iii) Input offset voltage  $V_{in} = V_{out} / 1000$
- (iv) Slew rate  $SR = \Delta V / \Delta T$

The maximum frequency at which the OP-amp can be operated without distortion

is  $f_{\max} = \frac{SR}{(2\pi V_p)}$ .

where SR = slew rate of OP- amp.

$V_p$  = Peak voltage of output sine wave.

## 16.5 Theory and Description

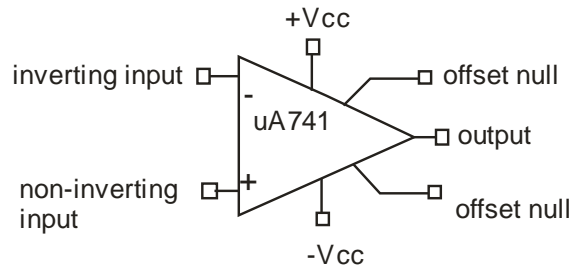
An op-amp is a high gain, direct coupled differential linear amplifier its response characteristics are externally controlled by negative feedback from the output to input, op-amp has very high input impedance, typically a few mega ohms and low output impedance, less than  $100\Omega$ .

Op-amps can perform mathematical operations like summation integration, differentiation, logarithm, anti-logarithm, etc., and hence the name operational amplifier op-amps are also used as video and audio amplifiers, oscillators and so on, in communication electronics, in instrumentation and control, in medical electronics, etc.

### Circuit symbol and op-amp terminals

The circuit schematic of an op-amp is a triangle as shown below in Fig. 16.7 op-amp has two input terminal. The minus input, marked (-) is the inverting input. A signal applied to the minus terminal will be shifted in phase  $180^\circ$  at the output. The plus input, marked (+) is the non-inverting input. A signal applied to the plus terminal will appear in the same phase at the output as at the input  $\pm V_{cc}$  denotes the positive and negative power supplies. Most op-amps operate with a wide range of supply voltages. A dual power supply of  $\pm 15V$  is quite common in practical op-amp circuits. The use of the positive and negative supply voltages allows the

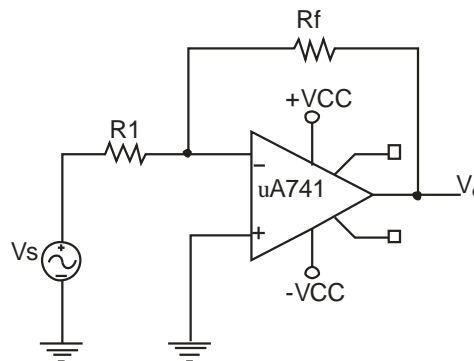
output of the op-amp to swing in both positive and negative directions.



**Fig. 16.7 Op-amp circuit symbol**

### Negative Feedback Control

The basic circuit connection using an op-amp is shown below in fig. 16.8



**Fig. 16.8 Op-amp circuit connection in the inverting mode**

An input signal,  $V_s$  is applied through resistor  $R_1$  to the inverting input. The output is then connected back to the same minus input through resistor  $R_f$ . The plus input is connected to ground since the signal is essentially applied to the minus input the resulting output is opposite in phase to the input signal. Note that the output is feedback to the minus input terminal (inverting input terminal) in order to provide negative feedback for the amplifier. This circuit arrangement is called inverting amplifier.

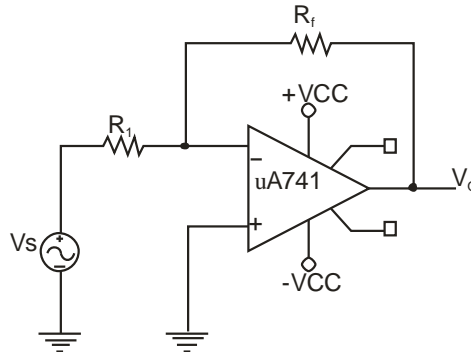
For this amplifier, the output can be defined as

$$V_o = \left( \frac{R_f}{R_1} \right) V_s \quad (1)$$

The minus sign indicates that the sign of the output is inverted as compared to the input. The equation for gain of this amplifier is

$$\text{Gain} = \left( \frac{R_f}{R_1} \right) \quad (2)$$

It is also possible to operate the op-amp as a non-inverting amplifier by applying the signal to the plus input (non-inverting input terminal), as shown below in fig. 16.9



**Fig. 16.9 Op-amp circuit connection in the non-inverting mode**

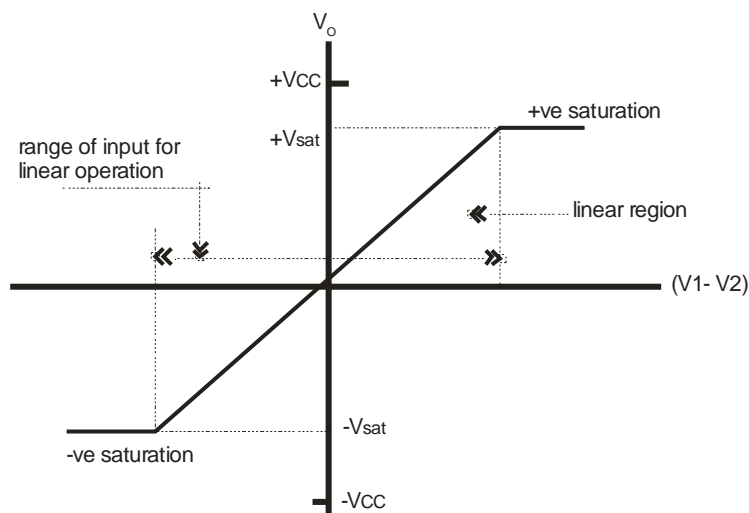
Note that the feedback network is still connected to the inverting input. For this amplifier circuit, that output of the amplifier is defined by

$$V_o = \left(1 + \frac{R_f}{R_1}\right) V_s \quad (3)$$

and its gain is  $Gain = 1 + \frac{R_f}{R_1}$  (4)

### The op-amp transfer characteristics

The transfer characteristics of a typical op-amp are sketched in fig. 16.10 and it shows three regions of operation, namely the linear region, the negative saturation region and the positive saturation region.



**Fig. 16.10 op-amp transfer characteristics**

In the linear region, the output voltage  $V_o$  is linearly related to the difference in the input voltage ( $V_1 - V_2$ ). The supply voltage limits the maximum value of the output voltage. The OUTPUT voltage is normally 2 to 3 volts lower than the power supply voltage, i.e.,  $|V_o| < |V_{cc}|$

$$\text{also, } V_o = A (V_1 - V_2) \quad (5)$$

$$\text{Therefore } |(V_1 - V_2)| < |V_{cc} / A| \quad (6)$$

$$\text{For } V_{cc} = 15V \text{ and } A = 10^5, |V_1 - V_2| < 150\mu V.$$

Thus, for very high gain op-amps, the input voltages  $V_1$  and  $V_2$  are almost equal. Unequal input voltages characterize the operation in saturation region. If  $V_1 > V_2$  by  $150\mu$ ,  $V_o$  will be saturated at a negative voltage  $-V_{sat}$ . Although the op-amp has distinct non-linear characteristics bias as a linear devices under certain conditions and the principles of linear circuit theory can be used to design and analyses op-amp is operated in the linear region. Since the magnitude of the input voltage for linear operations is quite small, op-amps are seldom used in open-loop configuration. Feedback from output to inverting (-) terminal tends to extend the range of input for linear operation.

Here  $R_{id}$  is the differential input resistance,  $A V_{id}$  is the Thevenin voltage source and  $R_o$  is the Thevenin equivalent output resistance looking back into output terminals. The output voltage  $V_o$  is

$$V_o = AV_{id} = A (V_1 - V_2) \quad (7)$$

Where  $A$  is the open-loop voltage gain of the op-amp,  $V_{id}$  is the differential input voltage, and  $V_1$  and  $V_2$  are the voltages w.r.t ground potential at the non-inverting and the inverting input terminals respectively.

Thus the op-amp amplifies the difference between the two input voltages. The input voltages  $V_1$  and  $V_2$  can be either ac or dc voltages.

In the open-loop configuration, no connection exists between the output and input terminals. When connected in an open-loop configuration, the op-amp works as a high gain amplifier. Any input signal slightly above zero volts drives the output  $V_o$  to saturation. For this reason, the op-amp is seldom used in open-loop configuration for linear applications. The property of op-amp output saturating under open-loop configuration is used in non-linear circuit applications of op-amp as a voltage comparator.

## Op-amp characteristics

An ideal op-amp draws no current from the source and its response is also independent of temperature. However, a real op-amp does not work this way. Current is taken from the source into op-amp inputs. Also the two inputs respond differently to current and voltage due to mismatch in transistors. A real op-amp also shifts its operation with temperature. These non-ideal characteristics are:

1. Input bias current
2. Input offset current
3. Input offset voltage
4. Slew rate
5. Input and output voltage range
6. Input Impedance
7. Output Impedance

### Input bias current

The op-amp's input is a differential amplifier, which may be made of BJT or FET. In either case the input transistors must be biased into this linear region by supplying currents into the bases. In an ideal op-amp, no current is drawn from the input terminals. However, practically, input terminals conduct a small value of dc current to bias the input transistors when base currents flow through external resistances, they produce a small differential input voltage or unbalance; this represents a false input signal. When amplified, this small input unbalance produces an offset in the output voltage.

The input bias current shown on data sheets is the average value of base currents entering into the terminals of an op-amp.

$$I_B = \frac{(I_B^+ + I_B^-)}{2} \quad (8)$$

For 741, the bias current is 500nA or less. The smaller the input bias current, the smaller the offset at the output voltage.

### Input offset current

The input offset current is the difference between the two input currents driven from a common source.

$$|I_{OS}| = |I_B^+ - I_B^-| \quad (9)$$

It tells you how much larger one current is than the other. Bias current compensation will work if both bias current  $I_B^+$  and  $I_B^-$  are equal. So, the smaller the input offset current the better the OP amp. The 741 op-amps have input offset current of 20nA.

### **Input offset voltage**

Ideally, the output voltage should be zero when the voltage between the inverting and non-inverting inputs is zero. In reality, the output voltage may not be zero with zero input voltage. This is due to un-avoidable imbalances, mismatches, tolerances, and so on inside the op-amp., In order to make the output voltage zero, we have to apply a small voltage at the input terminals to make output voltage zero. This voltage is called input offset voltage i.e., input offset voltage is the voltage required to be applied at the input for making output voltage to zero volts. The 741 op-amp has input offset voltage of 5mV under no signal conditions Therefore, we may have to apply a differential input of 5mV, to produce an output voltage of exactly zero.

### **Slew rate**

Among all specifications affecting the ac operation of the op-amp, slew rate is the most important because it places a severe limit on a large signals operation. Slew rate is defined as the maximum rate at which the output voltage can change. The 741 op-amp has a typical slew rate of 0.5 volts per microsecond ( $V/\mu s$ ). This is the ultimate speed of a typical 741; its output voltage can change no faster than 0.5 V/ $\mu s$ . If we drive a 741 with large step input, it takes 20 $\mu s$  ( $0.5 V/\mu s \times 10V$ ) for the output voltage to change from 0 to 10V.

### **Band Width**

Slew rate distortion of a sine wave starts at a point where the initial slope of the sine wave equals the slew rate of the op-amp. The maximum frequency at which the op-amp can be operated without distortion is

$$f_{\max} = \frac{SR}{(2\pi V_p)} \quad (10)$$

Where SR = Slew rate of op - amp,

VP = peak voltage of output sine wave.

Frequency  $f_{\max}$  is called bandwidth of op-amp. The 741 op-amp has a bandwidth of approximately 8 KHz. This means the undistorted band width for large signal operation is 8 KHz.

### **Input and output voltage ranges**

Maximum positive and negative input voltage applied to the op-amp for undistorted output gives the input voltage range. Maximum positive and negative undistorted output voltage of the op-amp gives the output voltage range.

### **Input Impedance**

Input impedance is defined as the impedance seen by the source at the input of the operational amplifier when output is short circuited, for an ideal OP-AMP it is finite.

First of all we measure the output voltage  $E_o$  for the input voltage  $E_{in}$ , When  $R_{in} = 0$ , thus we select the resistance  $R_{in}$  such that the output voltage falls to the half value of input voltage, thus that value of  $R_{in}$  is the required value of input impedance of operational amplifier. Hence for finding the high input impedance of operational amplifier we use Non-Inverting Configuration.

### **Output Impedance:**

Output Impedance is defined as the impedance at the output of operational amplifier when input is short circuited. For ideal operational amplifier is low. Here output resistance is selected such that the voltage connected at output terminal or output voltage becomes half the value of input voltage. Then that value of output resistance is known as low output impedance of the operational amplifier, here we use Non-Inverting configuration for the measurement of low output impedance of OP-Amp.

## **16.6 Self Learning Exercise-I**

- Q.1** What is an operational amplifier?
- Q.2** What is the voltage gain of an operational amplifier?
- Q.3** What is the input impedance of an operational amplifier?
- Q.4** What is the output impedance of an operational amplifier?
- Q.5** What are the main features of an operational amplifier circuit?



## 16.7 Procedure

### Input offset current:-

1. Connect the circuit of Figure 16.1
2. Using a DMM measure the dc voltage at the Negative terminal.
3. By ohm's law, calculate the input currents,  $I_B^+$  and  $I_B^-$ . Average these values to find out the input bias current. Also, find the differences between these two currents to know the input offset current.

### Input offset voltage:

1. Connect the circuit of figure 16.2.
2. Measure the DC output voltage at pin 6 using multimeter.
3. Calculate the input offset voltage using the formula.

$$V_i = V_{out} / 100$$

### Slew rate and band width:

1. Connect the circuit of figure 16.3
2. Using an audio frequency oscillator, provide a 1V peak to peak square wave with a frequency of 25 KHz.
3. With an oscilloscope, observe the output of OPAMP. Adjust the oscilloscope timing the get a couple of cycles.
4. Measure the voltage change  $\Delta V$  and time change  $\Delta t$  of the output wave from.
5. Calculate the slew rate using the formula.

$$SR - \Delta V / \Delta T$$

6. Set the Audio frequency oscilloscope at 1kHz. Adjust the signal level to get 20v Peak to Peak out of the OP-amp.
7. Increase the frequency and watch the waveform somewhere above 10 kHz, slew rate distortion will become evident. That maximum frequency  $f_{max}$  at which the op-amp can be operated is called band width of an op-amp.

### Input and Output voltage ranges

1. Assemble the voltage follower circuit as shown in Fig. with  $R_1 = R_2 = 100 \text{ K}\Omega$ . Use op-amp dc power supply voltages of  $\pm 9V$ .

2. Apply  $\pm 5V$ , 100 Hz sinusoidal inputs, Vs. Observe on a CRO the voltages at the non-inverting input and output pins simultaneously. Increase the signal amplitude until distortion is observed at the peak value of the output. Measure the positive and negative input voltage peak values. This gives the OP-amp input voltage range.
3. Connect  $R_1$  between the source and inverting input. Ground the non-inverting input. Choose  $R_1 = 10k\Omega$ ,  $R_2 = 100k\Omega$ . Repeat observation of step 8 starting with  $\pm 0.5$ , 100 Hz sinusoidal inputs. Measure the positive and negative output voltage peak values. This gives the Op-amp output voltage range.

### Input Impedance:-

Connect the circuit of figure 16.5. For an ideal op-amp input impedance is very high approx to infinity, hence called as high input impedance.

In practical it is found to be in range of mega ohms when output voltage falls to half value of input voltage we measure it as high input impedance of the circuit.

### Output Impedance:-

Connect the circuit of Figure 16.6. For an ideal op-amp output to low, hence called as low input impedance.

In practical it is found to be in range of few ohms, when output voltage falls to half value of input voltage. We measure it as low output impedance of the circuit.

In both the cases voltage gain is  $\frac{1}{2}$  which is critically not possible in op-amp but we consider this condition for the measurements of high input impedances and low-outputs impedances.

## 16.8 Observation

### (I). Input offset Current:

S. No	DC voltage at the non inverting terminal $V_+$	DC voltage at the inverting terminal $V_-$	$I_B^+ = \frac{V^+}{220K}$	$I_B^- = \frac{V^-}{220K}$	Input bias current $I_B = \left( \frac{I_B^+ + I_B^-}{2} \right)$	Input offset current $I_{OS} =  I_B^+ - I_B^- $
1						

2						
3						
4						
5						
6.						

**(II). Input offset Voltage**

S.No.	$V_{out}$	$V_{in} = V_{out} / 1000$
1		
2		
3		
4		
5		
6.		

**(III). Slew rate and band width**

S.No.	$\Delta V$	$\Delta T$	$SR = \Delta V / \Delta T$	BW
1				
2				
3				
4				
5				
6				

**(IV). Input Impedance**

S.No.	Applied Voltage $V_{in}$ (Volts)	Output Voltage $V_o$ (Volts)	Required resistance to reduced $v_o$ half of $V_{in}$
1			
2			

3			
4			
5			
6			

**(V). Output Impedance**

S.No.	Applied Voltage $V_{in}$ (Volts)	Voltage output $V_o$ (Volts)	Required resistance to reduce $V_o$ to half of $V_{in}$
1			
2			
3			
4			
5			
6			

**16.9 Calculations**

**I. Input offset current**

For  $I_B^+$ :  $R_B = 220\text{ K}\Omega$        $V^+ = \dots\dots\dots$

$$I_B^+ = \frac{V^+}{220\text{K}} = \dots\dots\dots \text{Amp.}$$

For  $I_B^-$ :  $R_B = 220\text{ K}\Omega$        $V^- = \dots\dots\dots$

$$I_B^- = \frac{V^-}{220\text{K}} = \dots\dots\dots \text{Amp.}$$

Input offset current  $I_{OS} = |I_B^+ - I_B^-| = \dots\dots\dots\text{Amp.}$

**II. Input offset voltage**

$V_{out} = \dots\dots\dots$  volt.

$$V_{in} = \frac{V_{out}}{1000} = \dots\dots\dots \text{Volt.}$$

**III. Slew rate and band width.**

$\Delta V = \dots\dots\dots$  volt.

$\Delta T = \dots\dots\dots$  Sec.  
Slew rate  $SR = \Delta V/\Delta T = \dots\dots\dots$

**16.10 Result**

1. Input offset current for 741 Op-amp configuration is measured to be  $\dots\dots\dots$  Amp approx.
2. Input offset voltage is measured to be  $V_{in} = \dots\dots\dots$  Volts.
3. Increase the frequency and measure the waveform somewhere above 10 kHz, slew rate distortion will become evident. The maximum frequency at which the Op-amp can be operated is called band width of an OP- amp.
4. In the measurement of high input impedance it is found to be in the range of mega ohms.
5. Input impedance is selected when output voltage falls to half the input voltage. When voltage varies from  $\dots\dots\dots$  volts to  $\dots\dots\dots$  volts then the high input impedance varies from  $\dots\dots$  to  $\dots\dots$  respectively.
5. In the measurement of low output impedance it is found to be in the range of few ohms. Output impedance is selected when output voltage falls to half the input voltage. When voltage varies from  $\dots\dots\dots$  volts to  $\dots\dots\dots$  volts then the low output impedance varies from  $\dots\dots\dots$  to  $\dots\dots\dots$  respectively.

**16.11 Discussion**

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**16.12 Precautions and Sources of Error**

1. Connect the wires carefully and correctly in different configurations.
2. Input and output voltages should be measured properly and least counts should also be taken in care.

3. Input voltage supply should be given in limits otherwise OP-amp reaches the saturation state.
4. Input offset current is measured with intense care.
5. Use the CRO very carefully.
6. Input resistance must be greater and output resistance must be lesser.

### 16.13 Self Learning Exercise-II

- Q.1 Write the formula of Input offset current?
- Q.2 Write the relationship between Q and bandwidth?
- Q.3 Define power supply voltage rejection ratio?
- Q.4 What is an Input offset current?
- Q.5 Define Input offset voltage?

### 16.14 Glossary

**Amplifier** - A device that increases the strength of an electrical signal by drawing energy from a separate source to that of the signal.

**Input offset current** - difference between the currents into the two input terminals.

**Slew rate** - when the input is over driven, such as with a square wave, the output of the amplifier will slew at some maximum rate.

### 16.15 Answers to Self Learning Exercises

#### *Answer To Self Learning Exercise -I*

- Ans.1:** An operational amplifier is basically a differential amplifier used to amplify the difference between two input signals.
- Ans.2:** The voltage gain is extremely high and ideally approaches infinity ( $A=\infty$ )
- Ans.3:** Input impedance of an operational amplifier is in finite, ( $Z_{in} = \infty$ )
- Ans.4:** Output impedance of an operational amplifier is zero ( $Z_{out} = 0$ ).
- Ans.5:** A simple operational amplifier is a direct coupled amplifier whose characteristics are controlled by feedback.

#### *Answer to Self Learning Exercise-II*

**Ans.1:**  $I_{OS} = |I_B^+ - I_B^-|$

**Ans.2:** 
$$\frac{F_c}{B_w} = \frac{f_c}{f_H - f_L}$$

**Ans.3:** It is defined as input offset voltage change per volt of supply voltage change.

**Ans.4:** The input offset current  $I_{os}$  is the difference of the current into the two input terminals with the output at zero volt.

**Ans.5:** It is input voltage which must be applied across the input terminals to obtain zero output voltage.

## 16.16 Viva Questions

- Q.1 What are characteristics of ideal OP AMP?
- Q.2 What are the main features of IC741?
- Q.3 What do you mean by input biased current?
- Q.4 What do you mean by differential input resistance?
- Q.5 What do you mean common mode rejection ratio (CMMR)?
- Q.6 What do you mean by supply voltage rejection ratio Q (SVRR)?
- Q.7 What do you mean by output resistance of an OP-amp?
- Q.8 What do you mean by slew rate of an OP - amp?
- Q.9 What do you mean by gain band width product of an OP - amp?
- Q.10 What do you mean by an open loop configuration of an OP-amp?
- Q.11 How many configurations in open-loop op-amp configuration?
- Q.12 What do you mean by voltage follower?
- Q.13 What are the areas of application of non-linear Op-amp circuits?
- Q.14 What happens when the common terminal of  $V_+$  and  $V_-$  sources is not grounded?
- Q.15 In practical OP-amps, what is the effect of high frequency on its performance?
- Q.16 What is the need for frequency compensation in practical OP-amps?
- Q.17 Define slew rate.
- Q.18 What is amplifier?

**Q.19** What is perfect balance in OP AMP?

**Q.20** Which OP AMP don't have feedback loop?

## 16.17 Answers to Viva Questions

**Ans.1:** An ideal OP-amp would have the following characteristics:-

- (i) Infinite voltage gain
- (ii) Infinite input resistance
- (iii) Zero output impedance
- (iv) Infinite input impedance
- (v) Infinite slew rate
- (vi) Infinite CMRR

**Ans.2:** The main features of IC741 are

- (i) No external frequency compensation required.
- (ii) Short circuit protection.
- (iii) Offset null capability.
- (iv) Large common mode and differential voltage range.
- (v) Low power consumption.
- (vi) No latch up problem.

**Ans.3:** Input biased circuit is the average of the current that flow into the inverting and non-inverting input terminals of the OP-amps.

**Ans.4:** Differential input resistance is the equivalent resistance that can be measured at either the inverting and non- inverting input terminal with the other terminal connected to ground.

**Ans.5:** Common mode rejection ratio defines as it is the ratio of the differential voltage gain to the common mode gain of an OP-amp.

$$\text{CMRR} = \text{differential gain} / \text{common mode gain}$$

**Ans.6:** The change in an OP-amp input-offset voltage caused by variation in supply voltage is called supply voltage rejection ratio (SVRR).

**Ans.7:** The output resistance is the equivalent resistance that can be measured between the terminal of the OP-amp and ground.

**Ans.8:** Slew rate is the maximum rate of change of output voltage per unit of time.

**Ans.9:** The gain band width of an op-amp is the "band width when the voltage gain is unity".

**Ans.10:** The open loop configuration of an OP-amp indicate the No connection,



exist between the output signal is not fed-back in any form into the input.

**Ans.11:** There are three open-loop op-amp configurations.

(i) Differential amplifier (ii) Inverting amplifier (iii) Non-inverting amplifier.

**Ans.12:** The lowest gain that can be obtained from a non-inverting amplifier with feedback is unity when the non-inverting amplifier is configured for unity, it is called a voltage followers.

**Ans.13:** (i) Industrial Instrumentation (ii) Communication (iii) Signal processing.

**Ans.14:** If the common point of the two supplies is not grounded, twice the supply voltage will get applied and it may damage the OP-amp.

**Ans.15:** The open-loop gain of OP-amp decreases at higher frequencies due to the presence of parasitic capacitance. The closed loop gain increases at higher frequencies and leads to instability.

**Ans.16:** Frequency compensation is needed when large bandwidth and lower closed loop gain is desired. Compensating networks are used to control the phase shift and hence to improve the stability.

**Ans.17:** The slew rate is defined as the maximum rate of change of output voltage caused by a step input voltage. An ideal slew rate is infinite which means that OP-amp's output voltage should change instantaneously in response to input step voltage.

**Ans.18:** Amplifier is a device that makes sound louder and signal level greater.

**Ans.19:** Perfect balance is the characteristics of ideal OP AMP and if there is some input applied then we will get the output zero. In this condition it is known as perfect balance.

**Ans.20:** Comparator OP AMP don't have feedback loop.

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1. Ram Kanth Gayakwad, OP- AMP and Linear Integrated Circuits, PHI Learning private limited, 2011.
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# UNIT-17

## LC Transmission Line

### Structure of the Unit

- 17.1 Aim
- 17.2 Apparatus
- 17.3 Diagram
- 17.4 Formula
- 17.5 Theory and Description
- 17.6 Self Learning Exercise-I
- 17.7 Procedure
- 17.8 Observation
- 17.9 Graph
- 17.10 Calculations
- 17.11 Result
- 17.12 Discussion
- 17.13 Precautions and Sources of Error
- 17.14 Self Learning Exercise -II
- 17.15 Glossary
- 17.16 Answers to Self Learning Exercises
- 17.17 Viva Questions
- 17.18 Answers to Viva Questions

### References and Suggested Readings

#### 17.1 Aim

To study the LC transmission line in means of

- (i) To study the characteristic impedance of lumped constant delay line.
- (ii) To study the voltage distribution along a lumped constant delay line in the

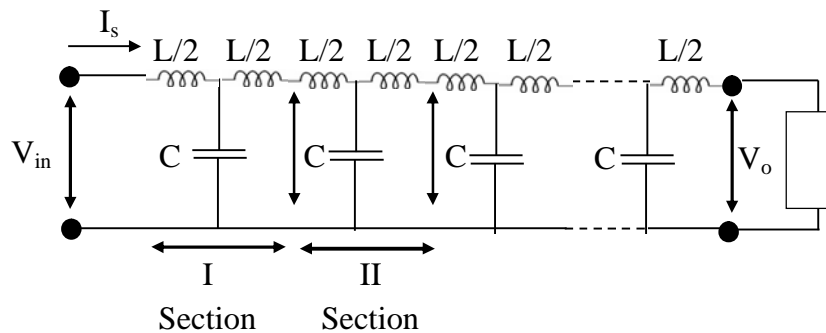
case when it is (a) open circuit, (b) short circuit, (c) determination in  $Z_0$  and hence determine  $\alpha$ ,  $\beta$  and  $v_p$ .

## 17.2 Apparatus

A lumped constant delay line having 20 to 25 sections, signal generator, resistance box and capacitors.

## 17.3 Diagram

A lumped delay (LC) transmission line can be shown as below:



**Figure 15:** A lumped delay transmission line.

Here:

$V_{in}$  is the input ac voltage provided by a signal generator at a selected frequency  $f_{in}$   
 $V_o$  is output voltage measured at the end of each section with the help of AC metre or Vacuum tube voltmeter (VTVM).

## 17.4 Formula

### (i) Attenuation Constant and Phase Constant:

Equation of ac current and voltage passing through LC unidirectional (along  $z$  direction) transmission line can be given by as

$$\frac{d^2 i(z)}{dz^2} - \gamma^2 i(z) = 0$$

and 
$$\frac{d^2 v(z)}{dz^2} - \gamma^2 v(z) = 0$$

Here

$v$  = Voltage measures at distance  $z$  of transmission line

$i$  = Current measured at distance  $z$  of transmission line

$\gamma$  = Transmission constant

$Z$  = Series impedance

$Y$  = Shunt admittance

We can define series impedance and shunt admittance as below

$$Z = R + j\omega L$$

and  $Y = G + j\omega C$

Here:

$R$  = Series resistance

$G$  = Shunt conductance

$L$  = Series inductance

$C$  = Shunt capacitance

If transmission coefficient  $\gamma$  is defined as

$$\gamma = \alpha + j\beta$$

Where Attenuation constant ( $\alpha$ ) and phase constant ( $\beta$ ) for this transmission line can be define as

$$\alpha = \frac{1}{\sqrt{2}} \left[ (RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2) + (G^2 + \omega^2 C^2)} \right]^{\frac{1}{2}}$$
$$\beta = \frac{1}{\sqrt{2}} \left[ (-RG + \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2) + (G^2 + \omega^2 C^2)} \right]^{\frac{1}{2}}$$

A LC transmission line will be an ideal transmission line if  $G=0$  and  $R=0$ . For low loss LC transmission line, it can be considered that  $R \ll \omega L$  and  $G \ll \omega C$ . In this case the values of attenuation constant, phase constant, characteristic impedance and phase velocity can be given by

$$\alpha_i = \frac{R}{2} \sqrt{\frac{C}{L}} \text{ neper/section}$$

$$\beta_i = \omega \sqrt{LC} \text{ rad/section}$$

$$v_p = \frac{1}{\sqrt{LC}} \text{ section/second}$$

$$Z_i = \sqrt{\frac{L}{C}} \Omega$$

**(ii) The characteristic impedance:**

Transmission line mostly not ideal and the resistance of coil cannot be neglected. The characteristic impedance of a transmission line can be calculated through

$$Z_o = \sqrt{Z_{oc} \times Z_{sc}}$$

Where, the impedance of open circuit transmission line can be given as

$$Z_{oc} = R_{oc} + jX_{oc}$$

Where, the impedance of short circuit transmission line can be given as

$$Z_{sc} = R_{sc} + jX_{sc}$$

Here, resistive part of open circuited and short circuited transmission line given by, respectively,

$$R_{oc} = \frac{V_{ro}}{I_s} \text{ and } R_{sc} = \frac{V_{rs}}{I_s}$$

Reactive part of open circuited and short circuited transmission line given by, respectively,

$$X_{oc} = \frac{V_{\theta o}}{I_s} \text{ and } X_{sc} = \frac{V_{\theta s}}{I_s}$$

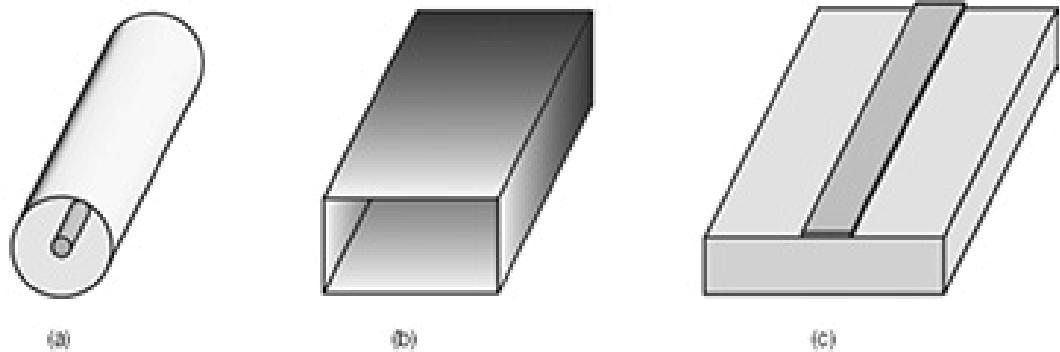
## 17.5 Theory and Description

In communications and electronic engineering, a transmission line is a specialized cable or a structure designed to carry electric energy from one point to another. Transmission lines are used for purposes such as connecting radio transmitters and receivers with their antennas, distributing cable television signals, computer network connections and high speed computer data buses [1-4]. Examples of transmission lines include wires, optical fibers, coaxial cables, rectangular closed waveguides, and dielectric slabs.

**(i) Significances of the study of the transmission line:** The loss of the transferred signal through a transmission line is minimum if, at each terminal, the line is matched to its characteristic impedance. Low frequency AC signals, and audio signal, in which the direction of current changes several hundred times per second can be carried by ordinary cable. However, for the higher frequency signals such as signals in the radio frequency range, these ordinary cables cannot be used

due to high power losses [1]. Also, at the junction point of cables, the radio frequency current reflects back and travel towards the source and It also prevents the signal from reaching the destination which causes further loss [1, 2].

In order to carry high frequency electromagnetic signals, transmission lines are constructed in a special such that the characteristic impedance matches at the end. These special structures so called transmission lines possess uniform characteristic



*Figure 16: Different shapes of waveguide. (a) coaxial cable, (b) waveguide, and (c) microstrip [5].*

impedance due to their uniform cross sectional dimensions along their length.

The example of transmission line are: parallel line (ladder line, twisted pair), coaxial cable, stripline, and microstrip [3, 4].

For long distance energy transfer the capacitances between current carrying conductors and their surroundings cannot be neglected. Hence when the wavelength of signal to be sent is larger than the spacing between the lines, the analysis to decide appropriate type of transmission lines become necessary.

Waveguides are used to transfer the signals in microwave frequency range and above because the power losses in transmission lines become very high. In the terahertz region and above, the transmission through the waveguide also becomes lossy. In that region optical methods, (such as lenses and mirrors), are used to guide electromagnetic waves [4].

**(ii) Equations for Transmission Line:** The transmission line can be analyzed either Maxwell's field equations or by the methods of distributed circuit theory [6]. By following uniformly distributed circuit theory, it is assumed that the parameters of transmission line are distributed uniformly throughout the line. The parameters

of a transmission line are defined per unit length of transmission line and these are: Resistance (R) Inductance (L) Capacitance (C) Conductance (G). Resistance (R) is defined as the loop resistance per unit length of the wire. Its unit is ohm/Km Inductance (L) is defined as the loop inductance per unit length of the wire. Its unit is Henry/Km Capacitance (C) is defined as the loop capacitance per unit length of the wire. Its unit is Farad/Km Conductance (G) is defined as the loop conductance per unit length of the wire. Its unit is mho/Km. Voltage and current equations for a LC transmission line can be expressed using Kirchhoff's law for one unit of the transmission line. One section of a transmission line which is extended in the z direction can be displayed as in figure 3.

(iii)

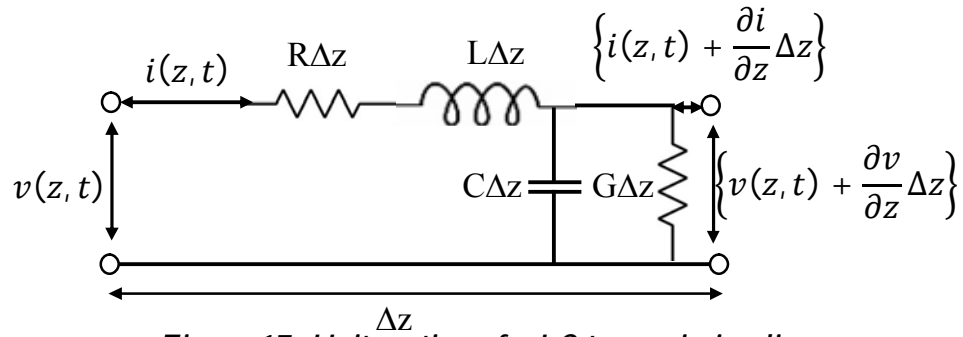


Figure 17: Unit section of a LC transmission line.

The circuit distribution theory involves one space variable in addition to time variable. In the one unit of transmission line the input voltage is considered  $v(z, t)$  and current  $i(z, t)$  [6]. If the change in voltage and current per unit length of transmission line is  $\frac{\partial v(z, t)}{\partial z}$ ,  $\frac{\partial i(z, t)}{\partial z}$  then at the end of the unit section of length  $\Delta z$ , the values of voltage and current will be  $v + \frac{\partial v(z, t)}{\partial z} \Delta z$  and  $i + \frac{\partial i(z, t)}{\partial z} \Delta z$ . Employing Kirchhoff law for voltage we can write

$$v(z, t) = i(z, t)R\Delta z + L\Delta z \frac{\partial I(z, t)}{\partial t} + \left\{ v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \right\}$$

Rearranging this equation

$$\frac{\partial v(z, t)}{\partial z} = - \left\{ iR + L \frac{\partial i(z, t)}{\partial t} \right\} \quad (1)$$

Similarly, using Kirchhoff's law for current

$$i(z, t) = G\Delta z \left\{ v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \right\} + C\Delta z \frac{\partial}{\partial t} \left\{ v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \right\} + \left\{ i(z, t) + \frac{\partial i(z, t)}{\partial z} \Delta z \right\}$$

Rearranging it we can obtain

$$\frac{\partial i(z,t)}{\partial z} = - \left\{ vG + C \frac{\partial v(z,t)}{\partial t} \right\} \quad (2)$$

On the other hand, the Instantaneous line voltage and current can be expressed as

$$v(z, t) = v(z) e^{j\omega t}$$

$$\frac{\partial v(z,t)}{\partial t} = j\omega v(z, t) \quad (3)$$

Similarly, the current

$$i(z, t) = i(z) e^{j\omega t}$$

$$\frac{\partial i(z,t)}{\partial t} = j\omega i(z, t) \quad (4)$$

By using equations 3 and 4 in equations 1 and 2 we can write the space part of voltage and current

$$\begin{aligned} \frac{dv(z)}{dz} &= -(R + j\omega L)i(z) \\ \frac{di(z)}{dz} &= -(G + j\omega C)v(z) \end{aligned} \quad (5)$$

Differentiating both equations w.r.t. z

$$\frac{d^2v(z)}{dz^2} = -(R + j\omega L) \frac{di(z)}{dz}$$

Or

$$\frac{d^2v(z)}{dz^2} = (R + j\omega L)(G + j\omega C)v(z)$$

Similarly, we can obtain

$$\frac{d^2i(z)}{dz^2} = (R + j\omega L)(G + j\omega C)i(z)$$

If we introduce  $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ , then we can write

$$\begin{aligned} \frac{d^2v(z)}{dz^2} - \gamma^2 v(z) &= 0 \\ \frac{d^2i(z)}{dz^2} - \gamma^2 i(z) &= 0 \end{aligned} \quad (6)$$

These are voltage and current wave equations through transmission line. Furthermore,  $\gamma$  is propagation constant [7], which is itself a complex quantity and can be expressed as below:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + i\beta \quad (7)$$

Here,  $\alpha$  is defined by attenuation constant and  $\beta$  is phase constant for voltage and current waves propagating through the transmission line.

#### (iv) Solutions of Equations for transmission line:

The voltage and current at any point z in a transmission line can be obtained by



following the possible solutions for wave equations (6),

$$\begin{aligned}v(z) &= v^+ e^{-\gamma z} + v^- e^{\gamma z} \\i(z) &= i^+ e^{-\gamma z} + i^- e^{\gamma z}\end{aligned}\quad (8)$$

Here  $v^+$  is representing the voltage coefficient of voltage wave propagating toward load while  $v^-$  is related to the wave reflected back. Similarly,  $i^+$  and  $i^-$  are current coefficients of current wave propagating toward load and reflected back respectively. These coefficients are also complex quantities.

Further we know that from equation 5

$$\frac{dv(z)}{dz} = -(R + j\omega L)i(z)$$

So, by using equation 8 in the above expression we can get

$$\begin{aligned}-\gamma v^+ e^{-\gamma z} + \gamma v^- e^{\gamma z} &= -(R + j\omega L)i(z) \\i(z) &= \frac{(R + j\omega L)}{\gamma} [v^+ e^{-\gamma z} - v^- e^{\gamma z}] \\i(z) &= \frac{1}{Z_0} [v^+ e^{-\gamma z} - v^- e^{\gamma z}]\end{aligned}\quad (9)$$

Here  $Z_0 = \frac{(R + j\omega L)}{\gamma}$  is defined as characteristic impedance [6, 7]. Putting the value of propagation constant

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o \pm j\omega X_o \quad (10)$$

(v) **Attenuation and Phase constants:** From equation 7, propagation constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

So we can write

$$\begin{aligned}\alpha + j\beta &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ \alpha - j\beta &= \sqrt{(R - j\omega L)(G - j\omega C)}\end{aligned}$$

Solving these two equations we can find the expression for attenuation and phase constants [6, 8]

$$\begin{aligned}\alpha &= \frac{1}{\sqrt{2}} \left[ (RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2) + (G^2 + \omega^2 C^2)} \right]^{\frac{1}{2}} \\ \beta &= \frac{1}{\sqrt{2}} \left[ (-RG + \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2) + (G^2 + \omega^2 C^2)} \right]^{\frac{1}{2}}\end{aligned}\quad (11)$$

(vi) **Phase velocity:** A plane wave can be represented in the form

$$\varphi = \varphi_o e^{j(\omega t - \beta z + \phi)}$$

The real Voltage wave in transmission line can be represented as

$$\begin{aligned} v(z, t) &= v(z)e^{j\omega t} \\ &= |v(z)|e^{\phi}e^{j\omega t} \\ &= |v^+|e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |v^-|e^{\alpha z} \cos(\omega t - \beta z + \phi^-) \end{aligned}$$

For a wavefront of voltage wave we can write

$$\omega t - \beta z + \phi^+ = 0$$

So phase velocity  $v_p = \frac{dz}{dt}$

$$v_p = \frac{1}{\beta} \frac{d}{dt} (\omega t + \phi^+)$$

$$v_p = \frac{\omega}{\beta}$$

**(vii) Line Impedance :** The complex ratio of voltage phasor and current phasor at any point of transmission line is known as line impedance at that particular point [6].

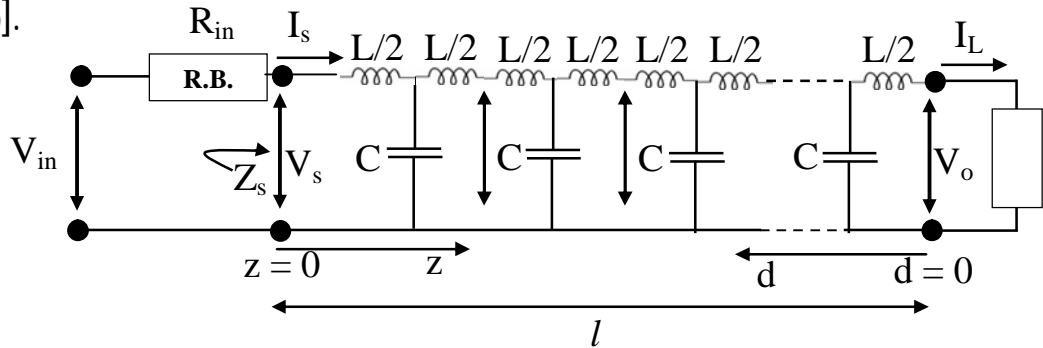


Figure 4: A LC transmission line of length 'l' is displayed with different parameters of the line.

From equation 9 we can write for load current (at  $z = l$ )

$$I_L = \frac{1}{Z_0} [v^+ e^{-\gamma l} - v^- e^{\gamma l}]$$

Also  $I_L = \frac{1}{Z_L} [v^+ e^{-\gamma l} + v^- e^{\gamma l}]$

Solving these two equations we can get

$$v^+ = \frac{I_L}{2} [Z_L + Z_0] e^{\gamma l}$$

$$v^- = \frac{I_L}{2} [Z_L - Z_0] e^{-\gamma l}$$

Using these values in equation 8 we can obtain

$$v(z) = \frac{I_L}{2} \{(Z_L + Z_0)e^{\gamma(l-z)} + (Z_L - Z_0)e^{-\gamma(l-z)}\}$$

$$i(z) = \frac{1}{Z_0} \frac{I_L}{2} \{(Z_L + Z_0)e^{\gamma(l-z)} - (Z_L - Z_0)e^{-\gamma(l-z)}\}$$

From figure 4, It is clear that for any point in transmission line ( $z + d = l$ )

$$v(d) = \frac{I_L}{2} \{(Z_L + Z_0)e^{\gamma d} + (Z_L - Z_0)e^{-\gamma d}\}$$

$$i(d) = \frac{1}{Z_0} \frac{I_L}{2} \{(Z_L + Z_0)e^{\gamma d} - (Z_L - Z_0)e^{-\gamma d}\}$$

So, the line impedance at any point can be expressed as (here d is distance measured from output terminals)

$$Z_d = Z_0 \left\{ \frac{(Z_L + Z_0)e^{\gamma d} + (Z_L - Z_0)e^{-\gamma d}}{(Z_L + Z_0)e^{\gamma d} - (Z_L - Z_0)e^{-\gamma d}} \right\}$$

Here  $Z_L$  is load impedance, while  $Z_0$  is characteristic impedance. So, the line impedance measured at input terminals ( $Z_s$ ) i.e. at  $d=l$  (here  $l$  is total length of transmission line) will be

$$Z_s = Z_0 \left\{ \frac{(Z_L + Z_0)e^{\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{\gamma l} - (Z_L - Z_0)e^{-\gamma l}} \right\}$$

Since  $e^{\pm\gamma l} = \cosh(\gamma l) \pm \sinh(\gamma l)$

So impedance at the input terminals

$$Z_s = Z_0 \left\{ \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right\}$$

(a) If the output terminals are short-circuited ( $Z_L = 0$ ) the impedance is

$$Z_s = Z_0 \left\{ \frac{0 - Z_0 \tanh(\gamma l)}{Z_0 - 0 \tanh(\gamma l)} \right\}$$

$$\mathbf{Z_{sc} = Z_0 \tanh(\gamma l)} \quad (12)$$

(b) If the output terminals are short-circuited ( $Z_L = \infty$ ) the impedance is

$$Z_{oc} = \lim_{Z_L \rightarrow \infty} \frac{Z_0}{\tanh(\gamma l)} \left\{ \frac{1 - \frac{Z_0 \tanh(\gamma l)}{Z_L}}{\frac{Z_0}{Z_L \tanh(\gamma l)} - 1} \right\}$$

$$Z_{oc} = \frac{Z_0}{\tanh(\gamma l)}$$

$$\mathbf{Z_{oc} = Z_0 \coth(\gamma l)} \quad (13)$$

(c) Characteristic Impedance:

$$Z_{oc} \times Z_{sc} = Z_o \coth(\gamma l) \times Z_o \tanh(\gamma l)$$

$$Z_o^2 = Z_{oc} \times Z_{sc}$$

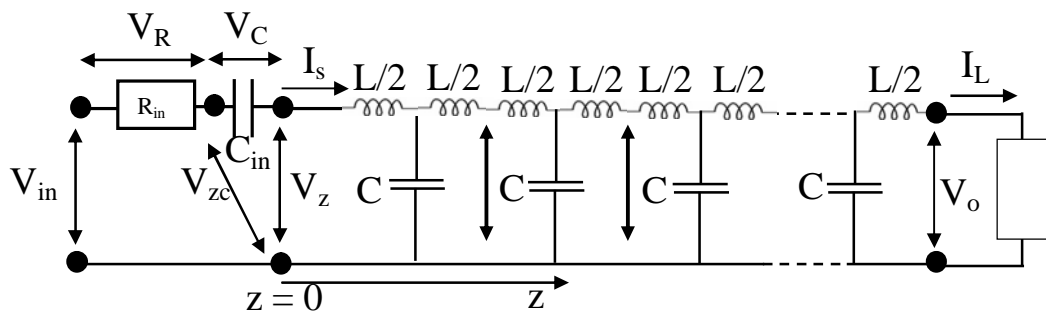
$$Z_o = \sqrt{Z_{oc} \times Z_{sc}} \quad (14)$$

## 17.6 Self Learning Exercise-I

- Q.1 What do you understand by a transmission line?  
 Q.2 What do you understand by a distortion less transmission line?  
 Q.3 What is meaning of loss less transmission line?  
 Q.4 How we can reduce the distortion in a transmission line?  
 Q.5 How can we obtain a low loss transmission line?

## 17.7 Procedure

- i. Connect the LC sections by connecting wire and make at least 20 segments LC transmission line
  - ii. Connect the input terminals by a function generator.
  - iii. A voltage ( $V_{in}$ ) with known frequency ( $f_{in}$ ) should be applied on these input terminals with the help of function generator
- (A) To find out characteristic impedance for given transmission line**
- iv. Characteristic impedance can be measured by  $Z_o = \sqrt{Z_{oc} \times Z_{sc}}$  where  $Z_{oc}$  is line impedance for open circuit and is  $Z_{sc}$  line impedance for short circuited.
  - v. Add a resistance  $R_{in}$  and a Capacitor  $C_{in}$  at the input terminals of transmission line with any arbitrary values as shown in figure below. The values of the capacitor and resistance should be selected in the way that impedance by capacitor ( $1/\omega C_{in}$ ) is in order with resistance ( $R_{in}$ ).



***A.1 To find out line impedance for open circuit ( $Z_{oc}$ ):***

- vi. Open the output terminals of the transmission line i.e.  $Z_L = \infty$
- vii. Measure the values of  $V_R$  and  $V_C$  and  $V_{Zc}$ , note down in table 1.
- viii. An another set of the measurements of these voltages can be taken by changing the values of  $R_{in}$  and  $C_{in}$  (to use the average values of  $V_{r0}$  and  $V_{\theta0}$ ).

***A.2 To find out line impedance for open circuit ( $Z_{sc}$ ):***

- ix. Open the output terminals of the transmission line i.e.  $Z_L = 0$
- x. Measure the values of  $V_R$  and  $V_C$  and  $V_{Zc}$ , note down in table 2.
- xi. An another set of the measurements of these voltages can be taken by changing the values of  $R_{in}$  and  $C_{in}$  (to use the average values of  $V_{rs}$  and  $V_{\theta_s}$ ).

**(B) To find out the wave pattern and various line parameters**

- xii. Remove the  $R_{in}$ ,  $C_{in}$  and connect the power supply from a signal generator at input terminals of transmission line.

***B.1: Measurements If output terminals are short***

- xiii. Short the output terminal with a conducting wire or conducting bar ( $Z_L = 0$ ).
- xiv. Measure the voltage at the end of each section with help of VTVM or ac voltmeter.
- xv. Note down and plot the graph between number of sections and measured voltage

***B.2: If output terminals are open***

- xvi. Let open the output terminals i.e.  $Z_L = \text{Infinite}$ .
- xvii. Measure the voltage at the end of each section with help of VTVM or ac voltmeter.
- xviii. Note down and plot the graph between number of sections and measured voltage

***B.3: If output terminals are loaded with characteristic impedance ( $Z_L = Z_0$ )***

- xix. Connect the resistance box at the output terminal and load these terminals with a resistance equal to characteristic impedance of the transmission line. The complex conjugate of calculated reactive impedance (capacitive or inductive) also should be added.
- xx. Measure the voltage at the end of each section with help of VTVM or ac voltmeter.

xxi. Note down and plot the graph between number of sections and measured voltage

Calculate the values of  $\alpha$ ,  $\beta$  and  $v_p$  from the plots and compare these values with theoretical values.

### 17.8 Observation

- Input voltage ( $V_{in}$ ) = ..... Volt
- Frequency of current ( $f_{in}$ ) = ..... Hz
- Total number of sections ( $n$ ) = .....
- $L$  = ..... H/section
- $C$  = ..... F/section

Section No. (n)	Voltage across section of lumped delay (in Volts) line when load end in:			Section No. (n)	Voltage across section of lumped delay (in Volts) line when load end in:		
	Open Circuit	Short Circuit	Terminated with $Z_0$		Open Circuit	Short Circuit	Terminated with $Z_0$
1				21			
2				22			
3				23			
4				24			
5				25			
6				26			
7				27			
8				28			
9				29			
10				30			

11				31			
12				32			
13				33			
14				34			
15				35			
16				36			
17				37			
18				38			
19				39			
20				40			

**(ii) For open circuit**

S. No.	$R_{in}(\Omega)$	$V_R$ (Volt)	$V_C$ (Volt)	$V_Z$ (Volt)	$V_{ZC}$ (Volt)
1					
2					

**(ii) For short circuit**

S. No.	$R_{in}(\Omega)$	$V_R$ (Volt)	$V_C$ (Volt)	$V_Z$ (Volt)	$V_{ZC}$ (Volt)
1					
2					

## 17.9 Graph

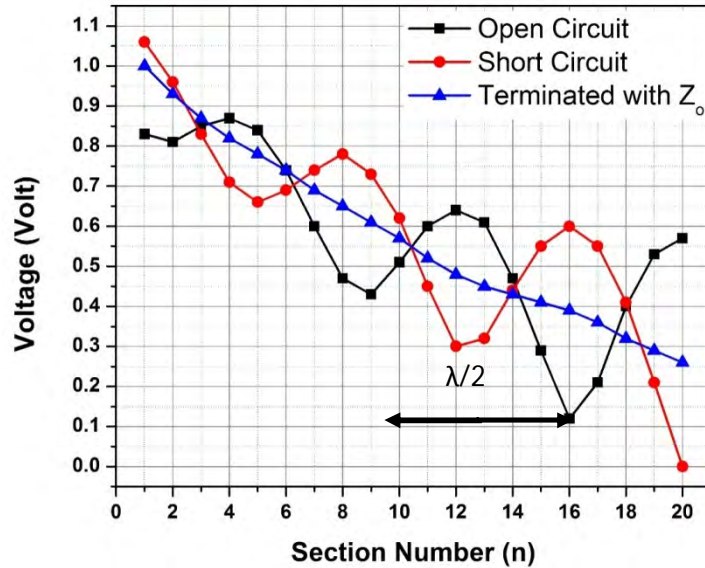


Figure 5: Plot between measured voltages for each section of the given LC transmission short-circuited, open circuited and line terminated with characteristic impedance.

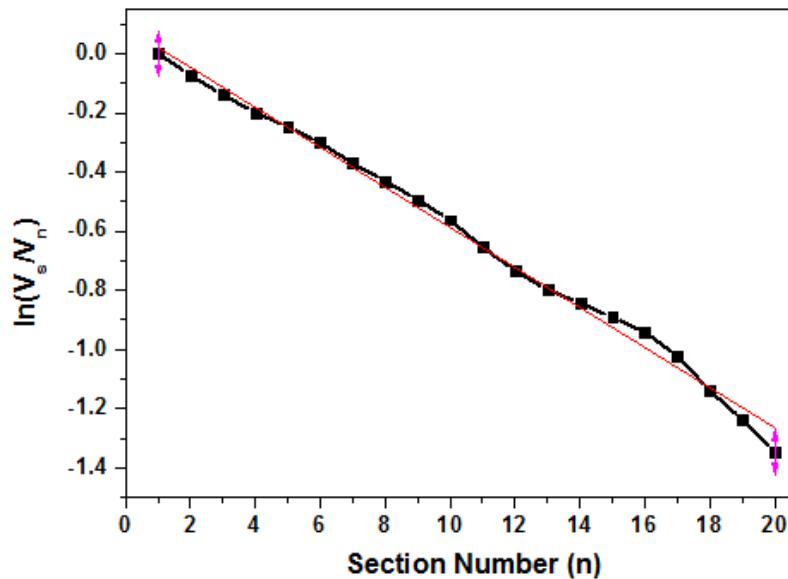


Figure 6: Plot between measured voltages for each section in the given LC transmission line terminated with characteristic impedance for calculation of attenuation constant.



## 17.10 Calculations

### Model calculation :

A signal generator is used with frequency  $f_{in} = 1.5$  KHz at input with  $V_{in} = 1.0$  volt. ( $L = 4.8$  mH;  $C = 0.47$   $\mu$ F)

#### (i) To calculate impedance for open circuit ( $Z_{oc}$ ):

The current flowing through transmission line can be calculate by the ratio of input resistance (a resistance added in the series of transmission line with help of a resistance box) and the voltage drop across this resistance. Hence,

$$I_s = \frac{V_R}{R_{in}}$$

Let, any value of  $R_{in} = 40\Omega$  and  $C_{in} = 2\mu$ F. The voltage measured across this resistance  $V_R = 0.41$  volt.

So the current following through transmission line

$$I_s = 0.01025 \text{ Amp}$$

Impedance for open circuit transmission line

$$Z_{oc} = R_{oc} + jX_{oc}$$

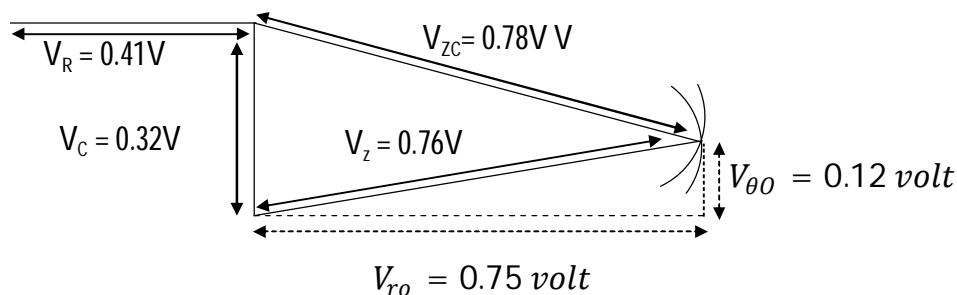
Here,  $R_{oc} = \frac{V_{r_o}}{I}$  and  $X_{oc} = \frac{V_{\theta_o}}{I}$

$V_{r_o}$  and  $V_{\theta_o}$  can be measured by Phasor diagrams.

If the measurements are in such a way that

$V_R = 0.41$  Volt,  $V_C = 0.32$  Volt,  $V_Z = 0.76$  Volt,  $V_{ZC} = 0.78$  Volt

If we take 0.1 volt = 1 cm



$V_{r_o} = 0.75$  volt and  $V_{\theta_o} = +0.12$  volt (+ value showing inductive)

So,  $R_{oc} = \frac{0.75}{0.01025} = 73.17 \Omega$

and  $X_{oc} = \frac{0.12}{0.01025} = 11.7073 \Omega$

$$Z_{oc} = 73.17 + j(11.7073) \Omega$$

(ii) To calculate impedance for short circuit ( $Z_{sc}$ ):

The current flowing through transmission line

$$I_s = \frac{V_R}{R_{in}}$$

Let, any value of  $R_{in} = 50 \Omega$  and  $C_{in} = 2 \mu F$ . The voltage measured across this resistance  $V_R = 0.50$  volt.

So the current following through transmission line

$$I_s = 0.01 \text{ Amp}$$

Impedance for open circuit transmission line

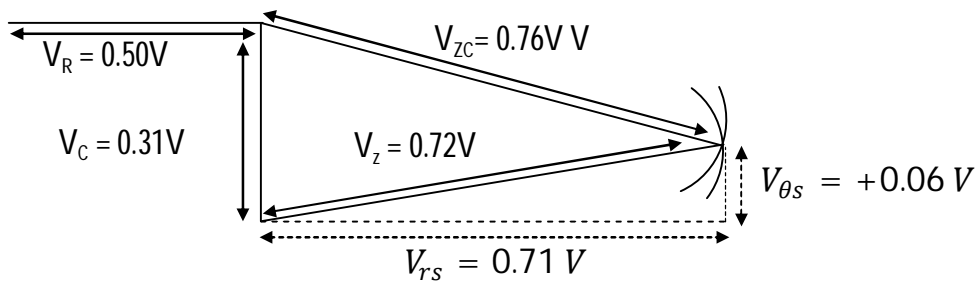
$$Z_{sc} = R_{sc} + jX_{sc}$$

Here,  $R_{sc} = \frac{V_{rs}}{I}$  and  $X_{sc} = \frac{V_{\theta s}}{I}$

$V_{r_o}$  and  $V_{\theta_o}$  can be measured by phaser diagrams

If measurements are in such a way that  $V_R = 0.50$  Volt,  $V_C = 0.31$  Volt,  $V_Z = 0.72$  Volt,  $V_{ZC} = 0.76$  Volt

If we take 0.1 volt = 1 cm



$V_{rs} = 0.71$  volt and  $V_{\theta_s} = +0.06$  volt

So,  $R_{sc} = \frac{0.71}{0.01}$

and

$$R_{sc} = 71 \Omega$$

$$X_{sc} = \frac{0.06}{0.01}$$

$$X_{sc} = 6 \Omega$$

$$Z_{sc} = R_{sc} + jX_{sc}$$

$$Z_{sc} = 71 + j(6) \Omega$$

(iii) To calculate Characteristic Impedance ( $Z_o$ ):

$$Z_o^2 = Z_{oc} \times Z_{sc}$$

$$Z_o^2 = \{73.17 + j(11.71)\}\{71 + j(6)\}$$

$$Z_o^2 = \{5195.67 + j(1270.22) - 70.26\}$$

$$Z_o^2 = \{5125.41 + j(1270.22)\}$$

$$\text{Let, } Z_o^2 = a + jb = \gamma e^{i\theta}$$

Here it can be easily defined

$$\gamma^2 = a^2 + b^2$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$Z_o = \sqrt{\gamma} e^{j\frac{\theta}{2}}$$

$$\text{So, } \gamma^2 = (5125.41)^2 + (1270.22)^2$$

$$\gamma = 5279.73$$

and

$$\theta = \tan^{-1}\left(\frac{1270.22}{5125.41}\right)$$

$$\theta = 0.2428^\circ$$

$$\text{So, } Z_o = \sqrt{5279.73} e^{j\frac{0.2428}{2}}$$

$$Z_o = 72.66 \cos(0.1214) + j \sin(0.1214)$$

$$Z_o = 72.1223 + j(8.7931)\Omega$$

It is characteristic impedance for given transmission line. We can terminate the transmission line by a resistance with value of  $R = 72.1223 \Omega$  and capacitor of value  $C = 12.06 \mu\text{F}$ . Since, 72.1223 is real part and can be treated as resistive part while imaginary part 8.7991 is a reactive part and it should be capacitive (since the imaginary part comes with plus sign and it indicating to inductive impedance and its complex conjugate part i.e. capacitive with the same value should be added). The value of capacitance can be calculated as follows

$$|X_c| = \frac{1}{2\pi f_{in} C}$$

$$8.7931 = \frac{1}{2\pi \times 1.5 \times 10^3 \times C}$$

$$C = 12.06 \mu\text{F}$$

#### (iv) Phase velocity and phase constant:

$$\text{From graph } \frac{\lambda}{2} = 7 \text{ sections}$$

$$\lambda = 14 \text{ sections}$$

So phase constant will be

$$\beta = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{14}$$

$$\beta = 0.4486 \text{ rad/section}$$

It is experimental value of phase constant while it can compare with its theoretical value for ideal LC transmission line

$$\beta_i = \omega\sqrt{LC} \text{ (0.447 rad/section in this case when } L = 4.8 \text{ mH and } C = 0.47 \text{ }\mu\text{F)}$$

phase velocity

$$v_p = f\lambda = 1.5 \times 10^3 \times 14$$

$$v_p = 21 \times 10^3 \text{ section/second}$$

It is experimental value of phase constant while it can compare with its theoretical value for ideal LC transmission line  $v_p = \frac{1}{\sqrt{LC}}$

**(v) Attenuation constant:**

Voltage at the n<sup>th</sup> section

$$V_n = V_s e^{-\alpha n}$$

$$\alpha = \frac{1}{n} \ln \left( \frac{V_s}{V_n} \right)$$

The slope of curve plotted in figure provide the attenuation

$$\alpha = \frac{0.06767}{20}$$

$$\alpha = 3.38 \times 10^{-3} \text{ Np/section}$$

Or  $\alpha = 8.686 \times 3.38 \times 10^{-3} \text{ decibel/section}$

$$\alpha = 29.36 \times 10^{-3} \text{ decibel/section}$$

It is experimental value of phase constant while it can compare with its theoretical value for ideal LC transmission line

$$\alpha_i = \frac{R}{2} \sqrt{\frac{C}{L}}$$

**17.11 Result**

1. The graphs for a given LC transmission line in short circuited, open circuited and  $Z_0$  terminated are found as like the plot attached.
2. The values of characteristic impedance, attenuation constant, phase constant, transmission coefficient and phase velocity are found

$$Z_0 = \dots\dots\dots \Omega$$

$$\alpha = \dots\dots\dots \text{Np/section}$$

$$\beta = \dots\dots\dots \text{rad/section}$$

$$\gamma = \dots\dots\dots$$
$$V_p = \dots\dots\dots \text{Section/second}$$

### 17.12 Discussion

.....

.....

.....

### 17.13 Precautions and Sources of Error

1. All the connections should be tight.
2. The cords to make the joints of the T sections should be thick.
3. The internal dc resistance of coil (R) should be very less ( $R \ll \omega L$ ).
4. Check all the connecting wires before making the power supply on.
5. Frequency should not in MHz and GHz range in transmission line.
6. The determination of the values of Resistance and Capacitor for characteristic impedance should be very carefully.
7. The input voltage and frequency should not be fluctuating during experiment.

### 17.14 Self Learning Exercise-II

- Q.1** Why there is standing wave pattern in study of LC transmission line?
- Q.2** What is the physical significance of characteristic impedance of the transmission line?
- Q.3** What is meaning of attenuation constant of the given transmission line?
- Q.4** What do you understand by line parameters? Define the line parameters.
- Q.5** What are the secondary constants of a transmission line?

### 17.15 Glossary

**Impedance:** The total opposition to alternating current by an electric circuit, equal to the square root of the sum of the squares of the resistance and reactance of the circuit and usually expressed in ohms.

**Short circuited:** Two terminals can be said shorted or short circuited electrically if both terminals are connected by a connecting wire with zero resistance.

**Pulse:** Brief sudden change in a normally constant quantity.

**Waveform:** The mathematical representation of a wave especially a graph obtained by plotting characteristics of the wave against time.

**Phasor:** Basically a rotating vector, simply called a “Phasor” is a scaled line whose length represents an AC quantity that has both magnitude and direction which is “frozen” at some point in time [11]. In analytic representation, which decomposes a sinusoid into the product of a complex constant and a factor that encapsulates the frequency and time dependence [12].

**Equalizers:** Equalizers in transmission are networks whose frequency and phase characteristics are adjusted to be inverse to those of the lines, which result in a uniform frequency response over the desired frequency band, and hence the attenuation is equal for all the frequencies.

## 17.16 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-I*

**Ans.1:** In communications and electronic engineering, a transmission line is a specialized cable or a structure designed to carry electric energy from one point to another.

**Ans.2:** If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line distorted. Transmission is said to be distortion-less if the input and output have identical wave shapes.

**Ans.3:** A transmission line is lossless if attenuation constant is zero.

**Ans.4:** It can be proved easily that phase velocity is approximately constant if the line is low-loss. So, distortion will be very less in low loss transmission line.

**Ans.5:** If  $R \ll \omega L$  and  $G \ll \omega C$  then we can consider a transmission line a low loss transmission line.

### *Answers To Self Learning Exercise-II*

**Ans.1:** If transmission line is end with load not equal to characteristic impedance,

the part of wave is reflected back from end terminals, gives standing wave pattern.

**Ans.2:** The maximum power can be transferred if the transmission line is terminated with its characteristic impedance.

**Ans.3:** :It defines the rate of reduction (in space) of the wave amplitude. It is measured in Neper/m or dB/m ( $1\text{Np} = 8.686\text{ dB}$ )

**Ans.4:** The parameters of a transmission line are: Resistance (R) Inductance (L) Capacitance (C) Conductance (G) these parameters are known as line parameters Since the line constants R, L, C, G are distributed through the entire length of the line, they are called as distributed elements. They are also called as primary constants.

Resistance (R) is defined as the loop resistance per unit length of the wire. Its unit is ohm/Km

Inductance (L) is defined as the loop inductance per unit length of the wire. Its unit is Henry/Km

Capacitance (C) is defined as the loop capacitance per unit length of the wire. Its unit is Farad/Km

Conductance (G) is defined as the loop conductance per unit length of the wire. Its unit is mho/Km

**Ans.5:** The secondary constants of a line are: Characteristic Impedance and Propagation Constant

## 17.17 Viva Questions

- Q.1** What do you mean by a transmission line?
- Q.2** What is the use to study an artificial transmission line?
- Q.3** Give the example of transmission line in daily life.
- Q.4** Why the different shapes of transmission lines are required?
- Q.5** Is it possible that a single stand wire act as a transmission line?
- Q.6** What is the reason of loss of power in a transmission line?
- Q.7** Is attenuation constant depend on the length of transmission line?
- Q.8** What is Neper?

- Q.9** What do you understand by phase constant in the transmission line?
- Q.10** Define Propagation constant.
- Q.11** What is a finite line?
- Q.12** What is an infinite line?
- Q.13** What is wavelength of a line?
- Q.14** What is Impedance matching?
- Q.15** What are standing waves?
- Q.16** What are the types of line distortions?
- Q.17** How to avoid the frequency distortion that occurs in the line?
- Q.18** What is delay distortion?
- Q.19** How the telephone line can be made a distortion less line?
- Q.20** How we can design an artificial transmission line?

### 17.18 Answers of Viva Questions

**Ans.1:** In communications and electronic engineering, a transmission line is a specialized cable or a structure designed to carry electric energy from one point to another.

**Ans.2:** To analyze the behaviour of a transmission line in laboratory (in a limited space)

**Ans.3:** Telephone cable, Electrical cable to transmit electrical energy from one city to another, waveguide etc.

**Ans.4:** To transmit different frequencies ranges.

**Ans.5:** No, to send an electrical energy minimum two wires required.

**Ans.6:** Stray capacitances, resistance and stray inductance.

**Ans.7:** No, it is measured per unit length of transmission line.

**Ans.8:** The ratio of voltage can be represented in Neper

$$\alpha = \ln \left( \frac{V_{in}}{V_{out}} \right)$$

**Ans.9:** It is the rate of variation of the phase of the wave along the z coordinate while keeping time constant.



- Ans.10:** Propagation constant is defined as the natural logarithm of the ratio of the input voltage (or current) to the output voltage (or current) of the line. It gives the manner in the wave is propagated along a line and specifies the variation of voltage and current in the line as a function of distance.
- Ans.11:** A finite line is a line, which is terminated, in its characteristic impedance ( $Z_L=Z_0$ ).
- Ans.12:** A transmission line can be said infinite if the input impedance is equivalent to the characteristic impedance ( $Z_s=Z_0$ ) since the wave will never reflected in this transmission line.
- Ans.13:** The distance, the wave travels along the line while the phase angle is changing through  $2\pi^c$  is called a wavelength of the transmission line.
- Ans.14:** For maximum power transfer, the impedances at input (impedance of source) and output terminals (impedance of load) should be matched. This is called impedance matching.
- Ans.15:** If the transmission line is not terminated in its characteristic impedance, then there will be two waves traveling in opposite directions i.e. with  $\pi^c$  phase along the transmission line which gives rise to standing waves having fixed maxima and fixed minima.
- Ans.16:** Waveform distortion is of two types: a) Frequency distortion b) Phase or Delay Distortion.
- Ans.17:** In order to reduce frequency distortion occurring in the line, a) The attenuation constant  $\alpha$  should be made independent of frequency. b) By using equalizers at the line terminals which minimize the frequency distortion.
- Ans.18:** If the signal having more than one frequencies is transmitted through a transmission line (as the phase velocity depends upon frequency), few frequencies have low phase velocity received at output with some delay and hence signal get distorted. This type of distortion is called phase distortion.
- Ans.19:** To make a distortion less line, the inductance value should be increased by placing lumped inductors along the line.
- Ans.20:** If the frequency to transmit through a transmission line, and the length or sections are already fixed, then first of all phase velocity is calculated

( $v_p = f\lambda$ ) while keeping wavelength ( $\lambda$ ) less than 10 (to achieve at least four loops of standing wave pattern in 20 sections). As we have phase velocity we can calculate the value of L for given C (or vice versa) using  $v_p = \frac{1}{\sqrt{LC}}$ .

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# UNIT-18

## Recovery Time of Diode

### Structure of the Unit

- 18.1 Aim
- 18.2 Apparatus
- 18.3 Diagram
- 18.4 Formula
- 18.5 Theory and Description
- 18.6 Self Learning Exercise-I
- 18.7 Procedure
- 18.8 Observation
- 18.9 Graph
- 18.10 Calculations
- 18.11 Result
- 18.12 Discussion
- 18.13 Precautions and Sources of Error
- 18.14 Self Learning Exercise -II
- 18.15 Glossary
- 18.16 Answers To Self Learning Exercises
- 18.17 Viva Questions
- 18.18 Answers To Viva Questions

References And Suggested Readings

### 18.1 Aim

To determine the recovery time of diode.

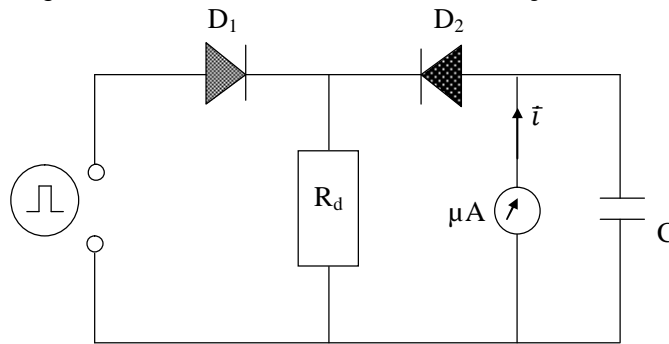
### 18.2 Apparatus

A diode for which the recovery time has to be determined ( $D_1$ ), fast switching

diode ( $D_2$ ), signal generator, high resistance box, capacitor and micro ammeter.

### 18.3 Diagram

A circuit diagram to measure the reverse recovery time for diode  $D_1$  is shown as below:



**Figure 18:** Circuit diagram to determine reverse recovery time of diode  $D_1$ .

$D_1$ : Diode for which reverse recovery time has to be measured

$D_2$ : Diode with comparatively fast recovery time

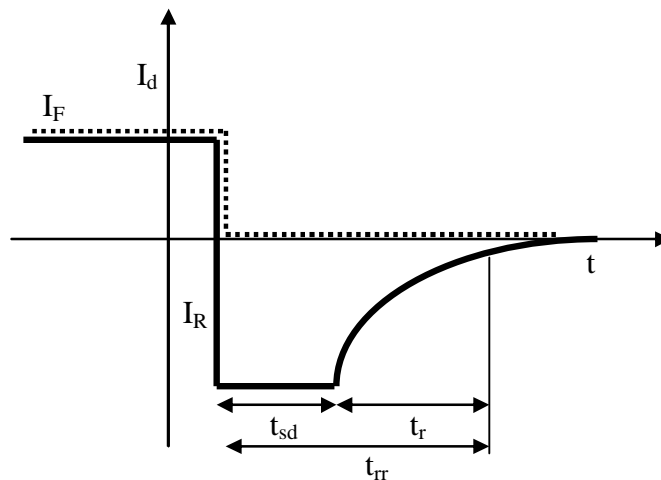
### 18.4 Formula

A pn junction diode possesses almost zero resistance and known as ON state of the diode and if it does not conduct at all, it is OFF state. The time taken by diode to switch from ON to OFF state is known as diode reverse recovery time.

The total reverse recovery time of a diode can be given by

$$t_{rr} = t_{sd} + t_r$$

Here,  $t_{sd}$  is storage delay time due to storage of charge and  $t_r$  is recovery time as shown in figure 2.



**Figure 2:** Change in the reverse current with time flowing through diode  $D_1$

Total reverse recovery time can be measured

$$t_{rr} = \frac{\bar{i}R_d}{fV_o}$$

Here,  $\bar{i}$  = Average current flowing through micro ammeter.

$R_d$  = Resistance to control reverse peak current

$f$  = Frequency of rectangular ac signal

$V_o$  = Peak voltage value of rectangular ac signal

## 18.5 Theory and Description

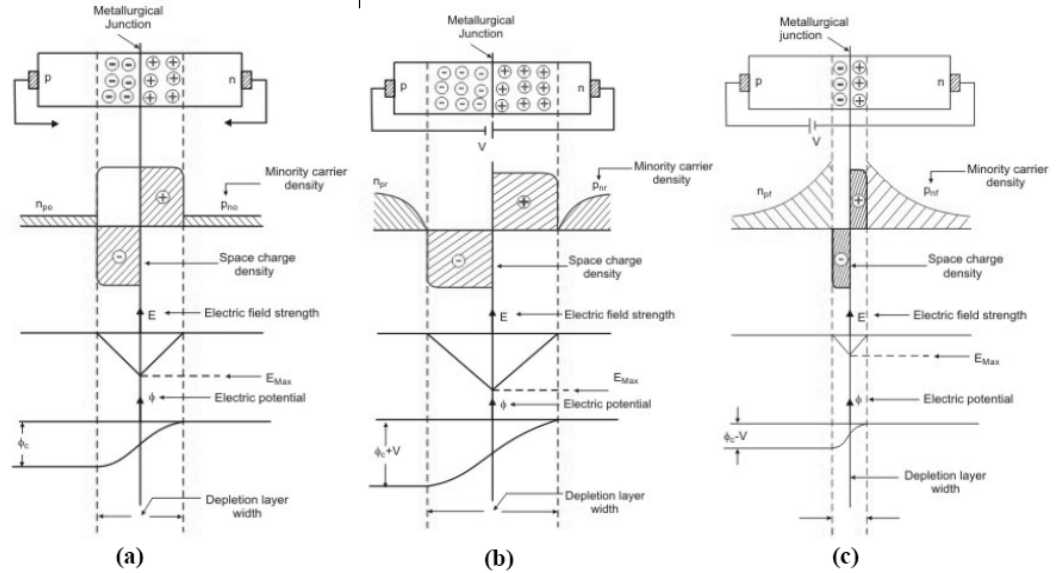
**18.5.1: Diode:** A diode is a semiconductor device with two electrodes called the anode and the cathode. The semiconductor materials, which are used to make the diode, are silicon, germanium, or selenium. Diodes in an electronic circuit used as an auto switch which conduct the current in ON state and interrupt the flow of current in OFF state. This application can be used in different way as rectifiers, signal limiters, voltage regulators, switches, signal modulators, signal mixers, signal demodulators, and oscillators.

A pn junction, formed by joining a p type semiconductor and an n type semiconductor, is a crucial part of semiconductor devices and a diode possesses one pn junction. In the absence of an external voltage at the junction, only immobile ions are available due to recombination of mobile charge carriers (holes and electrons). This region is known as depletion region or space charge region. A potential is thus created at the junction due to immobile charges and is known as potential barrier. This potential barrier does not allow the mobile charge carrier to cross the junction in the absence of external field as shown in figure 3.

The I-V characteristic curve for this pn junction device show non-ohmic behavior (shown in figure 4). This characteristic curve can be divided in two parts forward and reverse biased characteristic curve. If the potential is higher at p side in compare to n side, the diode is set in forward bias and if it reverses then diode is said to be in reverse bias.

In the forward bias condition the potential barrier is lowered. The minor charge carrier density in both p and n side follows exponential decay characteristics (figure 7a). Since, a very large number of minority charge carriers are injected across the junction and these injected minority carriers eventually recombine with

the majority charge carriers as they diffuse further into the electrically neutral drift region. The characteristic decay length is called the *minority carrier diffusion length* [1, 2].



**Figure 3:** Space charge density the electric field and the electric potential inside a p-n junction under (a) thermal equilibrium condition, (b) reverse biased condition, (c) forward biased condition [1].

When the diode is in reverse bias, majority charge carriers attracts toward the opposite terminals i.e. the hole from the p-side will get attracted towards the negative terminal of the supply and electrons from the n-side are attracted towards the positive terminal. This increased the area of the depletion region hence the electric field at the junction will also be increased. Free minority charge carrier densities ( $n_p$  in the p side and  $p_n$  in the n side) will be zero at the edge of the depletion region on both sides. This gradient in minority carrier density causes a small flux of minority carriers to diffuse towards the depletion layer. Then they are swept by the large electric field into the electrical neutral region of the opposite side. This will cause a leakage current from the **n** side to the **p** side across the junction. The generation of electron and hole pairs by thermal ionization process also contributes to the leakage current. These two components of current together is called the *reverse saturation current*  $I_s$  of the diode. The Value of  $I_s$  is extremely

sensitive to temperature variation but is independent of the reverse voltage magnitude [1].

For a given diode, when the applied reverse voltage exceeds a threshold value, the reverse current increases rapidly. The diode is said to have undergone *reverse break down*. Reverse break down is caused by *impact ionization or Avalanche multiplication*.

Carrier density gradients on either side of the junction are supported by a forward current  $I_F$  (flowing from p side to n side) which can be expressed as

$$I_F = I_S(e^{\frac{qV}{kT}} - 1)$$

Here,  $I_s$  = Reverse saturation current ( $\mu$  A)

$v$  = Applied forward voltage across the device (volts)

$q$  = Charge of an electron

$k$  = Boltzman's constant

$T$  = Temperature in Kelvin

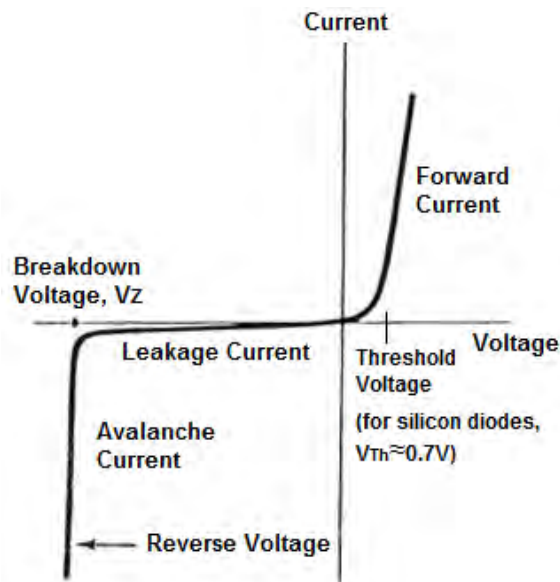


Figure 4: *I-V characteristics of a diode*

The *i-v* characteristics of a p-n junction diode can be obtained as shown in Fig 4.

The Ohmic drop has been neglected while drawing these characteristics of the semiconductor diode.

**18.5.2: Junction capacitances of a diode:** A PN junction diode possesses two types of capacitances:

(a) Transition or space charge capacitances ( $C_T$ ): In case of reverse biasing of the diode there are immobile space charges of opposite polarity around the junction. This depletion region does not possess free charge carriers so it act like a dielectric medium while free charge carriers (holes in p region and electrons in n region) act like conducting plates, so it behave like as a capacitor. The width of depletion region increases with the reverse voltage so this capacitance will be reduced with the increment in reverse voltage. This type of capacitance is effective in reverse biasing of diode.

(b) Diffusion or storage capacitance ( $C_D$ ): if the diode is forward biased the free charge carriers diffused through the junction. These holes and electrons recombine with each other across the junction. The density of electrons in p region is decreased with distance and in same way the density of holes in n region decreased with distance. So, we can say that with forward voltage there is accumulation of free charge carriers due to diffusion and this accumulation is increased with forward bias voltage. This gradient in concentration of free charge carriers behave like a capacitor and give raise the diffusion capacitance ( $C_D$ ). This capacitance is effective during the forward biasing of junction diode.

**18.5.3: Switching Characteristics of A diode:** A pn junction diode is used for switching action in electronic circuits. If the potential on p is greater than n, the diode possesses almost zero resistance (except resistance by material) and it is said to be in ON state. In OFF state, it does not conduct at all. A practical diode takes some time when it is switched from one state to another. The switching speed of a diode is important since in most of the applications, diodes are used as a switch in ac line.

Switching action of a junction diode is explained with following parameters in time:

- a. Forward recovery time ( $t_{fr}$ )
- b. The relaxation delay time( $t_{rd}$ )
- c. Storage delay time ( $t_{sd}$ )
- d. Reverse recovery time ( $t_r$ )
- e. Total reverse recovery time ( $t_{rr}$ )



If a step voltage is applied to pn junction diode, the output across the diode can be obtained as shown in figure 5. As soon as the step voltage is applied, the voltage across the diode can grow to amplitude of applied voltage but it gets down to steady state voltage, steady state voltage is actually the difference of peak voltage and voltage drop at junction. The **forward recovery time**,  $t_{fr}$  is the time from the switch on of the voltage source to the time in which the voltage across the diode has dropped to within 10% of the steady state value [2-4].

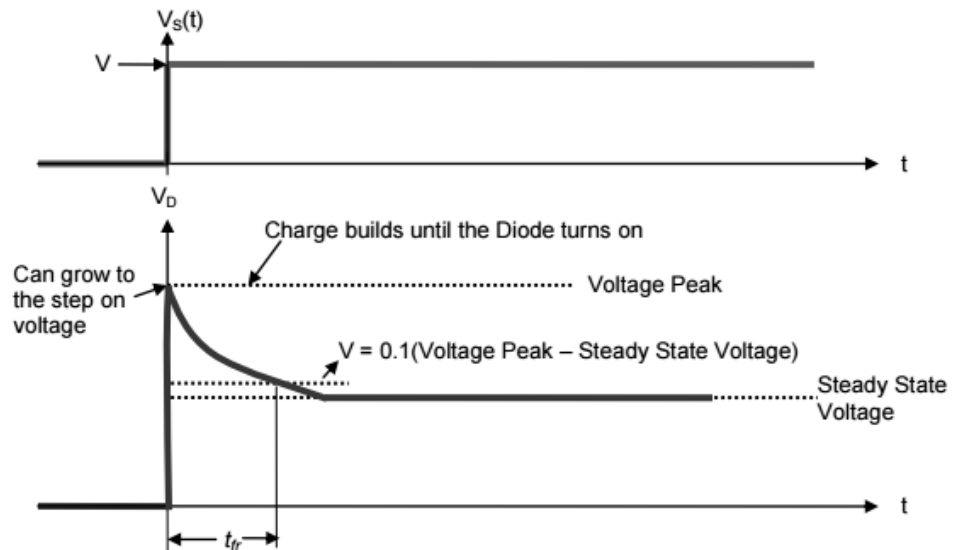


Figure 5: forward recovery time ( $t_{fr}$ ) during the diode getting ON state [2].

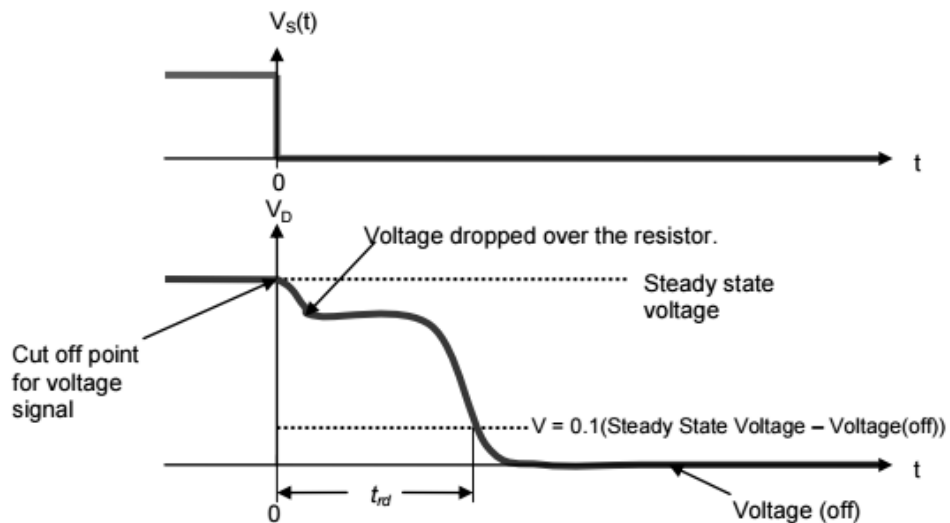
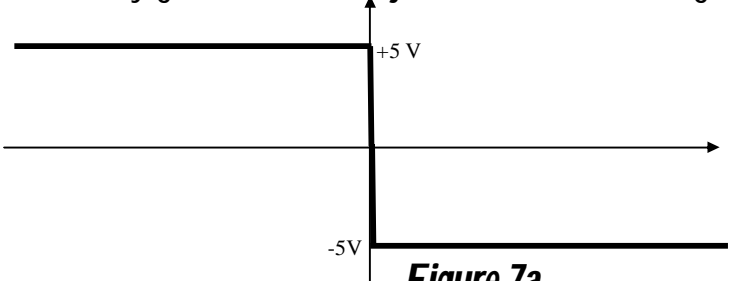


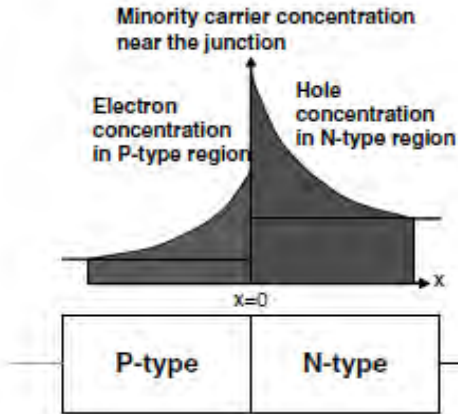
Figure 6: Relaxation delay time ( $t_{rd}$ ) as the external power supply is switched off [2].

When a voltage source is turned off, diode still conducts for few micro seconds since the stored charge is still present, until it recombines or is dissipated over the load resistance in the circuit. The **relaxation delay time** ( $t_{rd}$ ) for the diode is defined by the time in which the current value drops to 10% of the steady state value as shown in figure 6. Forward recovery and relaxation delay both times in junction diode are defined for a step voltage is applied and switched off [2].

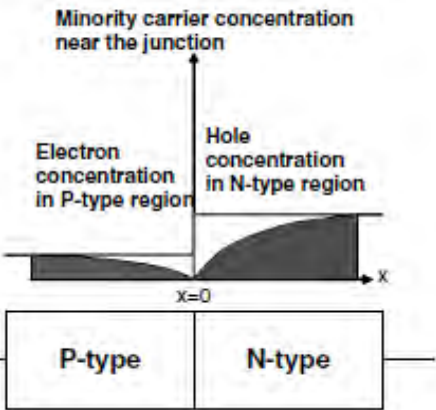
**Storage delay time** ( $t_{sd}$ ) and **reverse recovery time** ( $t_r$ ) are defined for an abrupt step voltage applied on diode in forward bias as well as in the reverse bias. When the voltage is switched from forward bias to reverse bias, reverse recovery switching will occur. During forward biasing of junction diode the minority charge carriers were already gathered near the junction as shown in figure 7b.



**Figure 7a**



**Figure 7b**



**Figure 7c**

*Figure 7: Transition of Minority charge carrier distribution during switching from forward to reverse biasing of PN junction diode (time taken in this transition is  $t_{sd}$ ) [3].*

As soon as the reverse voltage is applied this gathering of charge carriers leaves the junction and become as shown in figure 7c. Time take to achieve this

distribution known as storage delay time ( $t_{sd}$ ). During this period current flow as in the same manner as applied due to movement of minority charge carriers to be in redistribution (as per figure 7c) [2-4].

After achieving the distribution appropriated according to applied voltage the capacitance of pn junction ( $C_j = C_i + C_d$ ) started to charge and the current flows through diode to make it charge. This charging of junction capacitance occurs in exponential manner; hence the current flowing through the diode during this period reduces exponentially (as shown in figure 8b). Reverse recovery time is defined by the time in which the voltage signal across the diode to rise 10% of the steady state value from the reverse voltage experienced by the diode during the storage delay period. The **total reverse recovery time** ( $t_{rr} = t_{sd} + t_r$ ) is defined as the sum of storage delay time and reverse recovery time.

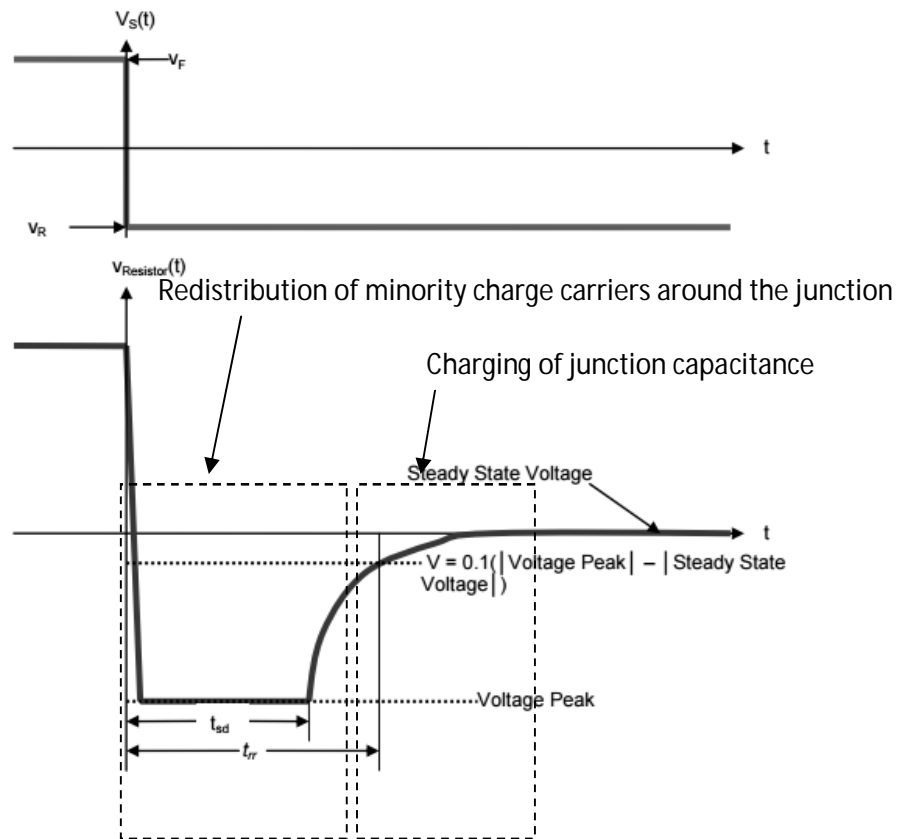


Figure 8: A typical turn off behaviour of a power diode assuming controlled rate of decrease of the forward current [2].

**18.5.4: Salient features of reverse recovery characteristics:** As it has already been discussed, a pn junction diode is used as a switch in electronic circuits and

most commonly in ac line. The reverse recovery time of the diode is very crucial parameter to for its performance as an electronic switch. It should be very less. The reverse recovery time becomes of extreme importance when frequency of ac is very high. The diode with appropriate reverse recovery time is selected as per frequency of ac line. Some salient features of this characteristic of pn junction are given as below:

- Instead of dropping at zero as expected in reverse bias the increases in the negative direction, which is known as peak reverse recovery current ( $I_{rr}$ ). In electronic circuits such as crimp - clamping, choppers, inverters etc., in addition to the load current this current also flows to the main power switch therefore while selecting the main switch for these electronic circuits the value of reverse recovery peak current must be accounted.

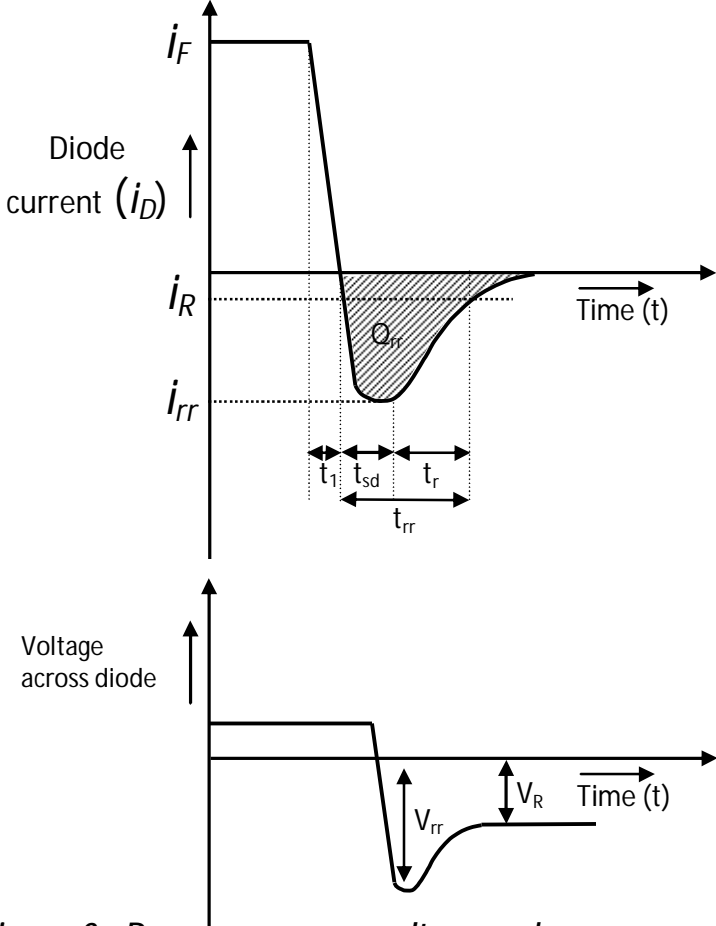


Figure 9: Reverse recovery voltage and reverse recovery charge in a reverse biased junction diode

- If the reverse current falls too sharply at the end of the reverse recovery period, stray circuit inductance may cause dangerous over voltage ( $V_{rr}$ ) across the device. It may be required to protect the diode using an RC snubber (see glossary).
- The charge stored within the diode when instantaneous switched from a specified forward current  $i_F$  (normally 0.5 A) to a specified reverse current  $i_R$  (normally 1.0 A) is known as reverse recovery charge ( $Q_{rr}$ ).
- During the period  $t_r$  large current and voltage exist simultaneously in the device. At high switching frequency this may result in considerable increase in the total power loss.
- Important parameters defining the turn off characteristics of Diode are, peak reverse recovery current ( $i_{rr}$ ), reverse recovery time ( $t_{rr}$ ), reverse recovery charge ( $Q_{rr}$ ) and the snappiness factor (S).
- Snappiness factor is defined as the ratio of recovery time and storage delay time

$$S = \frac{t_r}{t_{sd}}$$

The snappiness factor S depends mainly on the construction of the diode (e.g. drift region width, doping level, carrier life time etc.). Other parameters are interrelated and also depend on S.

**18.5.5: Fast recovery diode:** For high frequency circuits, fast recovery diodes are preferred. They offer significant reduction in both  $I_{rr}$  and  $t_{rr}$  (10% - 20% of a rectifier grade diode of comparable rating). It is typical of a particular type of diodes called "normal recovery" or "soft recovery" diode ( $S > 1$ ). In this case, the total recovery time is a few tens of microseconds. While this is acceptable for line frequency rectifiers (these diodes are also called rectifier grade diodes). High frequency circuits (e.g PWM inverters, SMPS) demand faster diode recovery. Diode reverse recovery time can be reduce by increasing the rate of decrease of the forward current (i.e, by reducing stray circuit inductance) and by using *snappy* recovery ( $S \ll 1$ ) diode.

In case of very high frequency applications ( $> 100\text{KHZ}$ ), improvement in the reverse recovery performance offered by normal fast recovery diode is not

sufficient. If the required reverse blocking voltage is less (<100v) Schottky diodes are preferred over fast recovery diodes. Compared to p-n junction diodes schottky diodes have very little Turn OFF transient and almost no Turn ON transient. On state voltage drop is also less compared to a p-n junction diode for equal forward current densities. However, reverse breakdown voltages of these diodes are less (below 200V) Power schottky diodes with forward current rating in excess of 100A are available [1, 2].

**18.5.5: Measurement of diode reverse recovery time:** Diode recovery time of any pn junction diode can be measured by comparing it with fast recovery diode by using rectangular shape ac current (with time period T of frequency  $f$ ). In circuit diagram shown in figure 1 if we assume and it is possible in practical way also, that

- $T \gg t_{rr}$
- $t_{sd} \ll t_r$  so  $t_{rr} \sim t_r$  (As  $t_{rr} = t_{sd} + t_r$ )
- forward recovery time  $t_{fr} = 0$

If a rectangular wave is implied on a diode then the reverse recovery charge can be given by

$$Q_{rr} = \int_0^T i_D dt$$

And diode current in reverse biasing can be given as (from figure 2)

$$i_D = -I_{rrm} e^{-\frac{t}{t_{rr}}}$$

So reverse recovery charge

$$Q_{rr} = -t_{rr} I_{rrm}$$

(Since  $T \gg t_{rr}$ )

The average value of current per cycle can be given by

$$\bar{i} = \frac{1}{T} \int_0^T i_D dt$$

$$\bar{i} = \frac{|Q_{rr}|}{T}$$

So, reverse recovery time can be measured by

$$t_{rr} = \frac{\bar{i} T}{I_{rrm}}$$

If the peak value of voltage of rectangular ac  $V_o$  and frequency of this current is  $f$  then reverse recovery time can be given by

$$t_{rr} = \frac{\bar{i}R_d}{fV_o}$$

By knowing the peak voltage value and frequency of input current and measuring the average value of current (across the capacitor) flowing through the diode in reverse bias we can measure the reverse recovery time.

## 18.6 Self Learning Exercise-I

- Q.1 What do you mean by ON state of diode?
- Q.2 What do you mean by OFF state of diode?
- Q.3 What is depletion region in a pn junction diode?
- Q.4 What is space charge capacitance?
- Q.5 What is diffusion capacitance?

## 18.7 Procedure

### a. Reverse recovery time as function of diode peak current

1. Connect the circuit as shown in figure 1.
2. Select any value of resistance  $R_d$
3. Select the rectangular output from signal generator.
4. Select the frequency of rectangular wave from signal generator.
5. Vary the peak voltage and measure the average current from micro-ammeter.
6. Calculate the reverse recovery time for the each value of frequency.

### b. Reverse recovery time as a function of frequency

1. Connect the circuit as shown in figure 1.
2. Select any value of resistance  $R_d$
3. Select the rectangular output from signal generator.
4. Select the output peak voltage of the supply.
5. Vary the frequency and measure the average current from micro-ammeter.
6. Calculate the reverse recovery time for the each value of frequency.

## 18.8 Observations

- Peak inverse voltage of diode:
- Peak forward current of diode:
- Load Resistance ( $R_d$ ) = ..... $\Omega$

S. No.	Reverse recovery time as function of diode peak current			S. No.	Reverse recovery time as a function of frequency		
	Peak voltage input signal ( $V_o$ ) Volts	Average current Measured by ammeter ( $\bar{i}$ ) $\mu A$	Reverse recovery time ( $t_r$ ) $\mu s$		Peak voltage input signal ( $V_o$ )= .....Volts	Frequency of rectangular wave current ( $f$ ) Hz	Average current Measured by ammeter ( $\bar{i}$ ) $\mu A$
1				1			
2				2			
3				3			
4				4			
5				5			
6				6			
7				7			
8				8			



9				9			
10				10			
11				11			
12				12			
13				13			
14				14			
15				15			
16				16			
17				17			
18				18			
19				19			
20				20			

## 18.9 Graph

Model Graph:

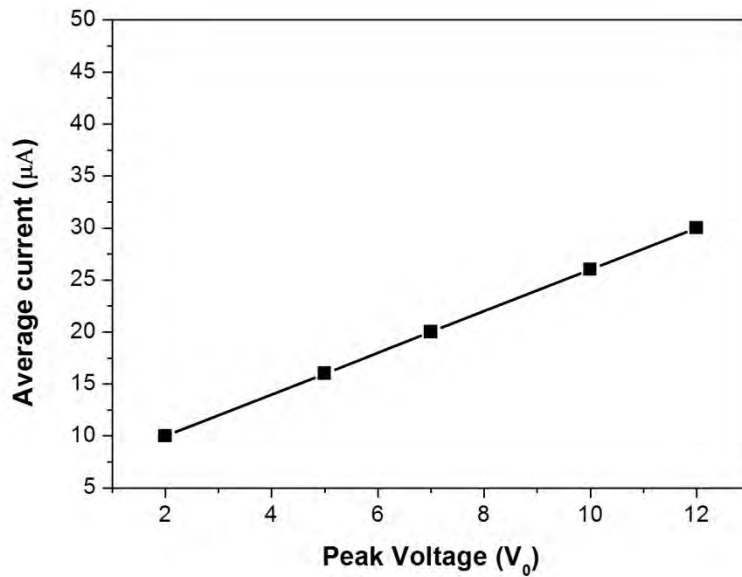


Figure 10: Plot between Average current and peak voltage

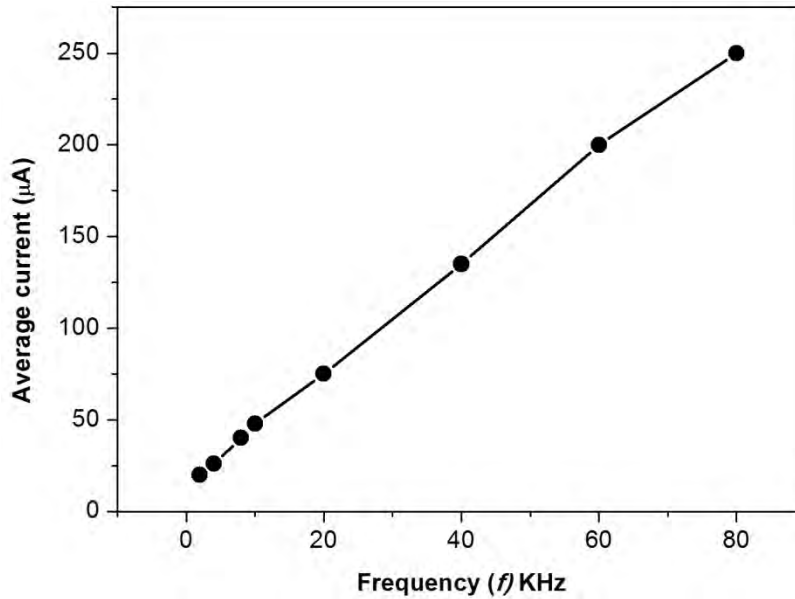


Figure 11: Plot between average current and frequency.

## 18.10 Calculations

### Model Calculations:

Noting down the values:

- Load Resistance ( $R_D$ ) = ..... $\Omega$
- Peak voltage of input rectangular shape voltage ( $V_0$ ) = ..... volt
- Frequency of input signal ( $f$ ) = ..... KHz

Measuring the value of average current by micro ammeter:

- $\bar{i} = \dots\dots\dots\mu\text{A}$

Reverse recovery time can be calculated as:

$$t_{rr} = \frac{\bar{i}R_d}{fV_0}$$

## 18.11 Result

- a. Average current as a function of peak voltage is plotted in figure 10.
- b. Average current as a function of frequency of rectangular wave is plotted in figure 11.

## 18.12 Discussion

.....

.....

.....

.....

## 18.13 Precautions and Sources of Error

8. All the connections should be tight.
9. The cords to make the joints of the T sections should be thick.
10. Reverse recovery time of fast diode should be very less (in nano seconds range) in comparison to diode under test.
11. The function generator should supply constant voltage and stable frequency.

## 18.14 Self Learning Exercise-II

- Q.1 Why current flows through the diode while even it is reverse biased?
- Q.2 Which are the parameters for a diode to be considered for an ac line?
- Q.3 What is forward recovery time?
- Q.4 What should be the value of forward recovery time for an ideal diode?
- Q.5 What is relaxation delay time for a diode?

## 18.15 Glossary

**Frequency:** Number of occurrences of a repeating event per unit time.

**Impedance:** The total opposition to alternating current by an electric circuit, equal to the square root of the sum of the squares of the resistance and reactance of the circuit and usually expressed in ohms.

**Pulse:** Brief sudden change in a normally constant quantity.

**Snappiness factor (S):** Snappiness factor is defined as the ratio of recovery time and storage delay time

$$S = \frac{t_r}{t_{sd}}$$

**Snubber circuit:** A snubber circuit, which is also known as capacitive protection circuit (as shown in figure 12). It is used to restrict the reverse recovery voltage ( $V_{rr}$ ). Snubber circuits increase the overall power loss in the circuit since the energy stored in the snubber capacitor is dissipated in the snubber resistance during turning ON of the diode.

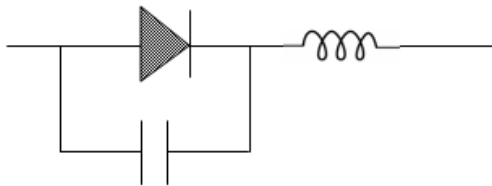


Figure 12: RC Snubber circuits.

## 18.16 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-I*

**Ans.1:** As the current flows through diode without experience of any resistance (or very less) is known as ON state of diode.

**Ans.2:** If diode do not allow to pass any current and possesses infinite resistance known as OFF state of diode.

**Ans.3:** What is depletion region in a pn junction diode?

**Ans.4:** See section 5.2

**Ans.5:** See section 5.2

### *Answers To Self Learning Exercise-II*

**Ans.1:** Leakage current due to recombination of minority charge carriers, charging of junction capacitance, redistribution of charge carriers.

**Ans.2:** Important parameters defining the turn off characteristics of Diode are, peak reverse recovery current ( $i_{rr}$ ), reverse recovery time ( $t_{rr}$ ), reverse recovery charge ( $Q_{rr}$ ) and the snappiness factor S.

**Ans.3:** When voltage across the diode has dropped to within 10% of the steady state value the junction is said to have recovered and time taken is known as

forward recovery time,  $t_{fr}$ .

**Ans.4:** Zero

**Ans.5:** As the external power is switched off the diode is relaxed if current value is within 10% of the steady state value and this time is called relaxation delay time,  $t_{rd}$ .

### 18.17 Viva Questions

- Q.1 What is role of pn junction diode in an electronic circuit?
- Q.2 Why there is capacitive effect found in a pn junction diode?
- Q.3 What is transition capacitance in a pn junction diode?
- Q.4 What is diffusion capacitance in a pn junction diode?
- Q.5 When the transition capacitance is effective?
- Q.6 When the diffusion capacitance is effective?
- Q.7 What do you understand by switching time?
- Q.8 What is forward recovery time?
- Q.9 What is relaxation delay time?
- Q.10 What is storage delay time?
- Q.11 What is reverse recovery time?
- Q.12 What is total reverse recovery time?
- Q.13 What is reverse recovery charge storage?
- Q.14 What is the reason of storage delay time?
- Q.15 What is reason of recovery time?
- Q.16 How can we reduce recovery time
- Q.17 What do you mean by fast recovery diode?
- Q.18 What is significance to study the reverse recovery time?

### 18.18 Answers to Viva Questions

**Ans.1:** As a electronic switch

**Ans.2:** Concentration gradient of mobile charge carriers across the pn junction.

**Ans.3:** For answer see section 5.2

**Ans.4:** For answer see section 5.2

**Ans.5:** In reverse bias

**Ans.6:** In forward bias

**Ans.7:** A practical diode incurred some time when diode is switched from one state to another.

**Ans.8:** When voltage across the diode has dropped to within 10% of the steady state value the junction is said to have recovered and time taken is known as forward recovery time,  $t_{fr}$ .

**Ans.9:** As the external power is switched off the diode is relaxed if current value is within 10% of the steady state value and this time is called relaxation delay time,  $t_{rd}$ .

**Ans.10:** Storage delay time is defined for an abrupt step voltage applied on diode in forward bias as well as in reverse bias. Time take in redistribution of minority charge carriers is known as storage delay time ( $t_{sd}$ ).

**Ans.11:** After achieving the distribution appropriated according to applied voltage the capacitance of pn junction started to charge and the current flows through diode to make it charge. This charging of junction capacitance occurs in exponential manner; hence the current flow through diode during this period reduces in exponentially. If current value is within 10% of the steady state value and this time is called recovery time ( $t_r$ ).

**Ans.12:** The sum of storage delay time and recovery time is called as total reverse recovery time.

**Ans.13:** The charged stored within the diode when instantaneous switched from a specified forward current  $i_F$  (normally 0.5 A) to a specified reverse current  $i_R$  (normally 1.0 A) is known as reverse recovery charge

**Ans.14:** Time taken in redistribution of mobile charge carriers as the abrupt switching voltage is applied to diode.

**Ans.15:** The junction capacitance is start to charge

**Ans.16:** Diode reverse recovery time can be reduce by increasing the rate of

decrease of the forward current (i.e, by reducing stray circuit inductance) and by using “snappy” recovery ( $S \ll 1$ ) diode

**Ans.17:** The diode which has very less recovery timings in compare to time period of applied ac voltage.

**Ans.18:** A diode as a electronic switch in ac line is selected as per proper Reverse recovery time in compare to frequency of ac.

## References And Suggested Readings

1. Power Semiconductor Devices, Version 2 EE IIT, Kharagpur
2. [http://www.utdallas.edu/~jblee/EE3110/Lab7\\_F15\\_TransientDiodes.pdf](http://www.utdallas.edu/~jblee/EE3110/Lab7_F15_TransientDiodes.pdf)
3. Understanding Diode Reverse Recovery and its Effect on Switching Losses by Peter Haaf, and Jon Harper, Fairchild Semiconductor Europe.
4. Cliff L. Ma and P.O. Lauritzen, A simple power diode model with forward and reverse recovery, IEEE transaction of power electronics, Vol. 8, No. 4, 1993.
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# UNIT-19

## Ultra Sonic Interferometer

### Structure of the Unit

- 19.1 Aim
  - 19.2 Apparatus
  - 19.3 Diagram
  - 19.4 Formula
  - 19.5 Theory and Description
  - 19.6 Self Learning Exercise-1
  - 19.7 Procedure
  - 19.8 Observation
  - 19.9 Calculations
  - 19.10 Results
  - 19.11 Discussion
  - 19.12 Precautions and Sources of Error
  - 19.13 Self Learning Exercise-2
  - 19.14 Glossary
  - 19.15 Answers to Self Learning Exercises
  - 19.16 Viva Questions
  - 19.17 Answers to Viva Questions
- References and Suggested Readings

### 19.1 Aim

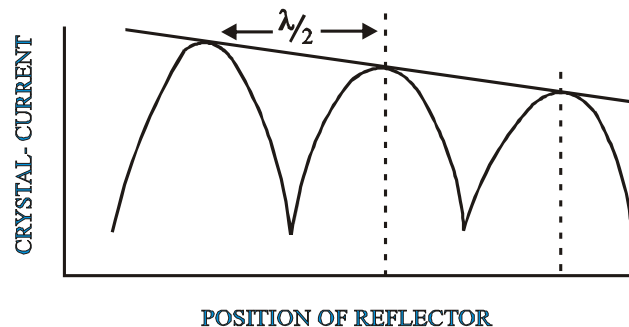
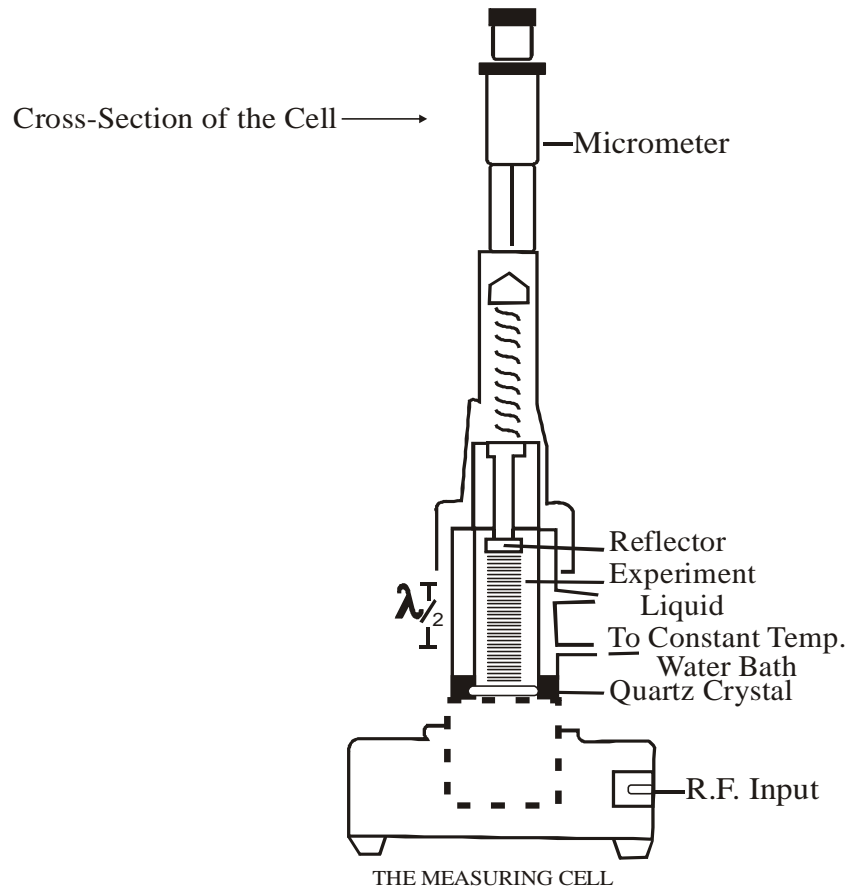
- (i) Determine the velocity of ultrasonic wave through different liquid by using ultra sonic Interferometer.
- (ii) To calculate the compressibility of the given liquid.



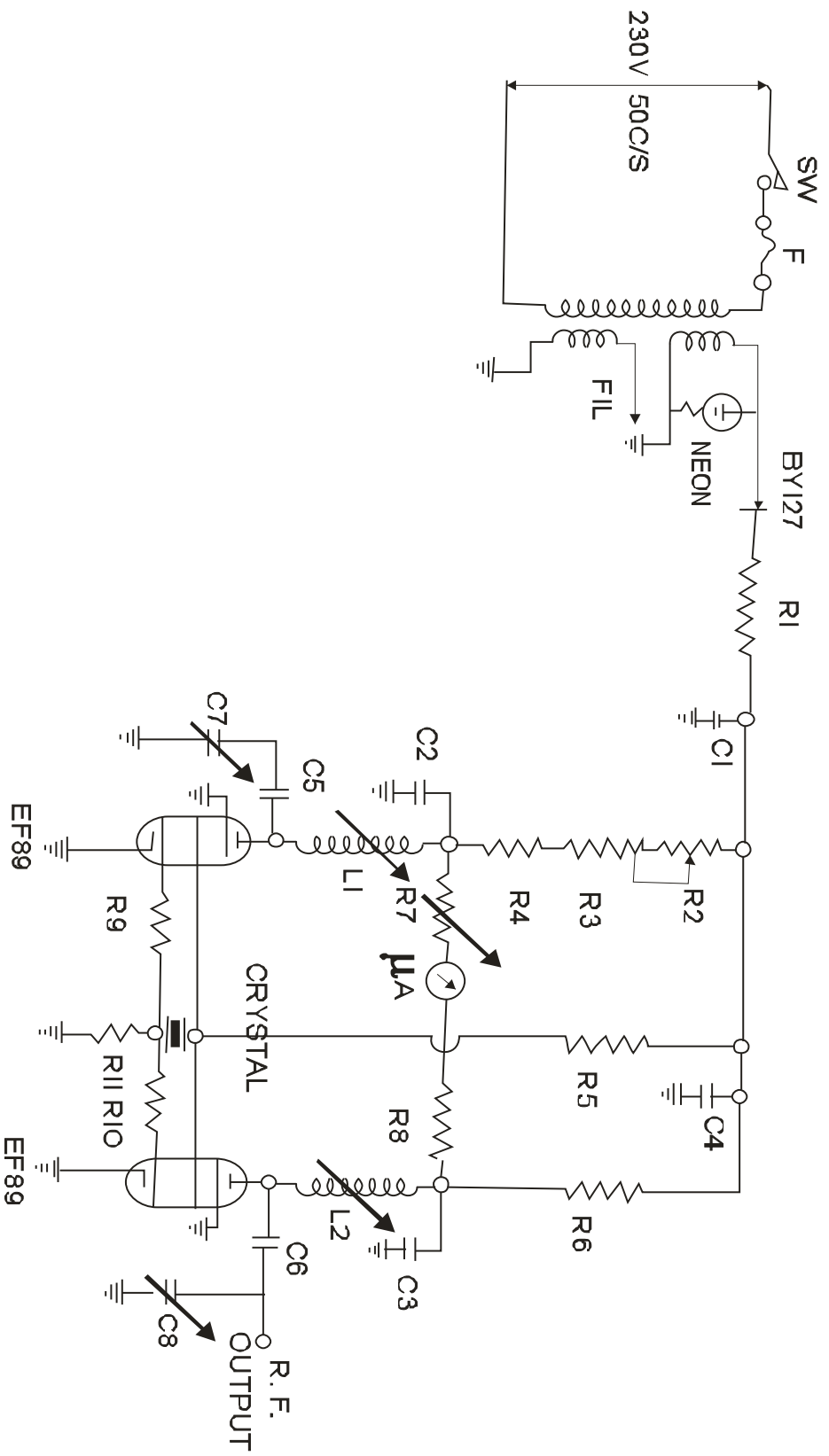
## 19.2 Appartus

Ultrasonic Interferometer, Sample liquid, high frequency etc.

## 19.3 Diagram



**Note:** Extra peaks in between minima & maxima occurs due to a number of reasons. But they don't affect the value of  $\lambda/2$  as shown in figure position of Reflector V/s Crystal Current.



## ULTRASONIC INTERFEROMETER

## 19.4 Formula

From the knowledge of wave length ( $\lambda$ ) the velocity ( $v$ ) can be obtained by the relation

Velocity = wave length  $\times$  Frequency

$$V = \lambda \times f$$

$$\text{Compressibility } \beta = \frac{1}{\rho v^2} \quad \text{where } \rho = \text{density of liquid, } v = \text{Velocity}$$

## 19.5 Theory and Description

There are three types of waves:-

- (i) **Mechanical Waves:** - They require medium to travel (like air/water, ropes etc.) The mechanical waves are of two types:-
  - (a) **Transverse Waves:** - Medium particles move perpendicular to the direction of wave propagation.
  - (b) **Longitudinal Waves:** - Medium particles move parallel to the direction of wave propagation.

When both transverse and longitudinal waves mixed in a medium then they are called surface waves.

- (ii) **Electro Magnetic Waves:** - They do not require any medium to travel. (Like light waves, radio waves etc.)
- (iii) **Matter Waves:** - They are produced by electrons and matter particles.

Ultrasonic interferometer is a simple and direct device to determine the ultrasonic velocity in liquids with a high degree of accuracy. The ultrasonic interferometer consists of the following parts:-

- (a) High Frequency Generator.
- (b) Measuring Cell.

**(A) High Frequency Generator:-** It is designed to excite the quartz crystal fixed at the bottom of the measuring cell at its resonant frequency to generate ultrasonic waves in the experimental liquid filled in the measuring cell. A micrometer to observe the change in current and to control for the purpose of

sensitivity regulation and initial adjustment of the micro meter are provided on the panel of high frequency generator.

**(b) The measuring Cell:-** It is a specially designed double walled cell for maintaining the temperature of the liquid constant during the experiment. A fine micrometer screw has been provided at the top which can lower or raise the reflector plate in the liquid in the cell through a known distance. It has a quartz crystal fixed at its bottom.

**Ultrasonic:-** Ultrasonic sounds refer to sound pressure with a frequency within the human audible range (20Hz to 20 kHz). When an ultra sonic wave propagates through a medium, the molecules in that medium vibrate over short distance in a direction parallel to the longitudinal waves. During this vibration is transferred among molecules. This cause the wave to pass through the medium wavelength.

**Generation of Ultra-Sound:-** Ultrasound can be produced by different methods. The most common method include:-

**(1) Mechanical Method:-** In this method, ultrasonic frequencies up to 100 KHz are produced. But this method is rarely used due to its limited frequency range.

**(2) Piezo-Electric Generator:-** This is the most common method used for the production of ultra sound. When mechanical pressure is applied to the opposite faces of certain crystals which are cut suitably, electric fields are produced. When subjected to electric field, these crystals contract or expand depending on the direction of field. Thus a property oriented rapid crystal to vibrate mechanically. This vibration is largest when the crystal is at resonance and is used to produced a longitudinal wave i.e. a sound wave.

**(3) Magnetostriction Generator:-** In this method the magnetostriction method is used for the production of ultrasonic frequency ranging from 8000 Hz to 20,000 Hz.

**Ultrasonic Interferometer:-** In an ultrasonic interferometer the ultrasonic waves are produced by piezoelectric method. In a fixed frequency variable path interferometer the wavelength of the sound in an experimental liquid medium is measured and from this one can calculate its velocity through that medium. The apparatus consists of an ultrasonic cell which is a double-walled brass cell with chromium plated surfaces having a capacity of 10ml. The double wall allows water

circulation around the experimental medium to maintain it at a known constant temperature.

The micrometer scale is marked in units of 0.01 mm and has an overall length of 25mm ultrasonic waves of known frequency are produced by a quartz crystal which is fixed at the bottom of cell. There is a movable metallic plate parallel to the quartz plate, which reflects the waves. The waves interfere with their reflections and if the separation between the plates is exactly an integer multiple of half wavelength of sound "Standing Waves" are produced in the liquid medium under these circumstances "Acoustic resonance" occurs. The resonant waves are a maximum in ampere causing a corresponding maximum in the anode current of the piezoelectric generator.

If we increase or decrease the distance by exactly one half of the wavelength ( $\lambda/2$ ) or an integer multiple of one-half wave length, the anode current again becomes maximum. If  $d$  is the separation between successive adjacent maxima of anode current. then

$$d = \frac{\lambda}{2} \quad (1)$$

We have the velocity ( $v$ ) of a wave is related to its wavelength  $\lambda$  by the relation

$$v = \lambda f \quad (2)$$

Where  $f$  is the frequency of the ultrasonic wave then from equation 1 and 2 we can write

$$v = 2 d f \quad (3)$$

The velocity of ultrasound is determined principally by the compressibility of medium. For a medium with high compressibility the velocity will be less. Adiabatic compressibility of a fluid is a measure of the fluid as a response to pressure change .Compressibility is the reciprocal of bulk modulus and is usually denoted by the  $\beta$ . The adiabatic compressibility of the material of the sample can be calculated using the equation.

$$\beta = \frac{1}{\rho v^2} \quad (4)$$

Where  $\rho$  is the density of material of the medium and  $v$  is the velocity of the ultrasonic wave in the medium.

**Wave Interference:** - When two or more waves from different sources are present at the same time they interact with each other to produce a new wave. The new wave is the sum of all the different waves. The interaction of all these waves produce interference. If intensity of resultant waves increase after the interaction of different waves the resultant wave formed represent constructive interference.

If intensity and amplitude of resultant wave is lower than the interfering waves it called destructive interference.

When path difference  $\Delta = n\lambda$

$$I_{\max} = (a_1 + a_2)^2 = \text{Constructive Interference}$$

$$\text{When path difference } \Delta = \left(n + \frac{1}{2}\right)\lambda$$

$$I_{\min} = (a_1 - a_2)^2 = \text{Destructive Interference.}$$

### 19.6 Self Learning Exercise-1

- Q.1 What are ultrasonic's Waves?
- Q.2 What is Piezoelectric effect?
- Q.3 What is inverse Piezoelectric effect?
- Q.4 How are ultrasonic are produced?
- Q.5 What are the properties of ultrasonic waves?

### 19.7 Procedure

1. Insert the quartz crystal in the socket.
2. Clamp to it with the help of a screw provided one of its side.
3. Unscrew the knurled cap of cell and lift it away from double walled construction of cell. In the middle position of it pour experimental liquid and screw the knurled cap.
4. Two chutes in double wall construction are provided for water circulation to maintain desired temperature.

5. Connect the high frequency generator with cell by co-axial cable provided with the instrument.
6. For initial adjustment two knobs are provided on high frequency generator, one is marked with 'Adj' the position of the needle on the ammeter is adjusted and the knob marked 'Gain' is used to increase the sensitivity of the instrument for greater deflection if desired.
7. The meter is used to notice the number of maximum deflections while micrometer is moved up and down in liquid as described in the method.
8. Continue to increase the micrometer setting, noting the reading at each maximum count any no. of maxima and call it as n. subtract the reading at the first maxima from the reading at the last maximum.

### 19.8 Observation

- (i) Density of Water = ..... Kg/m<sup>3</sup>
- (ii) Density of nitro benzene = ..... Kg/m<sup>3</sup>
- (iii) Density of Acetone = ..... Kg/m<sup>3</sup>.
- (iv) Least count of main scale = ..... cm.
- (v) Least count of vernier scale = ..... cm.
- (vi) Frequency of ultrasonic wave = ..... MHz

#### Observation table for water:-

S.No.	Anode Current	Micro Meter Reading d (cm)	d'1(cm)	$d = \frac{\epsilon d'_1}{n}$	Wavelength $\lambda = 2d$


**Observation table for Nitrobenzene:-**

S.No.	Anode Current	Micro Meter Reading d (cm)	d'1(cm)	$d = \frac{\epsilon d'_1}{n}$	Wavelength $\lambda = 2d$




**Observation table for Acetone :-**

<b>S.No.</b>	<b>Anode Current</b>	<b>Micro Meter Reading d (cm)</b>	<b>d'1(cm)</b>	$d = \frac{\epsilon d'_1}{n}$	<b>Wavelength</b> $\lambda=2d$

## 19.9 Calculations

**(i) For Water**

(a) Velocity  $v = \lambda f$   
 $v = \dots\dots\dots$  m/sec.

(b) Compressibility  $\beta = \frac{1}{\rho v^2}$   
 $\beta = \dots\dots\dots$  m kg<sup>-1</sup> Sec<sup>2</sup>

**(ii) For Nitrobenzene**

(a) Velocity  $v = \lambda f$   
 $v = \dots\dots\dots$  m/sec.

(b) Compressibility  $\beta = \frac{1}{\rho v^2}$   
 $\beta = \dots\dots\dots$  m kg<sup>-1</sup> Sec<sup>2</sup>

**(iii) For Acetone**

(a) Velocity  $v = \lambda f$   
 $v = \dots\dots\dots$  m/sec.

(b) Compressibility  $\beta = \frac{1}{\rho v^2}$   
 $\beta = \dots\dots\dots$  m kg<sup>-1</sup> Sec<sup>2</sup>

## 19.10 Result

The velocity and compressibility of given liquids at known frequency are as follows:-

S.No.	Liquid	Velocity(m/sec)	Compressibility(mkg <sup>-1</sup> Sec <sup>2</sup> )
1.	Water		

2.	Nitrobenzene		
3.	Acetone		

### 19.11 Discussion

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### 19.12 Precautions and Sources of Error

1. Do not switch on the generator without filling the experimental liquid in the cell.
2. Remove experimental liquid out of cell after use. Keep it dried and cleaned.
3. Avoid sudden rise or fall in temperature of circulated liquid to prevent thermal shock to the quartz crystal.
4. While cleaning the cell care should be taken not to spoil or scratch the gold plating on the quartz crystal.
5. Give 15 seconds to generator to warm up before observation.
6. Keep micrometer open at 25 mm after use.

### 19.13 Self Learning Exercise-2

- Q.1** Why ultrasonic waves inaudible to humans?
- Q.2** Give important applications of ultrasound.
- Q.3** What is the principle used for finding ultrasonic velocity using acoustic grille?
- Q.4** What is acoustic grille?

## 19.14 Glossary

**Acoustic** - having to do with the energy of sound waves.

**Crystal** - The piezo electric element (s) in a stack which expands and contracts in an alternating (charged) electrical field, thus inducing vibration.

**Generator** - that device which powers and controls the convertor of an electronically driven ultrasonic device or system. Mechanical generators also have been made.

**Magnetostrictive**- having to do with systems driven by the effect of certain metals/ especially nickel, and certain other materials such as ytterbium compounds, which expand and contract in an alternating magnetic field.

**Piezoelectric** - having to do with systems driven by the effect of certain crystals such as lead- zirconate titanate, and other materials which expand and contract in an alternating electrical field.

**Sonar** - an underwater acoustic means of determining distance.

**Stack** - the most basic combination of transducer and amplifying elements together forming a resonant body to be attached to (or including) a radiating surface or horn.

**Ultrasonic** - having to do with frequencies of sound above normal human hearing, generally accepted to be at 20 KHz to 2 MHz and above, but also extended down to the 5 KHz to 20KHz range in certain processing applications.

## 19.15 Answer To Self Learning Exercise

### ***Answer To Self Learning Exercise-1***

**Ans.1:** Ultrasonic waves are sound waves, whose frequencies are greater than 20 KHz. Their wavelength is less than 1.65 cm. They exhibit all the properties of sound waves and some additional properties.

**Ans.2:** When mechanical pressure is applied to one pair of opposite faces of a quartz crystal, then the other pair of opposite faces develop equal and opposite electrical charges on the crystal.

**Ans.3:** The piezoelectric effect is reversible. If an electric field is applied to one pair of opposite faces of quartz crystal, alternative mechanical expansion or contraction (Pressure) is produced across the other pair of opposite faces of the crystal.

**Ans.4:** Ultrasonic are produced by

- (i) Galton's whistle
- (ii) Magnetostriction oscillator
- (iii) Piezoelectric generator

Galton's whistle is primarily a mechanical device based on the principle of resonance of air column. It can produce ultrasonic waves about 30 KHz.

A magnetostriction oscillator is based on the property of magnetostriction exhibited by ferromagnetic materials. It can produce ultrasonic waves of 25 KHz.

Piezoelectric is also an electrical device, which makes use of the piezoelectric property of quartz. It generates ultrasonic waves of high frequencies.

**Ans.5:** (i) Ultrasonic waves are longitudinal wave of high frequency (greater than 20 KHz). Owing to their short wave length they can travel long distances as a sharp beam.

(ii) They undergo reflection and refraction.

(iii) They produce heating effect, mechanical effect and chemical effect.

(iv) They produce biological effect. Animals such as frogs and rats are killed when subjected to ultrasonic waves.

### ***Answer To Self Learning Exercise -2***

**Ans.1:** Ultrasonic waves inaudible to humans, as their wave frequency is very high and so they can't vibrate tympanic inner membrane in the human ear, which is necessary for the hearing.

**Ans.2:** (a) Detection of defects in metals.

(b) Sonar for submarine detection, iceberg and other objects in the ocean.

(c) Brazing and cutting metals.

(d) Diagnostic applications such as detection of tumors and defects in the human body.

**Ans.3:** When ultrasound is passed through a liquid as kerosene in the tank, due to changes in the pressure of the fluid acts as a speaker grille. Now, when monochromatic light source passes through a speaker grille, it produces a variety of orders of diffraction spectrum.

**Ans.4:** When ultra sonic waves passing through the liquid the liquid compression and depression is formed. The compression as opaque and dilution function as openings for light waves. This mechanism is called speaker grille.

### 19.16 Viva Questions

- Q.1 What are mechanical waves?
- Q.2 Define Transverse waves.
- Q.3 What do you mean by electromagnetic waves?
- Q.4 Define longitudinal waves.
- Q.5 Define matter waves.
- Q.6 What is the frequency range of ultrasonic waves?
- Q.7 What is the formula for compressibility of liquid?
- Q.8 What are the reference properties of the ultrasound?
- Q.9 What is constructive interference?
- Q.10 Write the formula of constructive interference?
- Q.11 What is destructive interference?
- Q.12 Write the formula of destructive interference?
- Q.13 What is the path difference of constructive interference?
- Q.14 What is the path difference of destructive interference?
- Q.15 What is the formula velocity of a wave is related to its wave length?
- Q.16 What is Piezoelectric generator?
- Q.17 Define Magnetostriction oscillator?
- Q.18 What is Galton's whistle?

**Q.19** Piezoelectric effect is a reversible or irreversible.

**Q.20** What are the examples of electromagnetic waves?

### 19.17 Answers To Viva Questions

**Ans.1:** They required medium to travel.

**Ans.2:** Medium particle move perpendicular to the direction of wave propagation.

**Ans.3:** They do not require any medium to travel.

**Ans.4:** Medium particle move parallel to the direction of wave propagation.

**Ans.5:** They are produced by electrons and matter particles.

**Ans.6:** 20Hz to 20 KHz

**Ans.7:**  $\beta = \frac{1}{\rho v^2}$

**Ans.8:** (i) They are very energetic

(ii) They go through great distances.

(iii) They are reflected, refracted and absorbed similar to regular sound waves.

(iv) When ultrasound is passed through a liquid, it produces a stationary wave pattern and makes the liquid behave as auditory lattice.

(v) When an object is exposed to ultrasound for longer periods of time, giving the effect of heating.

**Ans.9:** If intensity of resultant waves increases after the interaction of different waves the resultant wave formed represent constructive interference.

**Ans.10:**  $I_{\max} = (a_1 + a_2)^2$

**Ans.11:** If intensity and amplitude of resultant wave is lower than the interfering waves it called destructive interference.

**Ans.12:**  $I_{\min} = (a_1 - a_2)^2$

**Ans.13:**  $\Delta = n \lambda$

**Ans.14:**  $\Delta = (n + \frac{1}{2}) \lambda$

**Ans.15:**  $v = \lambda f$



**Ans.16:** Piezoelectric oscillator is also an electric device, which makes use of the piezoelectric property of quartz. It generates ultrasonic waves of high frequencies.

**Ans.17:** A magnetostriction oscillator is based on the property of magnetostriction exhibited by ferromagnetic material. It can produce ultrasonic waves of 25 KHz.

**Ans.18:** Galton's whistle is primarily a mechanical device based on the principle of resonance of air column. It can produce ultrasonic waves about 30 KHz.

**Ans.19:** Piezo electric effect is reversible.

**Ans.20:** Light waves, radio waves etc.

### References and Suggested Readings

1. Sadhu, ultrasonic velocity studies in liquids and their correlation, Gyan publishing house, 1987.
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# UNIT-20

## Band Gap using Four Probe Method

### Structure of the Unit

- 20.1 Aim
  - 20.2 Apparatus
  - 20.3 Diagram
  - 20.4 Formula
  - 20.5 Model Graph
  - 20.6 Theory and Description
  - 20.7 Self Learning Exercise-I
  - 20.8 Procedure
  - 20.9 Observation
  - 20.10 Graph
  - 20.11 Calculations
  - 20.12 Result
  - 20.13 Discussion
  - 20.14 Precautions and Sources of Error
  - 20.15 Self Learning Exercise-II
  - 20.16 Glossary
  - 20.17 Answers to Self Learning Exercises
  - 20.18 Viva Questions
  - 20.19 Answers to Viva Questions
- References and Suggested Readings

### 20.1 Aim

- (i) To study temperature variation of resistivity of a semi conductor material.
- (ii) To find the band gap of the semiconductors by using four probe method.

## 20.2 Apparatus

A four probe arrangement in which connecting pins are available for connecting wires, a constant power supply, milliammeter, a high resistance voltmeter, oven, and thermometer.

## 20.3 Diagram

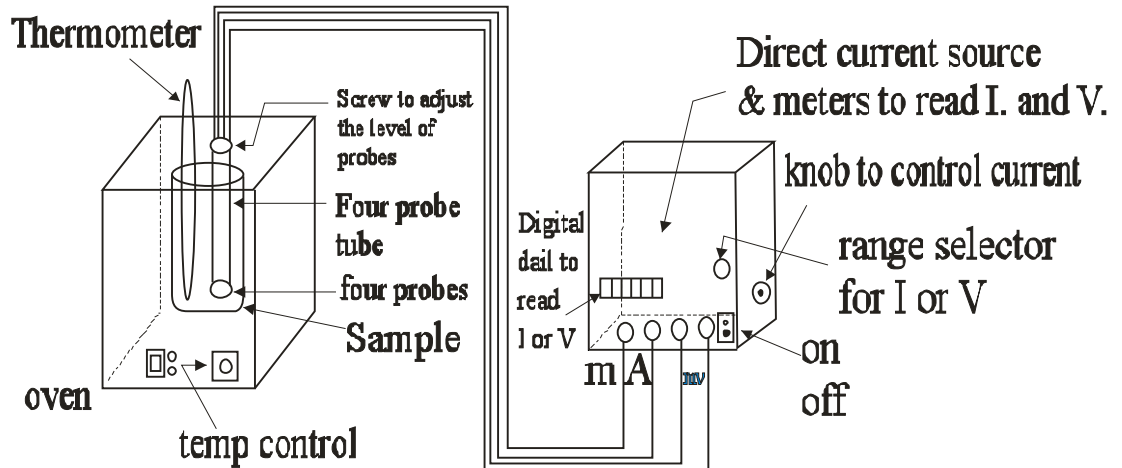


Fig. 20.1 Experimental arrangement for resistivity measurement

## 20.4 Formula

From the measured value of  $V$  and  $I$  resistivity  $\rho$  can be calculated using the following relation

$$\rho = \frac{V}{I} \frac{2\pi s}{G_7(w/s)}$$

where  $G_7(w/s) = 5.89$  (Note that Form of Formula depends on the type of apparatus that made by manufacturer)

$S$  = Distance between the consecutive probes

$W$  = Thickness of crystals.

$$\rho = \frac{V}{I} \times \frac{2\pi s}{5.89} = \frac{V}{I} \times \frac{1.256}{5.89}$$

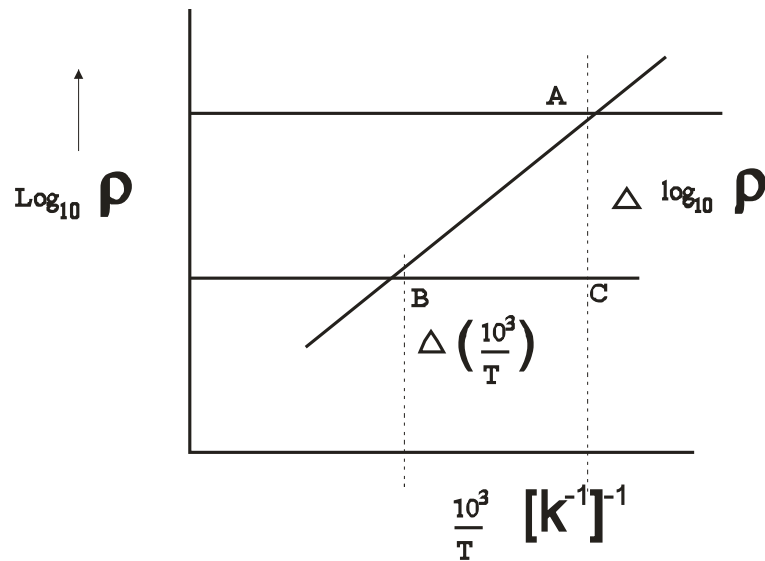
$$\rho = \frac{V}{I} \times 0.213 \quad \text{Thus } \rho \text{ may be calculated for various temperatures.}$$

Forbidden gap of semi conductor

$$E_g = \frac{2k \times 2.303}{10^{-3}} \times [\text{Slope of straight line between } \log_{10}\rho \text{ and } 10^3/T]$$

where  $K = 8.617 \times 10^{-5} \text{ eVK}^{-1}$

## 20.5 Model Graph



$$\text{Slope} = \frac{AC}{BC} = \frac{\Delta \log_{10} \rho}{\Delta (10^3/T)}$$

A graph between  $\log_{10}\rho$  and a straight line is obtained.

## 20.6 Theory and Description

**Description of four probe method study unit:** Normally this apparatus has two boxes, one in which current source, voltmeter and ammeter are arranged and keeping the value constant of necessary current by using different knobs, different range of voltmeter and ammeter reading scale can be selected or current and voltage can be measured. Other one is an oven whose temperature can be changed using a knob. This temperature can be measured with the help of thermometer. In this oven a sample (standard or experimental crystal) can be put. In the cap of this oven a four probe tube is arranged. By using screws the connecting pins can be shifted slightly up and down. One or more semiconducting crystal sample can be put in the oven and can be remove at the end of the experiment. It should be kept in

mind that these crystals are very delicate so these can break if we tight the screw. Contact of these with connecting pins should be done very carefully. It is very important that surface of crystal should be completely pure and clear.

**Theory and Formula For The Measurement of Resistivity :**

According to Ohm's law if physical conditions (temperature, pressure) are constant then the potential difference (V) at the ends of any material is directly proportional to flow of current (I) between them

i.e.  $V \propto I$   
 or  $V = IR$  (1)

Where resistance R is dependent on the length of material, area of cross section and nature of material in the following way—

$$R = \frac{\rho l}{A} \tag{2}$$

Here  $\rho$  is the resistivity of the material,  $l$  is length and  $A$  is area of cross section

so,  $V = \frac{\rho l I}{A}$  or  $\rho = \frac{VA}{lI}$  (3)

$\frac{I}{A} = J$  is also known as current density. Substituting this in equation (3)

$$\rho = \frac{V}{lJ} \tag{4}$$

Conductivity  $\sigma = \frac{1}{\rho} = \frac{lI}{V}$  (5)

Resistivity depends upon the characteristic properties and temperature of the material for example:

**(i) For metals and conductors:**

$$\rho = \frac{1}{\sigma} = \frac{m}{Ne^2\tau} \tag{6}$$

Where  $m$  is the mass of electron,  $e$  is charge of electron,  $N$  is the number density of free electron and  $\tau$  is relaxation time of electron i.e. the average time between two collisions. Value of relaxation time ( $\tau$ ) decreases by increasing temperature so that

resistivity of conducting materials increases with the increases with the increase in temperature.

**(ii) For semiconductors:** In semiconductors both electrons and holes contributes in flow of current. In intrinsic semiconductor the value of current density is given by the following formula:

$$J = n_i e v_n + p_i e v_p \quad (7)$$

Here  $n_i$  and  $p_i$  are electron and hole densities in intrinsic semiconductor respectively,  $e$  is electron charge and  $v_n$  &  $v_p$  are drift velocities of electrons and holes respectively. Value of drift velocity is directly proportional to intensity  $E$  and it is different for different carriers i.e.  $v_n = E \mu_n$  and  $v_p = E \mu_p$  where  $\mu$  is the mobility of charge carries. So equation (7) can be written in the following from:

$$J = (n_i e \mu_n + p_i e \mu_p) \frac{V}{l} \left( \because E = \frac{V}{l} \right) \quad (8)$$

On comparing with formula (4), it is clear that resistivity of semiconductor can be written in the following form -

$$\rho = \frac{1}{\sigma} = \frac{A}{l} \frac{V}{I} = \frac{1}{n_i e \mu_n + p_i e \mu_p}$$

Since for intrinsic semiconductors

$$n_i = p_i = n$$

$$\therefore \rho = \frac{1}{ne(\mu_n + \mu_p)} \quad (9)$$

For intrinsic semiconductor electron and hole densities depends on temperature and energy of forbidden energy band as follows -

$$n = AT^{3/2} e^{-(E_g/kT)} \quad (10)$$

Here  $A$  is a constant whose value depends upon the nature of material,  $T$  is temperature (in Kelvin)  $k$  is the Boltzmann constant and  $E_g$  is energy of forbidden energy band. It is clear from eq. (9) and (10) that on increasing temperature no. of carriers increases and resistivity decreases.

From above description it is clear for the evaluation of the value of resistivity

either there should be a detail microscopic knowledge of the several properties carrier density, carrier conductivity, relaxation time etc. For any actual material, it is very difficult to write down the theoretical formula for these quantities because their values depend very much upon presence of impurity and defects of crystals. Or there can be an another method in which according to formula (3) one can find  $I$ ,  $A$ ,  $V$ ,  $I$  and then, can calculate  $\rho$ . This is traditional experimental method. At different temperatures taking observations of  $V$  and  $I$ , study can be done for the dependence of resistivity  $\rho$  on temperature. For semiconductors by the same study, calculation of forbidden energy band can be done from the eq. (9) and (10).

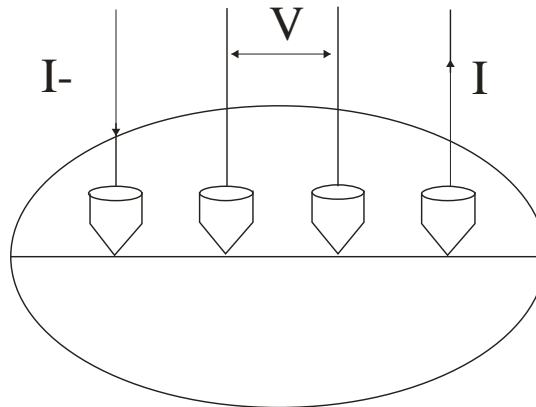
**Four Probe Method** - Commonly used method for the measurement of resistivity (i.e., finding value of resistivity of a conducting material in the form of wire by making electrical circuit with the help of voltmeter and ammeter and using ohm's law) is not satisfactory for the evaluation of resistivity of semiconductors because of following reasons:

1. Normally for the measurement of resistivity connecting wires are connected with material. Generally conductor and semiconductor contacts are rectifying so that at connecting terminals sufficient contact potentials are produced. So there may be a change in actual value of  $V$  and the value of  $\rho$  will not be correct.
2. This effect will be more in extrinsic semiconductor because normally from the contact point in any current carrying wire minority charge carriers inject from conductor to semiconductor because of it conductivity of material will change.

These are the following advantages of four probe method in comparison to two probe method -

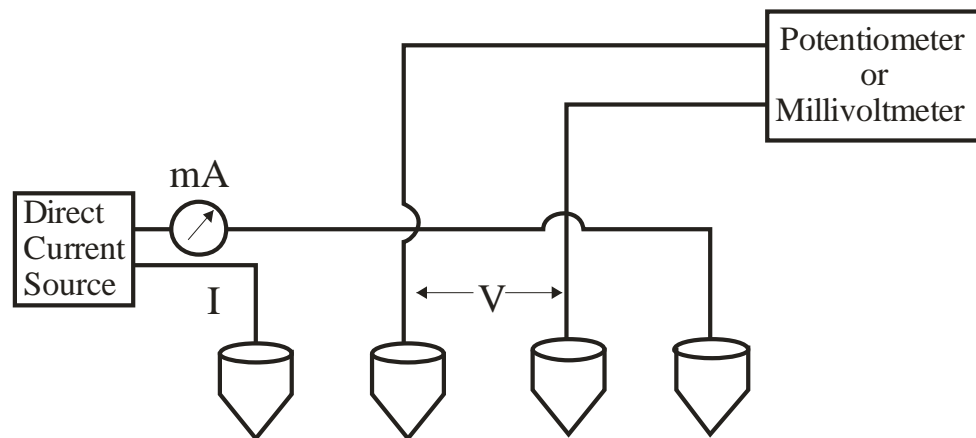
- (i) In the measurement of  $I$  and  $V$  the contact potential error will be negligible since in four probe method the contact area between the probe and the material is negligible.
- (ii) In most of the conductors the carrier density remains same in complete sample whereas in semiconductors especially in extrinsic semiconductors there may be different conductivity and carrier density at different regions. Actually use of this difference is done for the construction of different semiconductor devices. So there is a need of method in which the conductivity of material can be calculated at any part. Four probe methods is such a method.

(iii) From this method resistivity of different shape sample material can be calculated. Along with this in any given sample the resistivity can be calculated at different parts i.e. to calculate actual resistivity from both sides of any p-n junction, the semi conductor can cut into necessary small parts each one of them is a device. In this way the value of resistivity of many device made on single chip can be calculated.



**Fig. 20.2 Four Probe Method for the measurement of resistivity**

(iv) Since in this method current is made to flow in the sample by two fine probes and voltage is measured by two probes which are situated inside and all four pins are quite close to each other therefore, from this method resistivity is measured very accurately. Theoretically all the conditions of equation (1), See Fig. 20.3 are full filled.



**Fig. 20.3 Four Probe Measurement Apparatus**

(v) Form this method study of the effect of temperature on resistivity can be done and forbidden energy gap or band gap can also be found.



### Formula for Experiment:

From the measured values of V and I resistivity  $\rho$  can be calculated using the following relation:

$$\rho = \frac{V}{I} \frac{2\pi s}{G_7(w/s)} \quad (11)$$

Theoretical Formula for determination of band gap and study of dependence of resistivity on temperature.

From equations (9) and (10)

$$\rho = \frac{1}{AT^{3/2} e^{-Eg/kT} e^{(\mu_n + \mu_p)}}$$

Taking log of both sides

$$\log_e \rho = \log \left( \frac{1}{A.e.(\mu_n + \mu_p)} \right) - \frac{3}{2} \log_e(T) + \frac{E_g}{2kT}$$

If it is assumed that there is no special change in carrier conductivity with temperature and change in power as compare to exponential change is negligible then in the above formula first and second term can be taken as constant. So

$$\log_e \rho = \frac{E_g}{2kT} - \log_e K \quad (12)$$

Where K is constant

## 20.7 Self Learning Exercise-I

- Q.1 Give definition of resistivity?
- Q.2 What is the principle of measurement of resistivity?
- Q.3 Give the effect of temperature on resistivity of conductor, semiconductor and insulator?
- Q.4 Why normal methods of resistance measurement are not proper for semi conductors?
- Q.5 What are the advantage of four probe method in comparison to other methods?

## 20.8 Procedure

1. Carefully study the manual and observe the given study apparatus. Decide on the basis of points explained above in the principle how to evaluate the value of P. Note down the value of S and W.
2. Make the circuit as shown in Fig. 20.3 Outer probes are connected by direct current source and inner probes are connected with voltmeter. In apparatus these terminals are denoted by I and V and by using knobs on a single dial I and V can be read alternately.
3. Put the sample in the oven carefully and with help of screw connect four probe connecting pins with the sample (see fig. 20.1)
4. A thermometer, for the reading of temperature, which must be kept at a constant value, is fixed properly inside the oven through the cap of oven.
5. Switch on the direct current source and put reading knob at I and flow proper value of current (like 2 mA) through the sample. Note down the value of I.
6. Now put reading knob at V and note voltage reading.
7. Switch on the oven and note voltage and temperature at different temperatures.
8. Note the observations in observation table-1.
9. Draw a curve between  $\log_{10}\rho$  and  $10^3/T$ .
10. Find forbidden gap of semiconductor by formula (12) from expression (12)

$$E_g = \frac{2 \times k \times 2.303}{10^{-3}} \times (\text{Slope of straight line between } \log_{10}\rho \text{ and } 10^3/T.)$$

## 20.9 Observations

1. Given sample is .....  
(which type of sample intrinsic silicon/intrinsic Ge/ extrinsic semiconductor)
2. S = .....  
W = .....

### Observation Table 1

Value of Constant Current = .....mA

<b>S.No.</b>	<b>Temperature (TK)</b>	<b>Voltage V (mV)</b>	<b>103/T</b>	<b><math>\rho</math>(Formula 11)</b>	<b>log<sub>10</sub> <math>\rho</math></b>

**Observation Table 2**

Value of Constant Current = .....mA

S.No.	Temperature (TK)	Voltage V (mV)	103/T	$\rho$ (Formula 11)	log <sub>10</sub> $\rho$

**Observation Table 3**

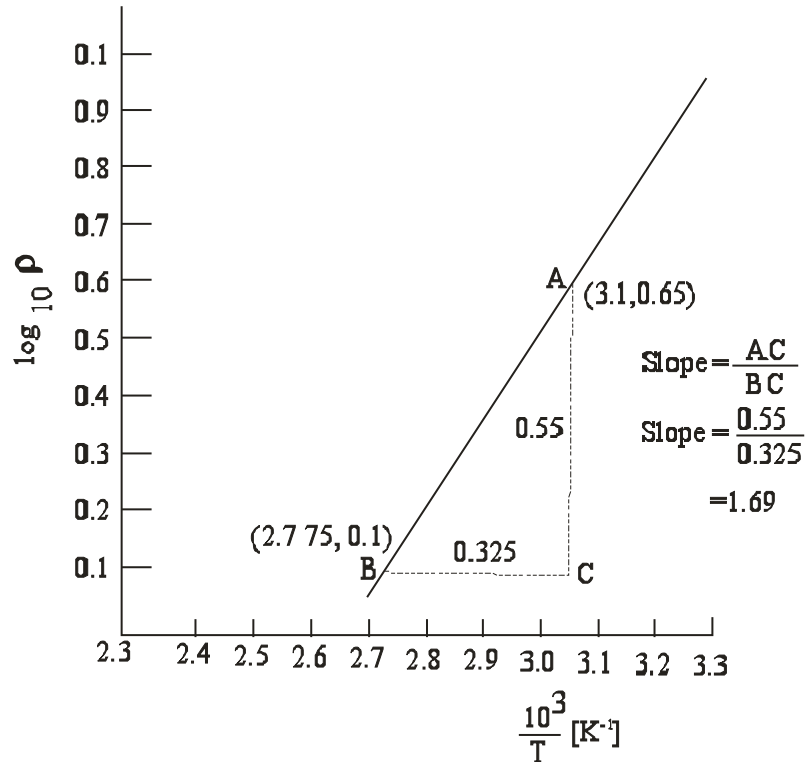
Value of Constant Current = .....mA

S.No.	Temperature (TK)	Voltage V (mV)	103/T	$\rho$ (Formula 11)	$\log_{10} \rho$

## 20.10 Graph

Graph between  $\log_{10}\rho$  and  $10^3/T$

$\log_{10}\rho$							
$10^3/T$ [K <sup>-1</sup> ]							



## 20.11 Calculations

For a given arrangement (w/s) = .....

(c) = .....

- Put the appropriate value of C in equation (11) and find the value of  $\rho$  and calculate  $\log_{10}\rho$  at different temperatures.
- Calculate the value of  $10^3/T$  at each temperature.
- Draw the graph between  $\log_{10}\rho$  and  $10^3/T$ .

(iv) From Formula (12)

$$E_g = \left( \frac{2 \times k \times 2.303}{10^{-3}} \right) \times \text{slope of straight line between } \log_{10} \rho \text{ and } 10^3/T$$

## 20.12 Results

1. Resistivity of the given semi conductor changes with temperature according to graph. Resistivity of semi conductors decreases with increase in temperature.
2. Energy band gap  $E_g = \dots\dots\dots$  eV of give semiconductor.  
Standard value for Germanium Crystal  $E_g = 0.69$  eV

## 20.13 Discussion

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## 20.14 Precautions and Sources of Error

1. Current through the sample should not be large.
2. Surface should be tapped to minimize effect.
3. Probes should always touch the semi conductor.
4. Probe should not be pressed tightly on the material otherwise it will damage.
5. Four probe method is a collinear.

## 20.15 Self Learning Exercise-II

- Q.1 Can we calculate forbidden energy band of semiconductor from this method?
- Q.2 Explain conductor, semiconductor and insulator on the basis of band energy?
- Q.3 Which one : germanium or silicon, has a lower forbidden energy gap?
- Q.4 Where does Fermi level lie in an intrinsic semiconductor?

Q.5 Which one : Ge or Si is more useful semiconductor material?

## 20.16 Glossary

**Band gap, energy gap ( $E_g$ ) :** Forbidden energy levels separating the valence and conduction bands. No electrons are allowed to have energies at these levels.

**Conduction band :** The upper energy band in a semiconductor separated from the valence band by the energy gap. The conduction band is not completely filled with electrons hence; electrons are free to "Conduct".

**Crystal :** Solid featuring periodic spatial arrangement atoms throughout the entire piece of materials.

**Probe G :** An electrode or other device that can be placed inside something to take and convey measurements.

**Resistivity:** A measure of the resisting power of a specified material to the flow of an electric current.

**Valence band:** the lower energy band in a semiconductor that is completely filled with electrons at ok, electrons cannot conduct in valence band.

## 20.17 Answer to Self Learning Exercises

### *Answer to Self Learning Exercise-I*

**Ans.1:** Resistance of a slab of material having unit length and unit area of cross section is numerically equal to its resistivity.

**Ans.2:** If due to flow of current  $I$  in any small part or any slab of material the potential difference between its ends is  $v$  then resistivity is directly proportional to the ratio of  $V$  and  $I$ . It can denoted by  $\rho = \frac{V}{I} p$  where  $p$  depends upon the geometry of material.

**Ans.3:** Resistivity of conductor increases with temperature, resistivity of semiconductor decreases with temperature and no effect is observed in the resistivity of insulator with temperature.

**Ans.4:** Because conductor- semiconductor contact is rectifying and at the place of contact sufficient contact potential is developed which leads to an error in the measurement of  $\rho$ .



**Ans.5:** In this actually the potential difference is measured between two points in which current is flowing. In this method connection of probes is over a very small region of material. From this method resistivity can be measured between the different parts of materials. So this method is used to measure the resistivity of those materials and devices in which the resistivity is different at different parts.

### ***Answer to Self Learning Exercise-II***

**Ans.1:** Yes, this is described in the method.

**Ans.2:** The materials in which there is overlapping between conduction and valence bands are known as conductor. The material in which there is a small energy gap between conduction band and valence band are known as semi conductor and materials in which the energy gap is very large are known as insulator.

**Ans.3:** Germanium has lower forbidden energy gap of 0.7eV as compared to silicon which has forbidden energy 1.1eV.

**Ans.4:** In an intrinsic semiconductor, Fermi level lies in the middle of the energy gap.

**Ans.5:** Si is more useful because thermal agitation produces fewer electrons and holes in Si than in Ge.

### **20.18 Viva Questions**

- Q.1** What is the resistance of a pure semiconductor at ok?
- Q.2** What is the net charge on a sample of semiconductor?
- Q.3** What is a semi conductor?
- Q.4** At room temperature, if a voltage is applied across a pure semi conductor, what is the direction of motion of electrons and holes?
- Q.5** What type of conductor is pure germanium?
- Q.6** What is valence band?
- Q.7** What is conduction band?
- Q.8** On band theory, how will you define conductor and Insulator?

- Q.9** What is the amount of impurity which is added in a pure semiconductor?
- Q.10** What are holes?
- Q.11** How resistivity of a semi conductor vary with temperature.
- Q.12** Give three examples of semi conductors?
- Q.13** What is the forbidden energy band gap of germanium semiconductor?
- Q.14** What do you mean by the band gap energy?
- Q.15** What is the lattice of the germanium semiconductor?
- Q.16** What are intrinsic semiconductors?
- Q.17** What is the forbidden energy band gap of silicon?
- Q.18** Out of pure semiconductor and impure semiconductor, of which the resistivity is affected more by the change in temperature?
- Q.19** What are the order of resistivity of the semi conductor?
- Q.20.** What are the conductivity of a semi conductor?

## **20.19 Answers To Viva Questions**

**Ans.1:** At 0K, the resistance of a pure semiconductor is infinite.

**Ans.2:** Zero.

**Ans.3:** A material which has electrical properties in between insulator and conductor is called as a semiconductor. According to band theory of solids a semiconductor is that material in which valence band and conduction band are separated by a small gap (less than 5eV).

**Ans.4:** At room temperature, on applying a voltage across a pure semiconductor, the electrons move towards the positive terminal (i.e. towards the higher potential) and holes move towards the negative terminal (i.e., towards the lower potential).

**Ans.5:** Pure germanium is a non-conductor of electricity due to covalent bonding.

**Ans.6:** Energy levels of an isolated atom are sharp and discrete. A very small portion of a solid has millions of atoms very close to each other. These atoms undergo mutual interaction as result energy levels of valence electrons are modified. The energy levels of valence electrons from a group

or a band known as valence band.

**Ans.7:** When an atom absorbs energy, its valence electrons jump to the higher orbit. Conduction band is a band of energy which arises when valence electrons have absorbed some energy such that they are available for conduction of electricity.

**Ans.8:** A conductor is a substance in which conduction and valence bands at room temperature overlap i.e. energy gap is zero. In insulator energy gap (separation between conduction band and valence band) is large.

**Ans.9:** In about  $10^8$  atoms of pure semiconductor, one atom of impurity is added to make it a doped semiconductor.

**Ans.10:** Electron deficiency in the band is called as the holes. Holes are supposed to be positively charged.

**Ans.11:** The resistivity of a pure semiconductor decreases with increase in temperature and is given as  $\rho = Ae^{E_g/kT}$

**Ans.12:** Silicon, Germanium and Gallium.

**Ans.13:** 0.72 eV

**Ans.14:** It is the energy gap in which there are no allowed energy states of electrons i.e. no electron can remain in this energy gap. All the energy states below this energy gap are completely filled by the electrons, while the energy states above this gap are allowed, but they are either completely empty or partially filled. The lower completely filled energy band is called the valence band and the upper completely empty or partially filled energy band is called the conduction band. The energy gap between these two bands is called the band gap energy.

**Ans.15:** Germanium has body-centered cubic lattice.

**Ans.16:** A pure semiconductor is called an intrinsic semiconductor.

**Ans.17:** 1.1 eV

**Ans.18:** Pure semiconductor.

**Ans.19:**  $10^{-3} \Omega\text{-m}$

**Ans.20:**  $10^5$  to  $10^{-4} \text{ S/m}$ .

## References and Suggested Readings

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# **UNIT-21**

## **Study of Horn Antenna Characteristics**

### **Structure of the Unit**

- 21.1 Aim
  - 21.2 Apparatus
  - 21.3 Diagram
  - 21.4 Formula
  - 21.5 Model Graph
  - 21.6 Theory and Description
  - 21.7 Self Learning Exercise-I
  - 21.8 Procedure
  - 21.9 Observation
  - 21.10 Graph
  - 21.11 Calculations
  - 21.12 Result
  - 21.13 Discussion
  - 21.14 Precautions and Sources of error
  - 21.15 Self Learning Exercise-II
  - 21.16 Glossary
  - 21.17 Answers to Self Learning Exercises
  - 21.18 Viva Questions
  - 21.19 Answers to Viva Questions
- References and Suggested Readings

## 21.1 Aim

- (i) To study the radiation pattern of given pyramidal horn by plotting it on a polar graph paper.
- (ii) To find the half power beam width and calculate its gains.

## 21.2 Apparatus

Microwave power source, source of power supply, Frequency meter, calibrated attenuator, pyramidal horn, tuned detector, indicating meter.

## 21.3 Diagram

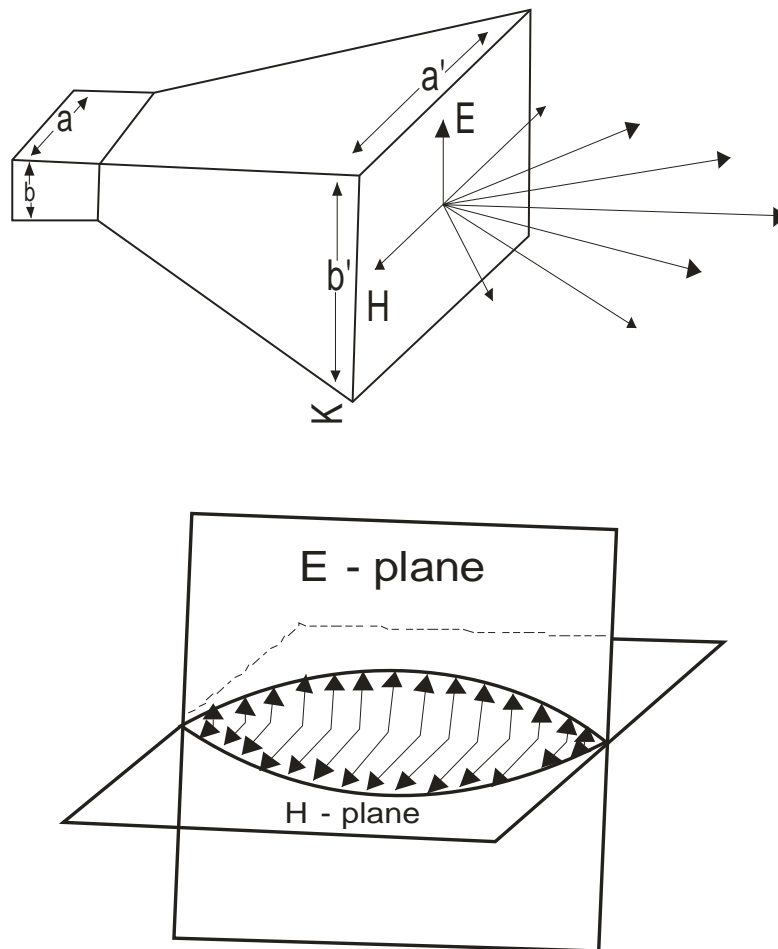
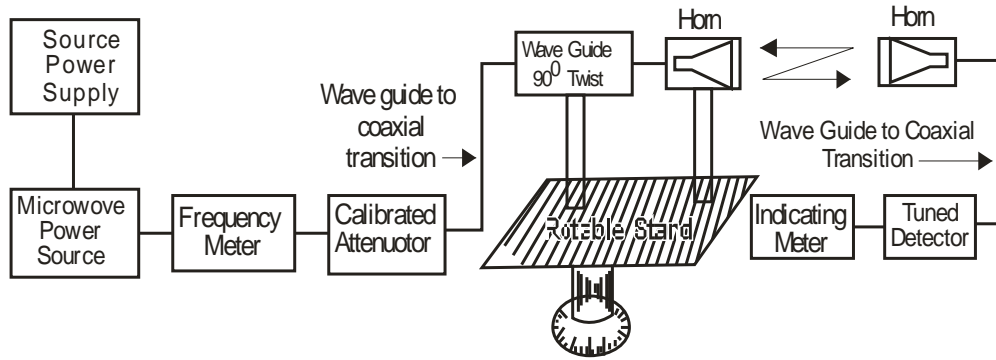


Fig. 21.1 Wave guide horn antennas for E-H Plane horn



**Fig. 21.2. Set-up for measuring gain of a horn antenna**

## 21.4 Formula

Beam Width are formed to be inversely proportional to their respective dimensions.

$$\theta_H = \frac{80\lambda_0}{a} \text{ and } \theta_E = \frac{53\lambda_0}{b}$$

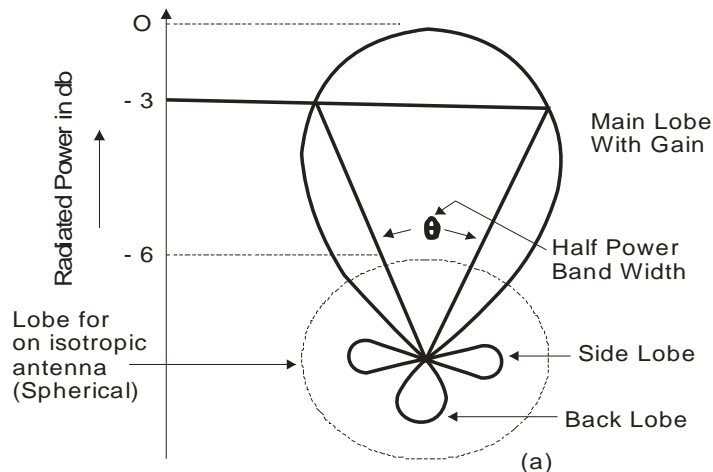
where  $\theta_H$  and  $\theta_E$  are respectively the beam widths of H and E plane radiation patterns of a pyramidal horn.

$\lambda_0$  is free space wavelength

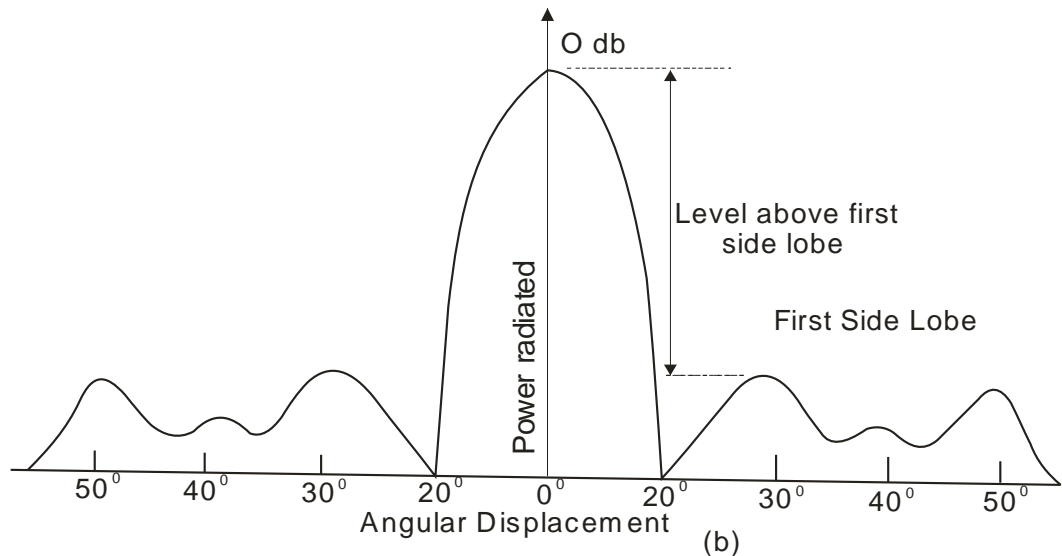
$a$  and  $b$  are respectively the broad and small dimensions of the wave guide

Gain (G) is given by  $G = \frac{2\pi ab}{\lambda_0^2}$

## 21.5 Model Graph



**Fig. 21.3(a) Antenna radiation pattern :Polar Plot**



**Fig. 21.3(b) Antenna radiation pattern :Linear plot.**

## 21.6 Theory and Description

In microwave communications, the transmission and reception of microwave power to/from space, is a primary necessity. The process is affected by an impedance transformer between the space and source, known as antenna. The basic characteristics of an antenna are expressed in terms of field pattern, directivity, bandwidth and gain. A transmission line shall act as an antenna if its output end is well matched to space. For example, if ends of a wave- guide are flared or tapered in such a way that fields of the propagating E-M wave expand in an ordinary manner thereby making the mismatch between guide and space a minimum, it will act as an antenna. Such antenna because of having shapes like horns are known as horn antennas.

### E-M HORNS

Horn antennas of EM horns may have various types of designs. When a circular waveguide is flared in a conical shape we have conical horn. In case of a rectangular waveguide, however, we can either flare broader side to have H-plane or H-sectorial horn, the flaring being parallel to magnetic field, or flare smaller side to have E-plane or E-sectorial horn, the flaring being parallel to the electric field. When both the sides of a waveguide are flared, we shall have a pyramidal horn Fig. 21.1 or E-H horn.



## RADIATION PATTERN : BEAM WIDTH

The basic source of radiations a short dipole and theoretically it radiates equal power in all the directions and so it is called an isotropic radiator. A practical antenna, however, does not radiate isotropically. The figure giving distribution of E-M fields around the antenna in the space around is called as radiation field pattern or simply radiation pattern. Practically, it is a diagram (most commonly polar) of the field strength, or the power intensity as a function of the aspect angle at a constant distance from the radiating antenna. The radiation pattern may be from E- or H-plane depending upon which field is parallel to the ground, but the general characteristics are the same. Further, if we have identical receiving and transmitting antennas and also the common polarization of the radiation, the receiving and transmitting radiation patterns shall also be identical. A typical field pattern, shown in Fig. (21.2), consists of several lobes, viz. main lobe, side lobes and back lobe. The major portion of the radiated power is concentrated in the main lobe and it is desired for a good antenna to have minimum power in the side lobes and in the back lobe. The angle (in a polar radiation pattern) between the two points on the main lobe where power intensity is half the power intensity at the maximum, is known as 3 db beam width or simply beam width of an antenna and it defines its basic directive characteristics. Beam widths are found to be inversely proportional to their respective dimensions.

$$\theta_H = \frac{80\lambda_0}{a} \quad \text{and} \quad \theta_E = \frac{53\lambda_0}{b} \quad (1)$$

Where and are  $\theta_H$  and  $\theta_E$  are respectively the beam widths of H and E plane radiation patterns of a pyramidal horn,  $\lambda_0$  is free - space wavelength and a and b are respectively the broad and small dimensions of the waveguide. To measure  $\theta_H$  and  $\theta_E$  experimentally, two identical horns are placed in the same orientation and in line. One is excited to microwave power while power received by the other is detected at various angles from the lines keeping the distance between the horns the same. The radiation pattern may then be plotted and  $\theta_H$   $\theta_E$ ) may be determined.

Another important characteristic of an antenna is its gain. It measures the directive character of a given antenna and is defined as the ratio of the power that must be radiated (or received) by an isotropic antenna to deliver particular field strength in

the desired direction to the power that must be radiated by actual antenna to obtain the same field strength in the same direction. In other words, it is the power intensity at the maximum of the main lobe compared to the power intensity from an imaginary unidirectional (isotropic radiator) antenna, with the same power fed to the antennas. For pencil-beam antennas (E-M horns) the gain can alternatively be defined as a ratio of the surface area of the isotropic radiator (sphere) to the radiation area (area over which the power density in the beam is at least half the power density on the beam axis) of the beam.

Thus gain G is given by

$$G = \frac{\text{Area of the spherical radiator}}{\text{Area of actual radiator}} \quad (2)$$

At microwave frequencies antenna dimensions being comparatively large (in terms of wavelengths), it can be assumed that all the power supplied to antenna is radiated. Then, using reciprocity theorem one can find that the maximum power transmitted by an antenna is given by

$$P_{\max} = \frac{GE^2\lambda_o^2}{480\pi^2} \quad (3)$$

Where G is gain of the antenna, E is the intensity of radiated electric field and  $\lambda_o$  is free-space wavelength. If we consider this power to be confined in a section of the wave front having area A called capture area or intercept area or absorption cross-section of the receiving antenna, and considering an isotropic dipole situated on the earth's surface, then power radiated by it per unit area is  $E^2/120$  (the surface being spherical). The capture area or aperture is thus (using definition)

$$A = \left( \frac{GE^2\lambda_o^2}{480\pi^2} \right) \left( \frac{E^2}{120\pi} \right)^{-1} = \frac{G\lambda_o^2}{4\pi}$$

or Gain,  $G = \frac{4\pi A}{\lambda_o^2}$

Capture area A is generally smaller than the actual aperture area. In case of pyramidal horn it is found to be 50 percent of the actual area, i.e.,

$$A = \frac{ab}{2}$$

$$\text{Thus, } G = \frac{2\pi ab}{\lambda_o^2} \quad (4)$$

Equation (4) suggests theoretical measurement of gain. Also it indicates that (i) gain is independent of the power level and (ii) gain is the same when antenna is used as a receiver or transmitter. Consider two-horn antennas, one as receiver and another as transmitter, then the power at a range (distance)  $r$  from the transmitting antenna is

$$P_{tr} G_{tr} / 4\pi r^2$$

$P_{tr}$  in power transmitted,  $G_{tr}$  is gain of the transmitter. The capture area of the receiving antenna is  $\frac{G_{rec}\lambda_o^2}{4\pi}$ . Thus power received by the receiving antenna

$$P_{rec} = \frac{P_{tr} G_{tr}}{4\pi r^2} \times \frac{G_{rec}\lambda_o^2}{4\pi}$$

When two horns are identical,

$$G_{tr} = G_{rec} = G, \text{ so}$$

$$\frac{G^2 \lambda_o^2}{(4\pi r)^2} \times P_{tr} = P_{rec}$$

$$\text{Or } G = \frac{4\pi r}{\lambda_o} \sqrt{\frac{P_{rec}}{P_{tr}}} \quad (5)$$

Equation (5) suggests the experimental measurement of the gain of an antenna.

In case two identical horns are not available, Equation (6) cannot be used. Under these conditions, too, we can measure gain. From Equation (1) and (4), we have

$$\begin{aligned} G &= \frac{2\pi ab}{\lambda_o^2} = \frac{2\pi}{\frac{\lambda_o}{a} \frac{\lambda_o}{b}} = \frac{2\pi}{\frac{\theta_H}{80} \times \frac{\theta_E}{53}} \\ &= \frac{2\pi \times 80 \times 53}{\theta_E \theta_H} = \frac{13325.71}{\theta_E \theta_H} \end{aligned} \quad (6)$$

Thus knowing  $\theta_H$  and  $\theta_E$  experimentally, gain can experimentally be determined.

In Equation (5), the accuracy of gain measurements depends upon the selection of

the proper value of  $r$ . When antennas are having different apertures and  $r$  is small, then there happens to be a phase lag between central and circum part of the radiated field causing error in gain measurement. The condition to minimize this error is

$$r_{\min} > \frac{2D^2}{\lambda_0} \quad 7(a)$$

When  $D$  is the aperture of the large antenna. Further, when the antennas are large, the intensity of the radiated field received at the edges of the antenna is small as compared to the centre. The effect is minimized when the separation happens to be

$$r_{\min} > 2D/\lambda_0 \quad 7(b)$$

Antennas are of equal aperture  $\left( \text{otherwise } r_{\min} > \frac{D+d}{\lambda_0} \right)$ . Further, to avoid mutual reflection and the interactions with other bodies, the separation should be optimum.

$$r_{\min} > 2a^2/\lambda_0 \quad 7(c)$$

Thus choosing  $r_{\min}$  as per above conditions, best results for gain may be obtained. ( $r_{\min}$  slightly greater than  $2D^2/\lambda_0$ ).

## 21.7 Self Learning Exercise-I

- Q.1 What is an antenna?
- Q.2 Why an oscillating dipole radiates in space isotropically?
- Q.3 Name Various types of antennas.
- Q.4 What is E-H Horn antenna?
- Q.5 Define beam width.

## 21.8 Procedure

### Antenna Radiation Pattern Plotting

1. Set up the equipments as shown in the fig. 21.2 keeping the axis of both antennas in same axis line.
2. Energize the Microwave source for maximum output at desired frequency with square wave modulation by tuning square wave amplitude and frequency of modulating signal of Gunn Power Supply and by tuning the detector.

3. Also tune the S>S Tuner in the line for maximum output (If S.S. Tuner is in the set up).
4. Obtain full scale deflection (0dB) on normal dB scale (0-10 dB) at any convenient range switch position of the VSWR Meter by gain control knob of VSWR meter or by variable attenuator.
5. Turn the receiving horn to the left in 20 or 50 steps up to  $40^\circ$  - $50^\circ$  and note the corresponding VSWR dB reading in normal dB range. When necessary change the range switches to next higher range and add 10 dB to the observed value.
6. Repeat the above step but this time turns the receiving horn to the right and note down the readings.
7. Draw a relative power pattern i.e. output V/S angle.
8. From diagram determine 3dB- width (beam width) of the horn antenna can be measured.

### **Gain Measurement**

1. Set up the equipments as shown in fig. Both horns should be in line.
2. Keep the range dB switch of VSWR meter at 50 dB position with gain control full.
3. Energize the Gunn Oscillator for maximum output at desired frequency with modulating amplitude and frequency of Gunn Power Supply and by tuning of detector.
4. Obtain full scale deflection in VSWR meter with variable attenuator.
5. The transmitting horn by detector mount and change the appropriate range db position to get the deflection 'ON scale (do not touch the gain control knob) Note and record the range db position and deflection of VSWR Meter.
6. Calculate the difference in dB between the power measured in step 4 and 5.

### **Example**

Suppose that a deflection of 5dB on 20dB range dB position was obtained in step 5, the difference between 4 and 5 is  $50 - (20 - 5) = 25$  dB

Convert the dB into power ratio. As for above example it will come 316 which will  $P_t P_r$  calculate gain by following equation.

$$G = \frac{4\pi S}{\sqrt{\lambda_o}} \sqrt{\frac{P_r}{P_t}}$$

In our above example suppose operating frequency us 9 GHz,  $\lambda_o = 3.33$  cm where C is velocity of light and is  $3 \times 10^8$  cm/ sec and distance between antennas is 150 cm. (Suppose).

7. Convert G into dB in above example  $G_{db} = 10 \log 318 = 15.02$  dB
8. The same set-up can be used for other frequency of operation.

### 21.9 Observation

Beam Voltage = ..... Volts  
 Beam Current = ..... mA  
 Reflector Voltage = ..... Volts  
 Broad dimension of wave guide (a) = ..... CM  
 Small dimension of wave guide (b) = ..... CM  
 Free space wave length  $\lambda_o$  = ..... CM

#### Observation Table :-

##### For H Plane

S.No.	Angular Position	Output Power deflection left ( $\mu$ A)	Output Power deflection right ( $\mu$ A)
1	$0^0$		
2	$2^0$		
3	$4^0$		
4	$6^0$		
5	$8^0$		
6	$10^0$		
7	$12^0$		
8	$14^0$		
9	$16^0$		

10	18 <sup>0</sup>		
11	20 <sup>0</sup>		
12	22 <sup>0</sup>		
13	24 <sup>0</sup>		
14	26 <sup>0</sup>		
15	28 <sup>0</sup>		
16	30 <sup>0</sup>		
17	32 <sup>0</sup>		
18	34 <sup>0</sup>		
19	36 <sup>0</sup>		
20	38 <sup>0</sup>		
21	40 <sup>0</sup>		

**For E- Plane :-**

S.No.	Angular Position	Output Power deflection left ( $\mu\text{A}$ )	Output Power deflection right ( $\mu\text{A}$ )
1	0 <sup>0</sup>		
2	2 <sup>0</sup>		
3	4 <sup>0</sup>		
4	6 <sup>0</sup>		
5	8 <sup>0</sup>		
6	10 <sup>0</sup>		
7	12 <sup>0</sup>		
8	14 <sup>0</sup>		
9	16 <sup>0</sup>		
10	18 <sup>0</sup>		

11	20 <sup>0</sup>		
12	22 <sup>0</sup>		
13	24 <sup>0</sup>		
14	26 <sup>0</sup>		
15	28 <sup>0</sup>		
16	30 <sup>0</sup>		
17	32 <sup>0</sup>		
18	34 <sup>0</sup>		
19	36 <sup>0</sup>		
20	38 <sup>0</sup>		
21	40 <sup>0</sup>		

## 21.10 Graph

**For H- Plane :-**

Angular Position								
Output Left ( $\mu A$ )								
Input Right ( $\mu A$ )								

**For - E Plane :-**

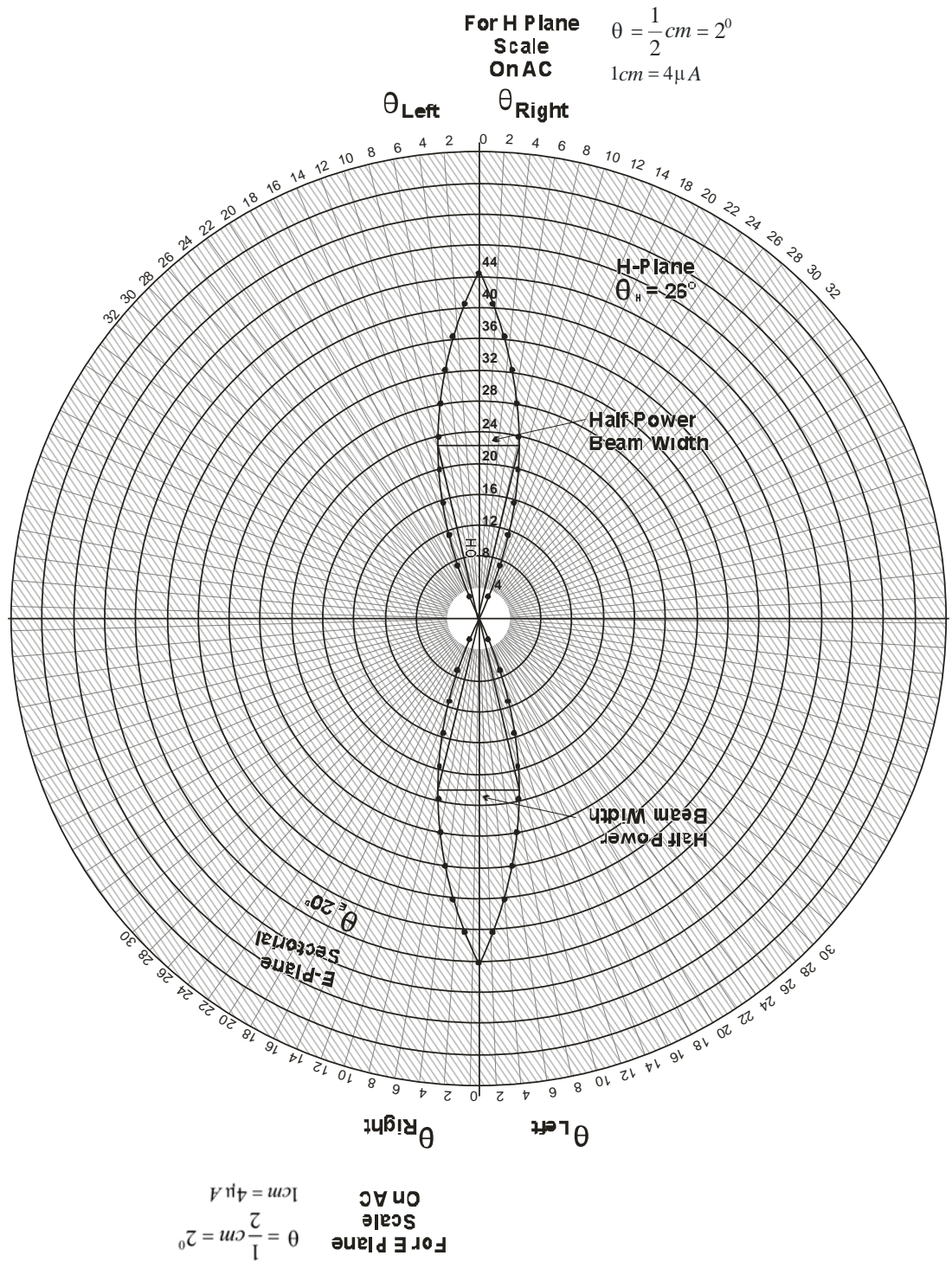
Angular Position								
Output Left ( $\mu A$ )								
Input Right ( $\mu A$ )								

**Scale**

ON x axis 1 cm = ..... Degree

On y axis 1 cm = ..... gm / cm<sup>3</sup>





## 21.11 Calculations

Theoretical values of Beam width of a horn antennas

$a = 9.8 \text{ cm}, \quad b = 7.8 \text{ cm}, \quad \lambda_0 = 2.9718 \text{ cm}$

For H plane  $\theta_H = \frac{80\lambda_0}{a} = \frac{80 \times 2.9718}{9.8} = 24.26^\circ$

For E plane  $\theta_E = \frac{53\lambda_0}{a} = \frac{53 \times 2.9718}{7.8} = 20.193^\circ$

and power gain  $G = \frac{2\pi ab}{\lambda_0^2} = \frac{2 \times 3.14 \times 7.8 \times 9.8}{(2.9718)^2} = 53.36$

From polar graph experimental values of beam width of a horn antennas

For H- Plane

$\theta_H = \dots\dots\dots$  Half band angle i. e. angle corresponding to 3db)

For E- Plane

$\theta_E = \dots\dots\dots$  (Angle corresponding to 3 db points)

Power gain  $G = \frac{13325.71}{\theta_E \theta_H} = \dots\dots\dots$

Thus knowing  $\theta_H$  and  $\theta_E$  experimentally, gain can experimentally be determined.

Percentage error =  $\frac{\text{Theoretical Gain} - \text{Experimental Gain}}{\text{Theoretical Gain}}$

Percentage error =  $\dots\dots\dots\%$

**21.12 Result**

The radiation power pattern for E- plane and H- plane for the pyramidal horn antenna are shown in polar graph.

By experimentally, the values of beam width for the half angle for E and H plane are

$\theta_E = \dots\dots\dots, \theta_H = \dots\dots\dots$  respectively.

and also, the power gain G is  $\dots\dots\dots$

Theoretical value of the power gain = 53.36

Percentage error =  $\dots\dots\dots\%$

**21.13 Discussion**

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## 21.14 Precautions and Sources of Error

1. The sensor should be parallel to the cross-section of antenna.
2. Do not see directly inside the horn antenna because power out of horns may damage retina of eye.
3. Materials present in the vicinity of the experimental set up should be absorbing ones. It's advised to perform this experiment in open space.
4. Angular displacement of  $2\theta$  must be given properly.
5. While modulating repeller should never become positive with respect to cavity.

## 21.15 Self Learning Exercise-II

- Q.1 What is power gain?
- Q.2 Write the formula of power gain.
- Q.3 What do you mean by isotropic radiator?
- Q.4 Write the formula of beam width of a horn antenna for E and H plane.
- Q.5 What is the purpose of an antenna?

## 21.16 Glossary

**Antenna:** For the transmission and reception of electromagnetic waves.

**Antenna power gain:** Antenna's maximum radiation intensity in a stated direction to the maximum radiation intensity of a reference antenna with identical power applied to both.

**Attenuation:** Loss in power of electromagnetic signals between transmission and reception points.

**Band Width:** A range of consecutive frequencies comprised of a band over which an antenna shall perform without the need of any adjustment.

**Directivity:** The theoretical characteristic of an antenna to concentrate power in only one direction. Whether transmitting or receiving.

**Gain:** Gain is the practical value of the directivity of an antenna. It takes into account the efficiency of the complete structure.

**Radiation pattern:** The graphical representation of the relative field strength radiated from an antenna in a given plane, plotted against the angular distance from a given reference.

**Isotropic:** (of an object or substance) having a physical property which has the same value when measured in different directions.

## 21.17 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** In microwave communications, the transmission and reception of microwave power to and from space, is primary necessity. The process is effected by an impedance transformer between the space and source, known as antenna.

**Ans.2:** The basic source of radiation is a short dipole and theoretically it radiates equal power in all the directions and so it is called an oscillating dipole radiates in space isotropically.

**Ans.3:** Conical antenna, H-Plane sectorial, E-Plane sectorial antenna.

**Ans.4:** When both sides the sides of a wave guide are flared we shall have a pyramidal E-H horn antenna.

**Ans.5:** The angle between the two points on the main lobe where power intensity is half the power intensity at the maximum is known as beam width.

### *Answers to Self Learning Exercise-II*

**Ans.1:** It is defined as the ratio of the power that must be radiated (or received) by an isotropic antenna to deliver a particular field strength in the desired direction to the power that must be radiated by actual antenna to obtain the same field strength in the same direction.

**Ans.2:**  $G = \frac{2\pi ab}{\lambda_0^2}$

**Ans.3:** The basic source of radiation is a short dipole and it radiates equal power in all the directions and it is called on isotropic radiator.

**Ans.4:**  $\theta_H = \frac{80\lambda_0}{a}$  and  $\theta_E = \frac{53\lambda_0}{b}$

**Ans.5:** An antenna is an electrical device which converts electric power into radio waves. It is usually used with a radio transmitter or radio receiver. In transmission, a radio transmitter supplies an electric current oscillating at radio frequency to the antenna's terminals, and the antenna radiates the energy from the current as electromagnetic waves. In reception, an antenna intercepts some of the power of an electromagnetic wave in order to produce a tiny voltage at its terminals that is applied to a receiver to be amplified.

### 21.18 Viva Questions

- Q.1 What is a radiation pattern?
- Q.2 What are the two types of radiation Pattern?
- Q.3 Define radiation efficiency.
- Q.4 What is the error introduced in direction finding of loop antenna?
- Q.5 What is the radiation resistance of multi turn loop antenna?
- Q.6 What is the principle of a pattern multiplication?
- Q.7 What is the general effect of earth on antennas.
- Q.8 What is the isotropic source?
- Q.9 What is the difference between transmitting and receiving patterns?
- Q.10 Describe the dipole antenna.
- Q.11 What are the uses of antenna?
- Q.12 Where horn antennas are used?
- Q.13 Define horn antenna.
- Q.14 What are their advantages?
- Q.15 They can operate over a wide range of frequencies, why?
- Q.16 Define gain of horn antenna.
- Q.17 What happened if a simple open-ended wave guide were to be used as an antenna, without the horn?
- Q.18 What is meant by an antenna polarization?

**Q.19** List some important applications of antennas.

**Q.20** What are the maximum power transmitted by an antenna?

## 21.19 Answers to Viva Questions

**Ans.1:** An antenna radiation pattern is a mathematical function or a graphical representation of the radiation properties of the antennas a function of space co-ordinates.

**Ans.2:** (i) Field Pattern

(ii) Power Pattern

**Ans.3:** Radiation efficiency = Maximum Power gain / directivity.

**Ans.4:** Antenna or vertical effect or night effect or Aeroplane effect or polarization error.

**Ans.5:** Radiation resistance =  $\frac{31,200 (NA) (2N)}{(2G)}$

where 2 N = Number of turns,

G = Wave length,

A = effective area.

**Ans.6:** In case of isotropic antenna arrays the total field of the antenna array is simply the vector sum of those of individual radiating sources. In case of non isotropic, but similar sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources, each located at the phase centre of individual sources and having relative amplitude and phase, where as the total phase pattern is the addition of the phase patterns of the individual sources and that of the array of isotropic point sources.

**Ans.7:** Effect of earth modifies the following:

1. Currents flowing in reflecting surface.

2. Radiation pattern.

**Ans.8:** An isotropic radiator is a theoretical point source of electromagnetic or sound waves which radiates the same intensity of radiation in all directions.

**Ans.9:** In the process of transmitting, one of the primary considerations of an antenna is efficiency. Noise is not a primary consideration in transmitting antenna design or deployment. On the receiving end, efficiency is of vanishing importance. Contrary to popular opinion, a Beverage antenna is extremely efficient but it's also a very effective antenna because of its low noise properties.

**Ans.10:** A dipole antenna is a straight electrical conductor measuring half wavelength from end to end and connected at the center to a radio frequency feed line. This antenna also called a doublet, is one of the simplest types of antenna, and constitutes the main radio frequency radiating and receiving element in various sophisticated types of antennas. The dipole is inherently a balanced antenna because it is bilaterally symmetrical.

**Ans.11:** They are used as feeders for larger antenna structures such as parabolic antennas, as standard calibration antennas to measure the gain of other antennas, and as directive antennas for such devices as guns, automatic, and micro wave radiometers.

**Ans.12:** A horn antennas are widely used as antennas at UHF and microwave frequencies, above 300 MHz.

**Ans.13:** A Horn antenna is an antenna that consists of a flaring metal waveguide shaped like a horn to direct the radio waves.

**Ans.14:** Their advantages are moderate directivity (gain) Low SWR, broad band width, and simple construction and adjustment.

**Ans.15:** Since they don't have any resonant elements, they can operate over a wide range of frequencies, a wideband width. The useable bandwidth of horn antennas is typically of the order of 10 : 1 and can be up to 20 :1.

**Ans.16:** Horns have very little loss, so the directivity of a horn is roughly equal to its gain. The gain  $G$  of a horn antenna is the ratio of the radiated power intensity along its beam axis of the intensity of an isotropic antenna with the same input powers.

**Ans.17:** If a simple open-ended wave guide were to be used as an antenna without The horn, the sudden end of the conductive walls causes an abrupt

impedance of free space, 377 ohms.

**Ans.18:** The polarization of an antenna is the polarization of the wave radiated by the antenna. At a given position, the polarization describes the orientation of the electric field.

**Ans.19:** Wire antenna, travelling wave Antenna, Micro strip Antennas.

**Ans.20:** 
$$P_{\max} = \frac{GE^2\lambda_0^2}{480\pi^2}$$

## References and Suggested Readings

1. M. L. Sisodia and G. S. Raghuvanshi, Basic Microwave Techniques and Laboratory Manual, new Age International (P) Limited, 2001.
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# **UNIT-22**

## **Characteristics of GM Counter and Study of Statistical Nature of Radioactive Decay**

### **Structure of the Unit**

- 22.1 Aim
  - 22.2 Apparatus
  - 22.3 Diagram
  - 22.4 Formula
  - 22.5 Model Graph
  - 22.6 Theory and Description
  - 22.7 Self Learning Exercise-I
  - 22.8 Procedure
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  - 22.17 Answers to Self Learning Exercises
  - 22.18 Viva Questions
  - 22.19 Answers to Viva Questions
- References and Suggested Readings

## 22.1 Aim

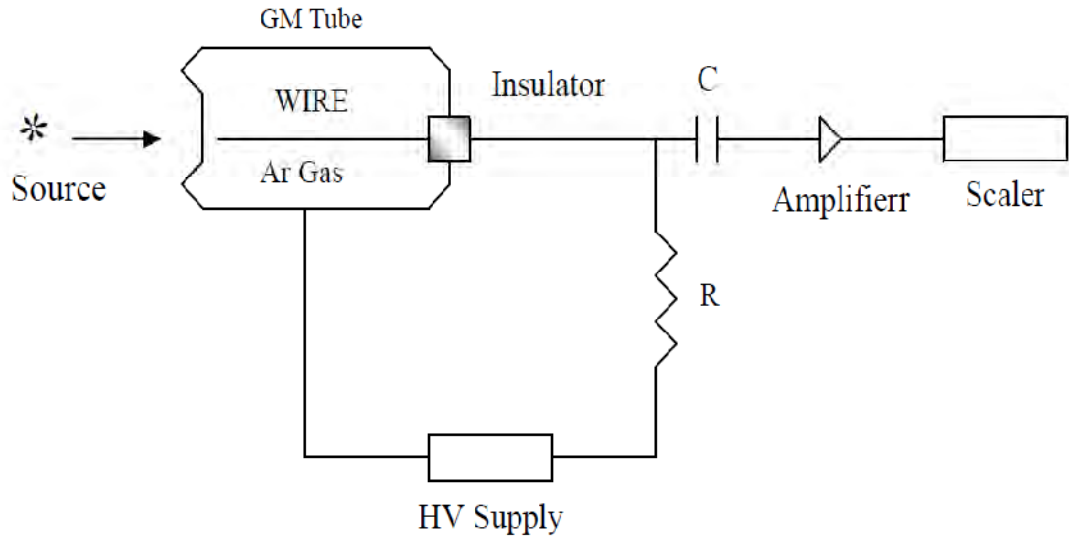
3. To determine the plateau and optimal operating voltage of a Geiger-Müller counter.
4. To determine the resolving time of a GM counter.
5. To investigate the statistics related to measurements with a Geiger counter. Specifically, the Poisson and Gaussian distributions will be compared.

## 22.2 Apparatus

- Set-up for ST-350 Counter
- GM Tube and stand
- shelf stand,
- serial cable, and
- a source holder
- Radioactive Source (e.g., Cs-137, Sr-90, or Co-60).

## 22.3 Diagram





## 22.4 Formula

Let  $n_1$ ,  $n_2$  and  $n_{12}$  be the true counts (sample plus background), with sources  $S_1$ ,  $S_2$  and  $(S_1 + S_2)$ , respectively. Let  $m_1$ ,  $m_2$  and  $m_{12}$  represent the corresponding observed rates. Also let  $n_b$  and  $m_b$  be the true and measured background rates with both the sources removed. Assuming the nonparalyzable model, the dead time  $\tau$  is given by

$$\tau = \frac{X(1 - \sqrt{1 - Z})}{Y}$$

Where

$$X = m_1 m_2 - m_b m_{12}$$

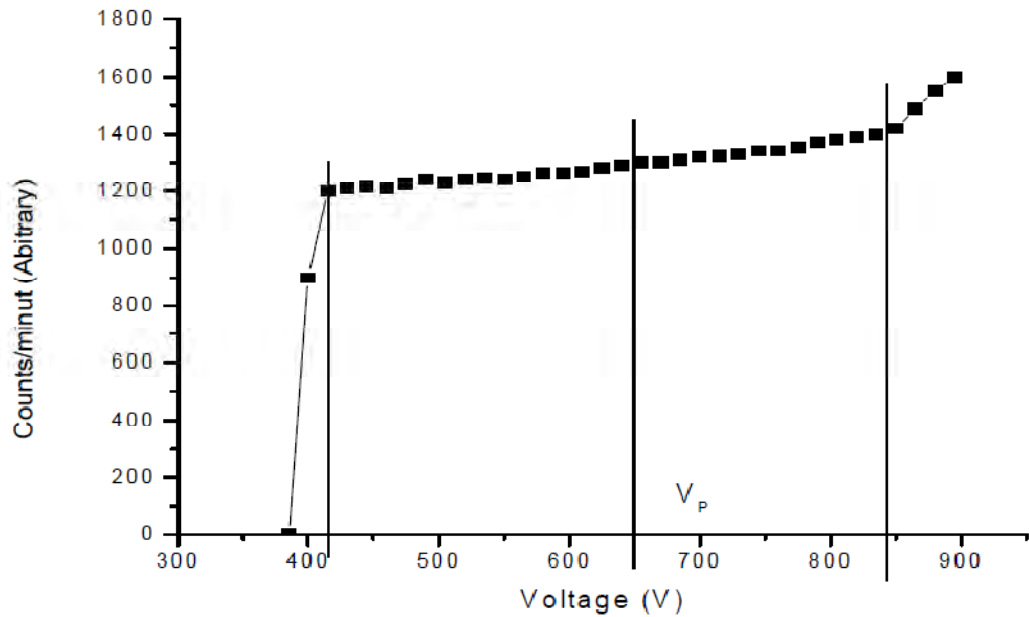
$$Y = m_1 m_2 (m_{12} + m_b) - m_b m_{12} (m_1 + m_2)$$

$$Z = \frac{Y(m_1 + m_2 - m_{12} - m_b)}{X^2}$$

## 22.5 Model Graph

Determination of Operating Voltage:

Model graph between count/minute and voltage is plotted here.



## 22.6 Theory and Description

Geiger-Müller (GM) counters were invented by H. Geiger and E.W. Müller in 1928, and are used to detect radioactive particles. A typical GM Counter consists of a GM tube having a thin end window (e.g. made of mica), a high voltage supply for the tube, a scalar to record the number of particles detected by the tube, and a timer which will stop the action of the scalar at the end of a preset interval.

The sensitivity of the GM tube is such that any particle capable of ionizing a single atom of the filling gas of the tube will initiate an avalanche of electrons and ions in the tube. The collection of the charge thus produced results in the formation of a pulse of voltage at the output of the tube. The amplitude of this pulse, on the order of a volt or so, is sufficient to operate the scalar circuit with little or no further amplification. The pulse amplitude is largely independent of the properties of the particle detected, and gives therefore little information as to the nature of the particle. Even so, the GM Counter is a versatile device which may be used for counting alpha particles, beta particles, and gamma rays, albeit with varying degrees of efficiency.

### Principle of the Method:

All nuclear radiations, whether they are charged particles or gamma rays, it will ionize atoms/molecules while passing through a gaseous medium. This ionizing

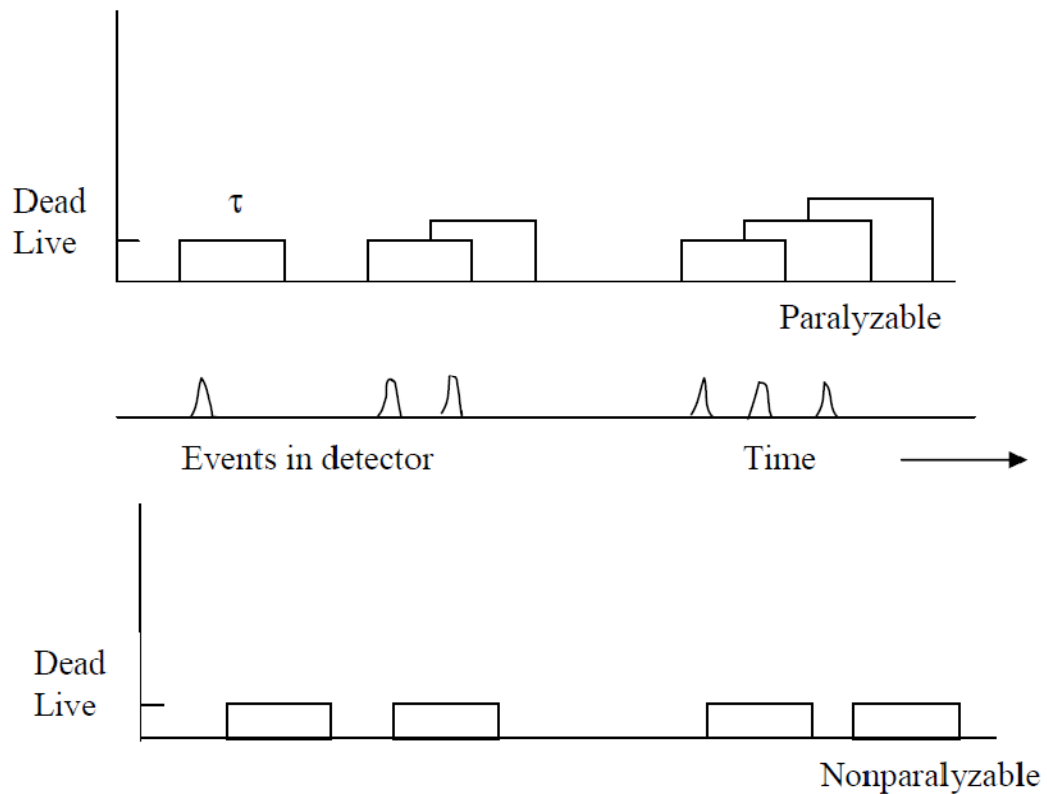
property of a nuclear radiation is utilized for its detection. Geiger-Muller counter, commonly called as GM counter or simply as Geiger tube is one of the oldest and widely used radiation detectors. It consists of a metallic tube with a thin wire mounted along its axis. The wire is insulated from the tube using a ceramic feed-through. The central wire (anode) is kept at a positive potential of a few hundreds of volt or more with respect to the metallic tube, which is grounded. The tube is filled with argon gas mixed with 5-10% of ethyl alcohol or halogens (chlorine or bromine).

When an ionizing radiation enters the Geiger tube some of the energy of the radiation may get transferred to a gas molecule within the tube. This absorption of energy results in ionization, producing an electron-ion pair (primary ions). The liberated electrons move towards the central wire and positive ions towards the negatively charged cylinder. The electrons now cause further ionization by virtue of the acceleration due to the intense electric field. These secondary ions may produce other ions and these in turn still other ions before reaching the electrodes. This cascading effect produces an avalanche of ions. In an avalanche created by a single original electron many excited gas molecules are formed by electron collisions in addition to secondary ions. In a very short time of few nanoseconds these excited molecules return to ground state through emission of photons in the visible or ultraviolet region. These photons are the key element in the propagation of the chain reaction that makes up the Geiger discharge. If one of these photons interacts by photoelectric absorption in some other region of the tube a new electron is liberated creating an avalanche at a different location in the tube. The arrival of these avalanches at the anode causes a drop in the potential between the central wire and the cylinder. This process gives rise to a very large pulse with amplitude independent of the type and energy of the incident radiation. The pulse is communicated to the amplifier through an appropriate RC circuit, and then to a counter which is called as scaler. Suitable arrangements are made to measure the counts for a preset time interval. The schematic diagram of the G-M tube and the associated electronic components is given in Figure.

### **Dead Time:**

In nearly all detector systems, there will be a minimum amount of time that separates two events in order that they may be recorded as two separate pulses. In

some cases the limiting time may be set by processes in the detector itself, while in other cases the limit may arise due to the delays associated with the electronics. This minimum time separation is usually called the *dead time* of the counting system. Because of the random nature of radioactive decay, there is always some probability that a true event will be lost because it occurs too quickly following a preceding event. Two models of dead time are in common use, categorized on the basis of paralyzable and nonparalyzable response of the detector. The fundamental assumptions of the two models are illustrated in Figure.



At the center of the figure, a time scale is shown on which six randomly spaced events in the detector are indicated. At the bottom of the figure is the corresponding dead time behavior of a detector to be nonparalyzable. A fixed time  $\tau$  is assumed to follow each true event that occurs during the "live period" of the detector. True events that occur during the dead period are lost and assumed to have no effect whatsoever on the behavior of the detector. In the example shown the nonparalyzable detector would record four counts from the six true events. In contrast, the behaviour of a paralyzable detector is shown along the top line of

Figure. The same dead time  $\tau$  is assumed to follow each true interaction that occurs during the live period of the detector. True events that occur during the dead period are not recorded but they extend the dead time by another period  $\tau$  following the lost event. In the example shown, only three counts are recorded for the six true events. The two models predict the same first order losses and differ only when true event rates are high. They are in some sense two extremes of idealized system behavior, and real counting system will often display a behavior that is intermediate between these extremes. The detailed behavior of a specific counting system may depend on the physical processes taking place in the detector itself or on delays introduced by the pulse processing and recording electronics.

If the system dead time is  $\tau$ , and the measured count rate is  $m$ , then the true count rate  $n$  predicted by the two models can be expressed as

Nonparalyzable Model

$$n = \frac{m}{1 - m\tau}$$

Paralyzable Model

$$m = ne^{-n\tau}$$

You may show that for low counting rates ( $n \ll 1/\tau$ ) both models give the same expression for  $n$ . In the present experiment we will utilize the nonparalyzable model in the calculation of true counts and the dead time  $\tau$ .

### Setting up the equipment

**Note that:** Dangerous voltages can exist at the GM and SCINT connectors. Ensure that the high voltage is set to zero or that the instrument is OFF before connecting or disconnecting a detector.

1. Connect the ST360 to its AC adapter.
2. Connect a GM tube to the GM connector via a BNC cable.
3. Enter the HIGH VOLTAGE mode and set the high voltage to the recommended value for the GM tube.
4. Place the radioactive source close to the GM tube's window.

5. Using the Operating Mode information described above, set the unit up to perform the desired function.
6. Press the COUNT button to start data acquisition, the STOP button to halt data acquisition (providing Preset Time is not being used), and the RESET button to reset the time and data to zero.

### **Operating Plateau for the Geiger Tube**

The purpose of this experiment is to determine the voltage plateau for the Geiger tube and to establish a reasonable operating point for the tube. Model graph 1, shows count rate vs. voltage curve for a typical Geiger tube that has an operating point in the vicinity of 500 V.

### **Measurement of Resolving (Dead) Time $\tau$**

A commonly used method for dead time measurements is known as *two source method*. The method is based on observing the counting rate from two sources individually and in combination. Because the counting losses are nonlinear, the observed rate due to the combined sources will be less than the sum of the rates due to the two sources counted individually, and the dead time can be calculated from the discrepancy.

### **Statistics of Radioactive Decay Process (Counting Statistics)**

Statistics of radioactive decay process (Counting statistics) Radioactive decay is a random process. Consequently, any measurement based on observing the radiation emitted in a nuclear decay is subject to some degree of statistical fluctuations. These inherent fluctuations are unavoidable in all nuclear measurements. The term counting statistics includes the framework of statistical analysis required to process the results of nuclear counting experiments and to make predictions about the expected precision of quantities derived from these measurements.

Although each measurement (number of decays in a given interval) for a radioactive sample is independent of all previous measurements (due to randomness of the process), for a large number of individual measurements the deviation of the individual count rates from the average count rate behaves in a predictable manner. Small deviations from the average are much more likely than



large deviations. These statistical fluctuations in the nuclear decay can be understood from the statistical models utilizing Poisson distribution or Gaussian (Normal) distribution. If we observe a given radioactive nucleus for a time  $t$  and define the success as “the nucleus decays during the process” then the probability of success “ $p$ ” is given by  $(1 - e^{-\lambda t})$ . The Poisson distribution applies when the success probability  $p$  is small and the number successes (i.e. number of counts measured) is also small (say  $< 30$ ). In practical terms, this condition implies that we have chosen an observation time that is small compared with the half life of the source. When the average number of successes becomes relatively large (say  $> 30$ ) we can utilize the Gaussian model of distribution. Since in most of the cases the count rates are reasonably large (few tens of counts per second) the Gaussian model has become widely applicable to many problems in counting statistics. On the other hand the Poisson distribution is applicable in the case of background counts. The details of experimental, Poisson and Gaussian distributions are given below.

### **Experimental Distribution Function**

We assume that we have a collection of  $N$  independent measurements of the same physical quantity. In this particular case the quantity is the number of counts recorded by the detector in a specific time interval. We denote the result of these  $N$  measurements as  $y_1, y_2, y_3, \dots, y_i, \dots, y_N$ .

The experimental mean is given by

$$\bar{y} = \frac{\sum_{i=1}^N y_i}{N}$$

The data set is conveniently represented by a frequency distribution function  $F(y)$ . The value of  $F(y)$  is the relative frequency with which the number appears in the collection of data. By definition

$F(y)$  = number of occurrence of the value  $y$  [ $\equiv(y)$ ]/number of measurements [ $=N$ ]

A plot of  $F(y)$  versus  $y$  gives the frequency distribution of the data (The number of occurrences can also be calculated by choosing a suitable interval for the values of  $y$ ). The standard deviation of the distribution is given by

$$\sigma_{\text{exp}} = \left( \frac{1}{N} \sum_1^N (y_i - \bar{y})^2 \right)^{1/2}$$

### Notes regarding $\sigma_{\text{exp}}$ and $y$

Remember that above equation is applicable to the quantities directly measured in the experiment and not to the derived quantities. To illustrate, in the present experiment if you measure the number of counts for a preset time interval (say 30 s) and call it  $y_i$ . Then Eq. is applicable to these counts only and not to the counting rates calculated using these values. To determine the deviations for the derived quantities proper error propagation methods should be used.

To be precise,  $\bar{y}$  is the true mean value determined from a set having infinitely large number of measurements and cannot be determined experimentally as such. However for a reasonably large set of measurements the value of  $y$  can be set equal to  $\bar{y}$ .

### The Poisson distribution

As mentioned above it is applicable when  $p \ll 1$  and the number of successes are very few

$$p(y) = \frac{(\bar{y})^y e^{-\bar{y}}}{y!}$$

In this case the standard deviation is given by

$$\sigma_p = \sqrt{\bar{y}}$$

### The Normal or Gaussian distribution

When  $p \ll 1$  and the successes are large one can model the experimental data using the Normal distribution which is also called Gaussian distribution (as per R.D. Evans it is erroneous to call this as Gaussian because its derivation by Gauss (1809) was antedated by those of Laplace (1774) and DeMoivre (1735)). This is given by

$$p(y) = \frac{1}{\sqrt{2\pi\bar{y}}} \exp\left(-\frac{(y - \bar{y})^2}{2\bar{y}}\right)$$

In this case the standard deviation is given by

$$\sigma_G = \sqrt{y}$$

We will denote both  $\sigma_p$  and  $\sigma_G$  as  $\sigma_{th}$ .

## Applications of Statistical Models In Nuclear Physics

There are two major applications of counting statistics in nuclear measurements. The first application involves the use of statistical analysis to determine whether a set of multiple measurements of the same physical quantity shows an amount of internal fluctuation that is consistent with statistical predictions. In this case the motivation is to determine whether a particular counting system is functioning normally. The second application is more important in which we examine these methods to make a prediction about the uncertainty one should associate with a single measurement. The following procedure and analysis will give you a feel as to how an experimental distribution in a nuclear counting experiment looks like and how does it compare with theoretical distributions.

### 22.7 Self Learning Exercise-I

- Q.1 What is ionizing radiation?
- Q.2 What is Radioactivity?
- Q.3 What are the Three Types of Radioactivity?
- Q.4 Is Radioactivity Good or Bad?

### 22.8 Procedure

#### Operating Voltage for the Geiger Tube

- a. After setting up the equipment as described above place a  $\gamma$  source ( $^{137}\text{Cs}$ ,  $^{90}\text{Sr}$ , or  $^{60}\text{Co}$ ) in the lead castle at a suitable distance from the window of the Geiger tube.
- b. Find the voltage at which the GM tube just starts counting ( $\sim 400\text{V}$ ). Starting from this voltage take the number of counts  $N(V)$  for 30 s, thrice at each voltage  $V$ , in steps of 20 volts. You will go over a plateau and should stop increasing the voltage as soon as the number of counts starts increasing very rapidly.

- c. Plot  $N(V)$  versus  $V$ . Choose a voltage on the central region of the plateau of  $N(V)$  vs.  $V$  curve. Perform all subsequent measurements (described below and in the next experiment) at this voltage.

### **Measurement of resolving (dead) time $\tau$**

- a. To find the dead time we have to use two  $\gamma$  sources say  $S_1$  ( $^{137}\text{Cs}$ ) and  $S_2$  ( $^{60}\text{Co}$ ). While performing the experiment as per the steps given below, care must be exercised not to move the source already in place and consideration must be given to the possibility that the presence of a second source will scatter radiation into the detector which would not ordinarily be counted from the first source alone. In order to keep the scattering unchanged, a dummy second source without activity is normally put in place when the sources are counted individually.
- b. Keep source  $S_1$  in one of the pits in the source holder made for this purpose. Keep a dummy source in the second pit. Record the counts for a preset time (say 300 s).
- c. Without removing source  $S_1$  remove the dummy source from the second pit and keep the source  $S_2$  in its place. Record the number of counts for the combined sources  $S_1$  and  $S_2$  for the same preset time as in b.
- d. Remove source  $S_1$  and measure the counts due to source  $S_2$  alone, for the same preset time as in b.
- e. Remove source  $S_2$  as well and record the background counts for the same period.
- f. Calculate the count rates in all the cases.

### **The statistics related to measurements with a Geiger counter**

- a. Set the operating voltage of the Geiger counter at its proper value.
- b. Don't put any source in the lead castle. Also remove all the sources in the vicinity of the castle.
- c. Take 100 independent readings of the background counts for a preset time of 10 s. (To set Preset time 10 sec. follow step b of initial procedure).
- d. Save the data by pressing STORE key. While taking 100 independent reading set ITERATION (Step c of initial procedure) to 1.

- e. Place one of the  $\gamma$  sources ( $^{137}\text{Cs}$  or  $^{60}\text{Co}$ ) far enough away from the window of the Giger tube so that approximately 2000 counts are recorded in a time period of 30 s. Take 100 independent readings of the counts for a preset time of 30 s.
- f. Save the data by pressing STORE key. While taking 100 independent reading set ITERATION (Step e of initial procedure) to 1.
- g. Transfer the data on PC and plot the required function.

### **Analysis of Background counts (data set c above)**

- a. Determine frequency of occurrence  $\nu(y)$  which is the number of measurements in which  $y = 0, 1, 2, 3, 4 \dots$  counts have been observed and plot the experimental distribution  $\nu(y)$  versus  $y$ .
- b. Calculate the average number of counts  $\bar{y}$  and the Poisson distribution

$$p(y) = \frac{(\bar{y})^y e^{-\bar{y}}}{y!}$$

- c. Compare this distribution with the experimental distribution.
- d. Calculate  $\sigma_{\text{exp}}$  and  $\sigma_{\text{th}}$  and compare. The comparison gives clue to the reliability of the measuring equipment. If  $\sigma_{\text{exp}}$  is larger than  $\sigma_{\text{th}}$ , it means that additional fluctuations have been introduced by the apparatus, such as spurious counts due to voltage surges, sparks in the tube or change of the background during the course of the experiment which can occur when you handle the sources (move from one place to another) while the measurements are going on.
- e. Determine the actual number of intervals for which the absolute value of the deviation from the average is larger than the standard deviation and the probable error  $0.6745\sigma$ . Compare with theory.

### **Analysis of the counts taken with the source data set (e) above**

Carry out the analysis following steps h to k above. However, in this case use Gaussian distribution. Also, in order to represent the distribution in the best possible manner, frequency of occurrence may be calculated by choosing equally spaced, non-overlapping, contiguous intervals for the counts. The width of the interval can be anywhere from 2 to 10 counts or more depending on the data set.

## 22.9 Observation

Operating voltage for the Geiger tube.

Voltage (V)	Counts	Count rate

Measurement of resolving (dead) time  $\tau$

	N	M	$\tau$	Error (%)
$S_1$				
$S_2$				
$S_{12}$				

Data Sheet for Statistics Lab

Run	Counts

Data set	Mean	Gaussian	Poission
Background			
Cs-137			

## 22.10 Graph

Plot the required graph.

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 .....

## 22.11 Calculations

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 .....  
 .....  
 .....

## 22.12 Result

1. The Optimal Voltage of the given Geiger Muller Counter is = ----- Volts
2. Counts Per Seconds Vs Voltage given graph has been drawn to identify the plateau region of the GM counter.
3. The Dead time of the GM counter is = ----- Seconds

## 22.13 Discussion

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 .....  
 .....  
 .....

## 22.14 Precautions and Sources of Error

1. While performing an experiment with one radioactive source other sources should not be present nearby. They should be put behind the lead shield.

2. Handle the radioactive sources with care. Don't touch in bare hand to the center of samples.
3. While handling the liquid radioactive samples please use hand gloves.
4. Don't put your mobile near to the detector. It may add some counts to the signal.

## 22.15 Self Learning Exercise -II

**Q.1** What is a Gamma Ray?

**Q.2** What are the Properties of a Gamma Ray?

**Q.3** Where do Gamma Rays come from?

**Q.4** How can the Direction of a Gamma Ray be Changed?

## 22.16 Glossary

**Radiation** :Energy emitted by atoms in a changing state. Radiation can take its form as electromagnetic waves, heat, light, X-rays, Gamma rays, or streams of alpha particles, beta particles, neutrons or protons.

**Background radiation:** The radiation of man's natural environment originating primarily from the naturally radioactive elements of the earth and from the cosmic rays. The term may also mean radiation extraneous to an experiment.

**Geiger counter:** A Geiger-Müller detector and measuring instrument. It contains a gas-filled tube which discharges electrically when ionizing radiation passes through it and a device that records the events.

**Ionizing radiation:** Radiation that is capable of producing ions either directly or indirectly.

**Scaler:** An electronic instrument for counting radiation induced pulses from radiation detectors such as a Geiger-Muller tube.

## 22.17 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** Ionizing radiation is energy that is given off by unstable atoms as they decay. In a nuclear power reactor core, radioactive uranium atoms split into



smaller radioactive atoms (like cesium, strontium, iodine) which give off alpha, beta and gamma radiation. These particles and waves have sufficient energy to cause damage to DNA and cells if they get into the environment and our bodies.

**Ans.2:** Sometimes the nucleus of an atom is unstable. A change will occur in the nucleus to make it more stable. The change is called a decay. When a nucleus decays it will emit (give out) some particles or waves. Emitting particles or waves from the nucleus is called radioactivity. Radioactive decay is a random process which gives out heat. The particles or waves that are emitted are called radiation. The same word radiation is also used for some electromagnetic waves which have not come from radioactive decay.

**Ans.3:** There are three types of radioactivity, called alpha, beta and gamma. A radioactive nucleus will emit either an alpha particle or a beta particle or a gamma ray. After an alpha particle or a beta particle has been emitted from a nucleus, the atomic number of the atom will have changed. The atom will have changed into a different element. An alpha or beta emitter might also emit gamma radiation.

**Ans.4:** Radioactivity can be both harmful and useful. For example, the rate of decay (called the half-life) can be used to find out how old things are. The energy released from a nucleus during fission can be used to generate electricity. Radioactivity can cause damage to living cells leading to cancer.

### ***Answers to Self Learning Exercise-II***

**Ans.1:** A gamma ray (also written as  $\gamma$ -ray) is an electromagnetic wave. It is at the small wavelength end of the electromagnetic spectrum. A gamma ray has no mass and no charge. It is written  $\gamma$ . Emitting a gamma ray makes no difference to the mass number or the atomic number but will make the nucleus more stable.

**Ans.2:** (i) A gamma ray has no charge (see above).

(ii) A gamma ray has no mass. Of the three types of radioactivity a gamma ray is the least likely to collide with other atoms which means that (a) gamma rays have the most penetrating ability and are not easily absorbed.

Many centimeters of lead or a few meters of concrete will absorb most gamma rays but some will still get through.

(b) gamma rays are the least ionising radiation.

**Ans.3:** Gamma rays are likely to be emitted at the same time as alpha particles or beta particles but some isotopes only emit gamma rays.

**Ans.4:** Unlike an alpha particle or a beta particle, a gamma ray is not deflected by a magnetic or electric field because it has no charge.

## 22.18 Viva Questions

- Q.1 What is an Alpha Particle?
- Q.2 What are the Properties of an Alpha Particle?
- Q.3 Where do Alpha Particles come from?
- Q.4 How can the Direction of an Alpha Particle be Changed?
- Q.5 Define Ionization?
- Q.6 What are the ionization radiations you know?
- Q.7 What is known as GM Region?
- Q.8 What is the expression for the voltage developed in a GM counter?
- Q.9 What is Efficiency of an ordinary GM Counter?
- Q.10 What are Efficiency affecting factors?
- Q.11 What is a Dead Time of Counter?
- Q.12 What is a Beta Particle?

## 22.19 Answers to Viva Questions

**Ans.1:** An alpha particle (also written as  $\alpha$ -particle) consists of 2 protons and 2 neutrons. An alpha particle is the same thing as the nucleus of a helium atom. It is written  ${}^2_2\text{He}^4$ .

**Ans.2:** An alpha particle has a positive charge because the 2 protons have a positive charge. An alpha particle has by far the most mass of the three types of radioactivity. It is the type most likely to collide with other atoms which means that alpha particles have the least penetrating

ability of the three types and are easily absorbed by paper, skin or a few centimeters of air. alpha particles are the most ionising of the three types.

**Ans.3:** Alpha particles are most likely to be emitted during the decay of heavy nuclei which have a large number of protons and neutrons. After an alpha particle has been emitted the atomic number and the mass number changes and a new element is made.

**Ans.4:** The positive charge on the alpha particle means that it will be deflected by a magnetic field or an electric field (like the charged droplets in an inkjet printer). An alpha particle will be deflected less than a beta particle because the alpha particle has a bigger mass. An alpha particle has a positive charge and will be deflected in the opposite direction from a beta particle which has a negative charge.

**Ans.12:** A beta particle (also written as  $\beta$ -particle) is the same as an electron. It is often called a high energy electron because it is very fast moving. A beta particle comes from the nucleus of an atom! It has a charge of  $-1$  and very little mass (only  $1 \div 1840$  as big as a proton). It is written  ${}_{-1}e^0$ . The e shows that the particle is an electron. A beta particle is given a mass number of zero because the mass is very small compared to a proton or a neutron.

## References and Suggested Readings

1. Particle Physics by A. Kamal 2014.
2. Engineering Physics, by Raghuvanshi, G. S. Raghuvanshi.
3. Nuclear and particle physics by B. R. Martin 2006.
4. Nuclear Energy by Raymond L. Murray, 2000.
5. R.R. Roy and B.P. Nigam: Nuclear Physics, 1979.
6. Radiation Detectors and Measurement by Glenn F. Knoll.
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8. Introduction to Experimental Nuclear Physics by R. M. Singru.
9. Instruction Manual of G.M. Counting System.

# **UNIT-23**

## **The Characteristics of a GM Counter and Verification of Inverse Law**

### **Structure of the Unit**

- 23.1 Aim
  - 23.2 Apparatus
  - 23.3 Diagram
  - 23.4 Formula
  - 23.5 Model Graph
  - 23.6 Theory And Description
  - 23.7 Self Learning Exercise-I
  - 23.8 Procedure
  - 23.9 Observation
  - 23.10 Graph
  - 23.11 Calculations
  - 23.12. Result
  - 23.13. Discussion
  - 23.14 Precautions and Sources of Error
  - 23.15 Self Learning Exercise-II
  - 23.16 Glossary
  - 23.17 Answers to Self Learning Exercises
  - 23.18 Viva Questions
  - 23.19 Answer To Viva Questions
- References and Suggested Readings

## 23.1 Aim

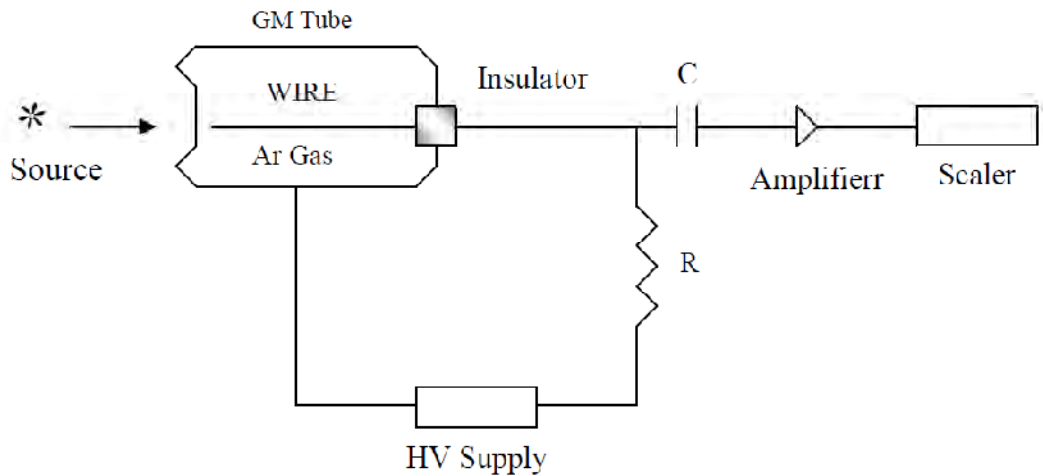
1. To determine the efficiency of a Geiger-Muller counter.
2. To investigate the relationship between absorber materials (atomic number), absorption thickness and backscattering.
3. To verify the inverse square relationship between the distance and intensity of radiation.
4. To investigate the attenuation of radiation via the absorption of beta particles.

## 23.2 Apparatus

- Set-up for ST-350 Counter
- GM Tube and stand
- shelf stand,
- serial cable, and
- a source holder
- Radioactive Source (e.g., Cs-137, Sr-90, or Co-60).

## 23.3 Diagram





## 23.4 Formula

### 1. Efficiency of a Geiger-Muller counter :

To find the disintegration rate, change from micro Curies ( $\mu\text{Ci}$ ) to disintegrations per minute (dpm). The disintegrations per minute unit is equivalent to the counts per minute from the GM tube, because each disintegration represents a particle emitted.

The conversion factor is:

$$1 \text{ Ci} = 2.22 \times 10^{12} \text{ dpm}$$

$$1 \text{ mCi} = 2.22 \times 10^6 \text{ dpm}$$

Multiply this by the activity of the source and you have the expected counts per minute of the source. We will use this procedure to find the efficiency of the GM tube, by using a fairly simple formula. You want to find the percent of the counts you observe versus the counts you expect, so you can express this as

$$\% \text{ Efficiency} = r(100)/CK$$

In this formula,  $r$  is the measured activity in dpm,  $C$  is the expected activity of the source in  $\mu\text{Ci}$ , and  $K$  is the conversion factor.

### 2. Absorption of gamma rays in materials:

When gamma rays pass through a foil a certain fraction of them is removed from the primary beam. If  $N$  is the number of gamma rays which enter a medium the number that emerge through thickness  $x$  is

$$N = N_0 e^{-\mu x}$$

where  $\mu$  is called as the absorption coefficient of the medium. We therefore have

$$\ln N = -\mu x + \ln N_0$$

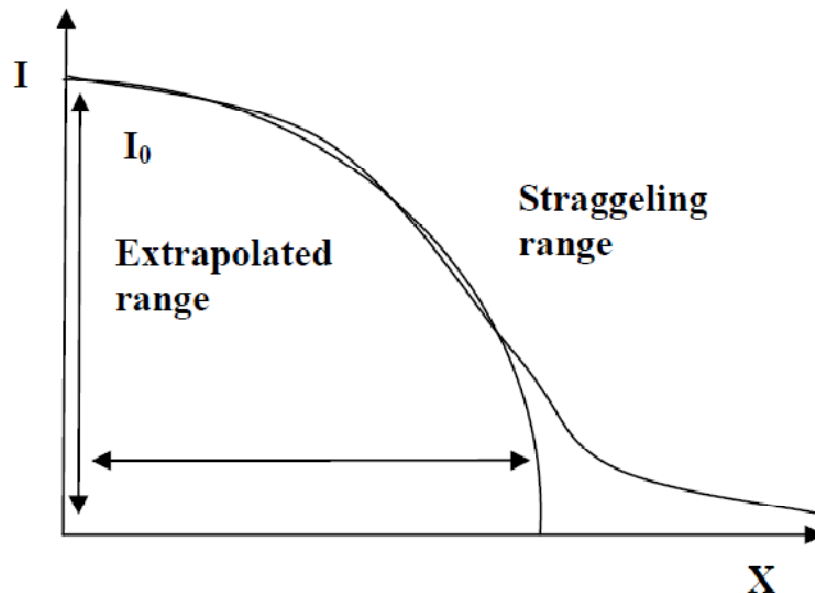
A plot of  $\ln N$  vs.  $x$  is thus a straight line where the slope gives  $\mu$ . Find  $\mu$  for Al and Cu using Cs-137 and Co-60 source.

$\mu$  is also called as linear attenuation coefficient. The absorption of gamma rays in matter can also be characterized by mass absorption coefficient ( $\mu_m$ ). Then

$$N = N_0 e^{-\mu_m \rho_s x}$$

where  $\mu_m$  is the mass attenuation coefficient ( $\text{m}^2\text{kg}^{-1}$ ) and  $\rho_s$  is the mass per unit area of the absorber.

## 23.5 Model Graph



## 23.6 Theory and Description

### Efficiency Of A Geiger-Muller Counter:

From earlier experiments, you should have learned that a GM tube does not count all the particles, which are emitted from a source (like, dead time). In addition, some of the particles do not strike the tube at all, because they are emitted uniformly in all directions from the source. In this experiment, you will calculate the efficiency of a GM tube counting system for different isotopes by comparing the measured count rate to the disintegration rate (activity) of the source.

### Absorption Of Gamma Rays In Materials:

The probability of interaction is given in terms of the total cross-section  $\sigma$  which is the summation of individual cross-sections related to the photoelectric effect, the Compton effect and the pair production and is written as

$$\sigma = \sigma_{PE} + Z\sigma_c + \sigma_{pp}$$

The factor  $Z$  in the second term embodies the assumption that all the atomic electrons contribute individually (and incoherently) to Compton scattering. For gamma ray energies above K-shell ionization energy of the atom, the Compton effect is more dominant and for the purpose of the present experiment the first and the third terms of above equation can be neglected. The cross-section  $\sigma_c$  is related to the absorption coefficient  $\mu$  by

$$\sigma_c = \frac{\mu}{n_e}$$

where  $n_e$  is the number of electrons per unit volume in the material. Using above equation find the value of  $\sigma_c$  (express it in the units of barn ( $1 \text{ barn} = 10^{-28} \text{ m}^2$ )). Compare the values obtained for Cu and Al and give your comments on the findings.

Data:

Density of Aluminum:  $2.7 \times 10^3 \text{ kg m}^{-3}$ ,  $Z = 13$ .

Density of Copper:  $8.92 \times 10^3 \text{ kg m}^{-3}$ ,  $Z = 29$ .

Avogadro Number :  $6.023 \times 10^{26}$  molecule per kg mole .



Radiation is emitted from a source in all directions. The radiation emitted within the angle subtended by the window of the GM tube is the only radiation counted. Most radiation is emitted away from the tube but it strikes matter. When it does so, the direction of its path may be deflected. This deflection is known as scattering. Most particles undergo multiple scattering passing through matter. Beta particles especially may be scattered through large angles.

Radiation scattered through approximately 180° is said to be backscattered. For Geiger counters, the fraction of radiation emitted away from the GM tube that strikes the material supporting the sample. It is deflected toward the tube window and is counted.

A beta particle entering matter undergoes a series of collisions with mostly nuclei and sometimes orbital electrons. A collision between particles does not occur in the same manner we picture

them in the macroscopic world. There are very little contact collisions, but instead the term, collision, refers to any interaction, Coulombic or otherwise. (A Coulombic interaction is an electrical attraction or repulsion that changes the path of the particle's motion.) A collision may be elastic or inelastic, but in either case the result is not only a change in direction but usually a decrease in the energy of the beta particle as well (inelastic).

In the first part of the experiment, we will investigate the effect of the size of the nucleus on backscattering since this would seem most likely effect the number of backscattered beta particles. This can be determined by using absorbers as backing materials. Even though the metal pieces you will use are called absorbers, in this experiment, they will also be used to deflect radiation upward into the GM tube. To eliminate any other effects, the absorbers will be the same thickness but different atomic number (and thus different sized nuclei). In the second part of the experiment, you will use absorbers of the same material (atomic number) and different thickness. This is a common experimental technique, fixing one variable and varying another to investigate each one individually

Normally when we think of thickness, we think of linear thickness that can be measured in a linear unit such as inches or centimeters. However, in nuclear and particle physics, thickness refers to areal thickness. This is the thickness of the absorbers in  $\text{mg}/\text{cm}^2$ . The absorptive power of a material is dependent on its

density and thickness, so the product of these two quantities is given as the total absorber thickness, or areal thickness. We determine areal thickness with the equation

$$\text{Density (mg/cm}^3\text{)} \times \text{thickness (cm)} = \text{Absorber thickness (mg/cm}^2\text{)}$$

You will determine the dependence that absorber thickness has on backscattering.

### **Inverse Square Relationship Between The Distance And Intensity Of Radiation:**

As a source is moved away from the detector, the intensity, or amount of detected radiation, decreases. You may have observed this effect in a previous experiment. If not, you have observed a similar effect in your life. The farther you move away from a friend, the harder it is to hear them. Or the farther you move away from a light source, the harder it is to see. Basically, nature provides many examples (including light, sound, and radiation) that follow an inverse square law.

What an inverse square law says is that as you double the distance between source and detector, intensity goes down by a factor of four. If you triple the distance, intensity would decrease by a factor of nine. If you quadruple the distance, the intensity would decrease by a factor of 16, and so on and so on. As a result, if you move to a distance  $d$  away from the window of the GM counter, then the intensity of radiation decreases by a factor  $1/d^2$ .

### **Absorption Of Beta Particles:**

Beta particles are electrons that are emitted from an atom when a neutron decays by the weak force. The neutron (n) becomes a proton (p), an electron ( $e^-$ ), and an anti-neutrino. When an electron originates in the nucleus, it is called a beta particle.

Unlike alpha particles which are emitted from the nucleus with the same energy (~5 MeV), beta particles are emitted with a range of energies lying between 0 MeV and the maximum energy for a given radioactive isotope. The velocity of a beta particle is dependent on its energy, and velocities range from zero to about  $2.9 \times 10^8$  m/s, nearly the speed of light. So the beta particles do not all have the same kinetic energy and thus they do not all have the same range. When the range varies over different values, this is called range straggling. It represents the

different energy losses all of the beta particles have given their different initial values. Model graph shows a typical absorption curve for beta particles, which illustrates the range straggling.

### **23.7 Self Learning Exercise-I**

- Q.1** How is Radioactivity Measured?
- Q.2** What is a Geiger-Müller Tube?
- Q.3** What is Background Radiation?
- Q.4** What is External Radiation?

### **23.8 Procedure**

#### **Efficiency Of A Geiger-Muller Counter:**

- a. Setup the Geiger counter as you have in the previous experiments. Set the Voltage of the GM tube to its optimal operating voltage, which should be around 900 Volts.
- b. Set Runs to zero and set Preset Time to 60 to measure activity in cpm.
- c. First do a run without a radioactive source to determine your background level.
- d. Next, place one of the radioactive sources (Po-210, Sr-90, Co-60) in the top shelf and begin taking data.
- e. Repeat this for each of your other two sources. Remember that the first run is a background number.
- f. (OPTIONAL) From the Preset menu, change the Preset Time to 300, and take data for all three sources again.
- g. Save your data to disk or to a data table before exiting the ST350 program.

#### **Absorption Of Gamma Rays In Materials :**

- a. Setup the Geiger counter as you have in the previous experiments. Set the Voltage of the GM tube to its optimal operating voltage, which should be around 900 Volts.
- b. From the Preset menu, set Runs to zero and set Preset Time to 60.

- c. First do a run without a radioactive source to determine your background level.
- d. Next, place the radioactive source in the second shelf from the top and take data.
- e. Place an absorber piece, or disk, in the source holder and place the source directly on top of it.
- f. Repeat this for all of the various absorbers (different materials) including the unknown. (Plastic is poly-carbonate\*\* -  $Z = 6$ , Aluminum -  $Z = 13$ , Copper -  $Z = 29$ , Tin -  $Z = 50$ , and Lead -  $Z = 82$ )
- g. Record the data to a file on disk or into a data table.

### **Thickness Dependence**

- h. Repeat 1-5 Setup the Geiger counter as you have in the previous experiments.
- i. Place the source on a different thickness of the absorber and insert into the second shelf.
- j. Repeat for at least three other absorber thicknesses and record the data to a file.

### **Inverse Square Relationship Between The Distance And Intensity Of Radiation:**

- a. Setup the Geiger counter as you have in the previous experiments. Set the voltage of the GM tube to its optimal operating voltage, which should be around 900 Volts.
- b. From the Preset menu, set Runs to zero and set Preset Time to 30.
- c. First do a run without a radioactive source to determine your background level.
- d. Next, place the radioactive source in the top shelf and begin taking data. In this position, the source is 2 cm from the GM tube's actual detector components.
- e. Move the source down one shelf each time and take another run. You should see the data accumulating in the Data window. After all ten shelves have been used, save the data onto disk or record in a data table. Remember that the first run is a background number.

### Absorption Of Beta Particles:

- a. Setup the Geiger counter as you have in previous experiments. Set the Voltage of the GM tube to its optimal operating voltage, which should be around 900 Volts.
- b. From the Preset menu, set Runs to zero and set Preset Time to 60.
- c. First do a run without a radioactive source to determine your background level.
- d. Next, place the radioactive source in the second shelf from the top and begin taking data.
- e. Place an absorber piece in the top shelf and take another run of data.
- f. Repeat this minimum of 5 more times with absorbers of increasing thickness.

## 23.9 Observation

### Efficiency of a Geiger-Muller counter

Source	Counts	Corr. Counts	Expected Counts	Efficiency

### Absorption of gamma rays in materials

Element (Z)	Counts	Corr. Counts	% backscattering


Thickness (mg/cm <sup>2</sup> )	Counts	Corr. Counts	% backscattering

**Inverse square relationship between the distance and intensity of radiation**

Counts	Corr. Counts	Distance	1/d <sup>2</sup>


**Absorption of beta particles**

<b>Counts</b>	<b>Corr. Counts</b>	<b>Thickness</b>	<b>In (counts)</b>

**23.10 Graph**

Plot the required Graph

### 23.11 Calculations

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### 23.12 Result

1. The Efficiency of a GM counter is = ----- (%)
2. The background scattering of a GM counter is = ----- (%)

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### 23.13 Discussion

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### 23.14 Precautions and Sources of Error

1. While performing an experiment with one radioactive source other sources should not be present nearby. They should be put behind the lead shield.
2. Handle the radioactive sources with care. Don't touch in bare hand to the center of samples.
3. While handling the liquid radioactive samples please use hand gloves.
4. Don't put your mobile near to the detector. It may add some counts to the signal.



## 23.15 Self Learning Exercise -II

- Q.1 What is the Most Harmful Type of External Radiation?
- Q.2 What is Internal Radiation?
- Q.3 What is the Most Harmful Type of Internal Radiation?
- Q.4 What Safety Precautions are needed when working with Radioactivity?

## 23.16 Glossary

**Radiation** : Energy emitted by atoms in a changing state. Radiation can take its form as electromagnetic waves, heat, light, X-rays, Gamma rays, or streams of alpha particles, beta particles, neutrons or protons.

**Background radiation**: The radiation of man's natural environment originating primarily from the naturally radioactive elements of the earth and from the cosmic rays. The term may also mean radiation extraneous to an experiment.

**Geiger counter**: A Geiger-Müller detector and measuring instrument. It contains a gas-filled tube which discharges electrically when ionizing radiation passes through it and a device that records the events.

**Ionizing radiation**: Radiation that is capable of producing ions either directly or indirectly.

**Scaler**: An electronic instrument for counting radiation induced pulses from radiation detectors such as a Geiger-Muller tube.

## 23.17 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-I*

**Ans.1:** Radioactivity is measured in Becquerels, symbol Bq.

1 Bq = 1 decay per second.

A Geiger-Müller tube displays the amount of radioactivity in Bq.

**Ans.2:** A Geiger-Müller tube detects the ions which are formed by radioactivity. The tube is filled with argon gas and has a very thin piece of mica at the end. It is called a mica window, and it lets all types of radioactivity penetrate through it, even alpha particles. Below is a picture of a Geiger-Müller tube.



The tube is filled with argon gas. When an electron is knocked off an argon atom, a positive ion is formed. The ion is attracted to the negative inside lining of the tube. When the ion collides with the tube it collects an electron and becomes an argon atom again.

The electron that was knocked off is attracted to the central positive wire. When the electron collides with the positive wire, the tiny amount of electricity causes a click from the loudspeaker of the counter.

The counter has a digital display of the number of clicks per second. This is the same as the number of radioactive waves or particles that have entered the tube and made ions in one second.

The Geiger-Müller tube and the counter together are often called a Geiger counter. The count rate (reading) obtained from a Geiger counter depends on the distance of the tube from the radioactive source. The closer the tube is to the source, the more radioactivity will enter it and the higher the reading will be. The reading is also affected by the background count.

**Ans.3:** Radioactive nuclei (the source of radioactivity) are all around us. Radioactive nuclei come from both natural and man made sources.

Natural radioactivity is found in the air, the ground and the sea. Everything living is radioactive and food is also radioactive. Radioactivity comes from rocks, particularly granite. Radiation also comes from space. Some comes from the Sun. Some comes from other galaxies in space as gamma ray bursts. These bursts may come from a distant star during a supernova.

Some man made radioactivity in the environment comes from nuclear weapons testing and accidents at nuclear power stations. All of this radioactivity is called the background radiation. The level of this radiation (called the background count) is low. If you switch on a Geiger

counter it will detect this background radiation and give a reading in Becquerels (Bq) for the level.

**Ans.4:** Radiation that has come from radioactive sources outside of the body is called external radiation. All three types of radioactivity are harmful and safety precautions must always be taken.

### ***Answers To Self Learning Exercise-II***

**Ans.1:**Of the three types of radioactivity, alpha particles are the least harmful form of external radiation. Alpha particles can not penetrate through the skin and so will not cause damage to the cells of internal organs of the body. This does not mean that alpha particles are safe. Alpha particles can cause cancer of the skin and in some cases this can be fatal. Both beta particles and gamma rays will penetrate through all of the tissues in a body and can cause damage to the cells of any internal organs. Beta particles are more ionising than gamma rays and will cause more damage for the same exposure and intensity. People who work with radioactivity must follow special safety precautions and wear a film badge.

**Ans.2:** Radiation which has come from radioactive sources inside of the body is called internal radiation. All three types of radioactivity are harmful and safety precautions must always be taken.

**Ans.3:**Of the three types of radioactivity, alpha particles are the most harmful form of internal radiation. Alpha particles can not penetrate through body tissue and so will be absorbed by the cells which are near to the radioactive source. This can cause damage and lead to cancer of the internal organs of the body. For example, granite rock contains radium-224 which decays to form the gas radon-220. Radon-220 is an alpha emitter and can be present in some buildings which have been built on granite. People who live in the buildings can breath in radioactive radon-220 which then continues to emit alpha particles from inside the lungs. This increases the risk of getting lung cancer.

Both beta particles and gamma rays will penetrate through all body tissues and are less likely to cause damage than alpha particles. Beta particles are more ionising than gamma rays and will cause

more damage for the same exposure and intensity. Gamma rays are the least harmful form of internal radiation. Gamma emitters with a short half-life are the most suitable for use as medical tracers.

**Ans.4:** Safety Precautions in School.

Radioactive sources that are used in school are usually very weak. They can only be used in the presence of an authorized teacher. They are kept in a sealed container except when they are being used in an experiment or demonstration. They are immediately returned to the container when the experiment or demonstration is finished. When using the radioactive source it should be

1. Handled with tongs or forceps, never with bare hands.
2. Kept at arms length, pointing away from the body.
3. Always kept as far as possible from the eyes.

Hands must be washed after the experiment and definitely before eating.

Safety Precautions in Industry.

Radioactive sources that are used in industry may be very strong and additional precautions must be followed. Fully sealed protective suits can be used which prevent any direct contact with radioactive materials. The suit can be lead lined to prevent any penetration by alpha particles or beta particles. Robots that are operated by remote control can be used to deal with very contaminated areas or high intensity gamma emitters. People who work with radioactive sources wear a film badge.

## 23.18 Viva Questions

- Q.1 Where do Beta Particles come from?
- Q.2 How can the Direction of a Beta Particle be Changed?
- Q.3 Which counter is more sensitive ionization chamber or GM Counter?
- Q.4 What is the Expression for finding the dead time of a counter?
- Q.5 What Element would be formed if Bismuth emitted a Beta Particle?
- Q.6 What are the Properties of a Beta Particle?

## 23.19 Answer To Viva Questions

**Ans.1:** Beta particles may be emitted from the nucleus during the decay of either big or small nuclei.

**Ans.2:** The negative charge on the beta particle means that it will be deflected by a magnetic field or an electric field (like the charged droplets in an inkjet printer). A beta particle will be deflected more than an alpha particle (because of its smaller mass) and in the opposite direction (because of its negative charge).

**Ans.5:** it would have a mass number 212 and atomic number 84. This element is polonium.

**Ans.6:**

1. A beta particle has a negative charge because it is an electron.

2. A beta particle has less mass than an alpha particle but more mass than a gamma ray. See the three types of radioactivity. This means that (i) beta particles have more penetrating ability than alpha particles but less penetrating ability than gamma rays.

(ii) beta particles are less ionising than alpha particles but more ionising than gamma rays.

## References and Suggested Readings

1. Particle Physics by A. Kamal 2014.
2. Engineering Physics, by Raghuvanshi, G. S. Raghuvanshi.
3. Nuclear and particle physics by B. R. Martin 2006.
4. Nuclear Energy by Raymond L. Murray, 2000.
5. R.R. Roy and B.P. Nigam: Nuclear Physics, 1979.
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7. The Atomic Nucleus by R.D. Evans.
8. Introduction to Experimental Nuclear Physics by R. M. Singru.
9. Instruction Manual of G.M. Counting System.

# UNIT-24

## Cauchy Dispersion Relation

### Structure of the Unit

- 24.1 Aim
- 24.2 Apparatus
- 24.3 Diagram
- 24.4 Formula
- 24.5 Theory and Description
- 24.6 Self Learning Exercise -I
- 24.7 Procedure
- 24.8 Observation
- 24.9 Calculations
- 24.10 Result
- 24.11 Discussion
- 24.12 Precautions and Sources of Error
- 24.13 Self Learning Exercise -II
- 24.14 Glossary
- 24.15 Answers to Self Learning Exercises
- 24.16 Viva Questions
- 24.17 Answers to Viva Questions

### References and Suggested Readings

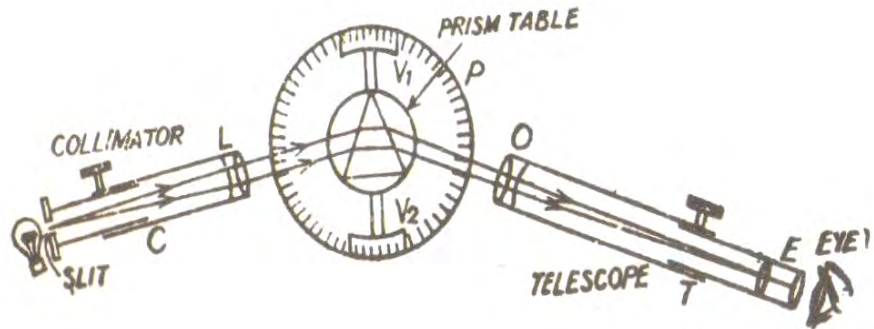
#### 24.1 Aim

To verify the Cauchy's dispersion relation and determine their constants.

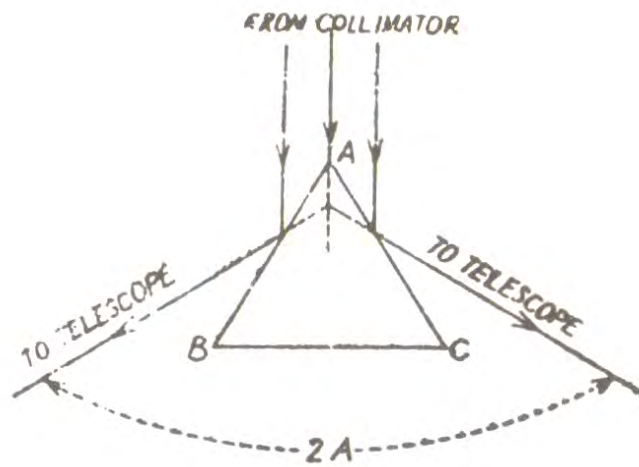
#### 24.2 Apparatus

Spectrometer, mercury lamp, prism and reading lens, spirit level.

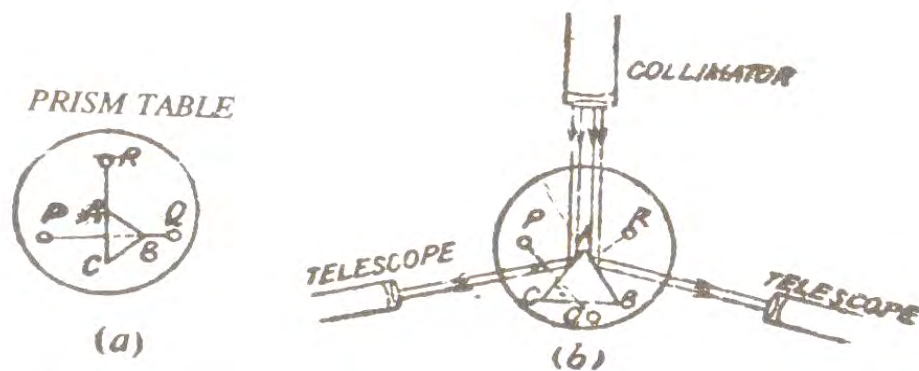
## 24.3 Diagram



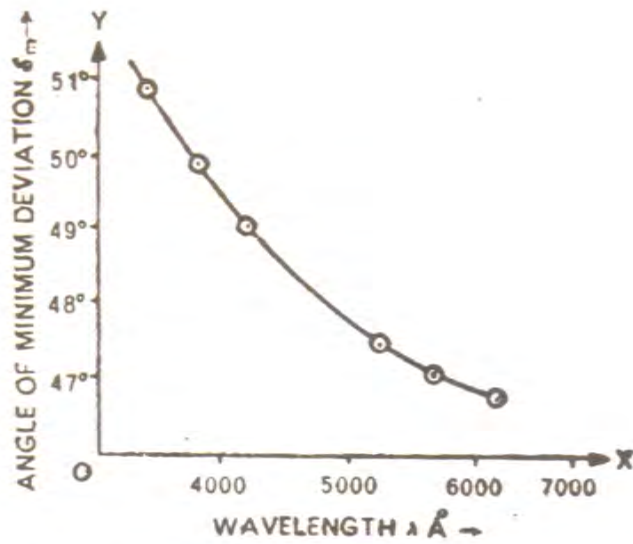
**Figure 24.1:** The Spectrometer



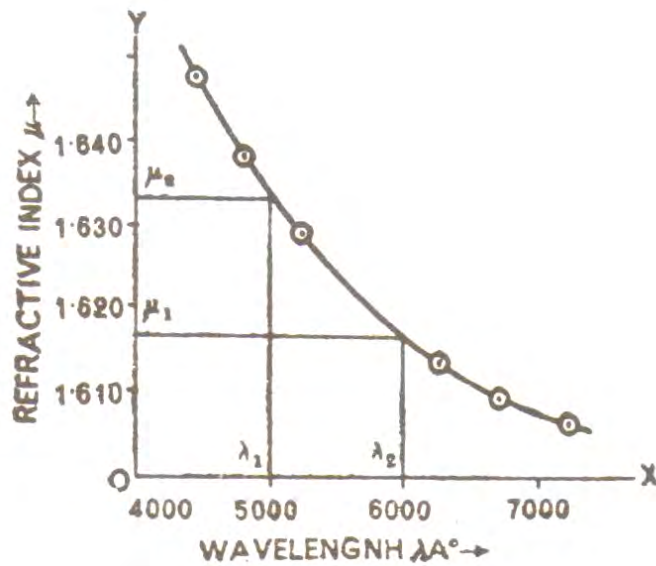
**Figure 24.2:** Measurement of angle of Prism (A).



**Figure 24.3:** Position of prism on prism table



**Figure 24.4 (a):** Verification Graph



**Figure 24.4 (b):** Verification Graph

## 24.4 Formula

Cauchy showed that the variation of the refractive index  $\mu$  with wavelength  $\lambda$  can be represented by the relation

$$\mu = A + B/\lambda^2$$

Where, A and B are two constants whose values are given by



$$B = \frac{\mu_1 - \mu_2}{\left(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\right)}$$

and

Here,  $\mu_1$  and  $\mu_2$  are the refractive indices for wavelength  $\lambda_1$  and  $\lambda_2$  respectively.

The value of  $\mu$  at a particular wavelength can be calculated by using the following formula

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where  $A$  = Angle of prism

and  $\delta_m$  = Angle of minimum deviation.

By knowing the values of  $A$  and  $B$  with the help of equations (2) and (3), the value of  $\mu$  can be calculated at any other wavelength with the help of eq. (1). The calculated value is compared with the help of eq. (1). The calculated value is compared with the experimental value. An agreement between theoretical and experimental value of  $\mu$  for certain value of  $\lambda$  verifies the Cauchy's dispersion relation.

## 24.5 Theory and Description

Dispersive power of a substance is usually denoted by the symbol  $\omega$  to define it. Consider three wavelengths  $\lambda_B, \lambda_Y$  and  $\lambda_R$  respectively in the blue.

Suppose the polychromatic light beam is passed through a prism of very small refracting angle  $A$ .

Let  $\delta_B, \delta_Y$  and  $\delta_R$  be the angle of deviation for the ray of light of wavelength  $\lambda_B, \lambda_Y$  and  $\lambda_R$ . The angular separation is  $(\delta_B - \delta_R)$  between the emergent rays of these colours. Therefore for small value of angle of prism deviations are given as-

$$\delta_B = (\mu_B - 1)A$$

$$\delta_Y = (\mu_Y - 1)A$$

$$\delta_R = (\mu_R - 1)A$$

The angular dispersion is therefore-

$$\delta_B - \delta_R = (\mu_B - 1)A - (\mu_R - 1)A = (\mu_B - \mu_R)A$$

and

$$\frac{\delta_B - \delta_R}{\delta_Y} = \frac{(\mu_B - \mu_R)A}{(\mu_Y - 1)A} = \frac{(\mu_B - \mu_R)}{(\mu_Y - 1)} = \omega$$

This is a dispersive power of a substance.

Now we adjust the prism at normal incidence. The following procedure is adopted:

- (i) Illuminate the slit of the collimator with the given source of light. Collimator and telescope are arranged in a line and the image of the slit is focussed on the vertical cross wire. The reading of the telescope is noted on the circular scale with the help of one of the vernier attached to it. Let this reading be  $\theta$ .
- (ii) The telescope is now rotated through  $90^\circ$  i.e. the new reading of the telescope is  $(\theta + 90^\circ)$ . In this position, the telescope and collimator are mutually perpendicular. Now the telescope is clamped.
- (iii) Mount the prism on the prism table and rotate the prism table so that the reflected is seen on the cross wire of the telescope. The face of the prism now makes an angle  $45^\circ$  with the collimator axis. Note down the position  $\phi$  of the prism table on the circular scale with the help of one of the verniers attached to the prism table.
- (iv) Turn the prism table from this position through  $45^\circ$  i.e. its reading is  $(\phi + 45^\circ)$ . Now the prism face is exactly normal to the direction of the incident light. The angle of incidence in this position is zero. Read both the verniers of the prism table.

### **Schuster's Method of Focusing:**

It is the method of focusing the telescope and the collimator simultaneously, when it is not possible take the spectrometer out of the lab. Place the prism coincide with the centre of the prism table and its ground surface is parallel to one of the parallel lines present of the prism table. Ideally the prism should be inscribed within the circle drawn on the prism table. Turn the prism table such that entire light from collimator falls on the one reflecting face and the image is viewed. From

the telescope on the other side. A spectrum with naked eyes and bring the telescope in front of your eyes. On rotation of prism table spectrum moves in one direction and often sometimes it stops and retraces its path. This particular position where the spectrum stops is called position of minimum deviation. Now fixed the telescope at the position which greater than that of minimum deviation position. Rotate the prism table towards the telescope (move refracting edge towards the telescope) and focus the spectrum. Now rotate the prism table in the opposite direction (move refracting edge towards the collimator) till the image is again visible through the telescope. Focus the collimator. Repeat the process three or more times so that the collimator and telescope can be adjusted for parallel light. If all the above operations have been performed correctly, you will find that spectrum will always be in sharp focus, no matter in which direction the prism is turned.

The above process is known as Schuster's method of focusing the telescope and collimator.

## 24.6 Self Learning Exercise -I

- Q.1** Give the analysis of elliptically polarized light using Babinet compensator.
- Q.2** A shift of 200 fringes is observed when movable mirror of Fabry-Perot interferometer is shifted by 0.0295 mm. Calculate the wavelength used.

## 24.7 Procedure

### 1. Adjustment of Spectrometer:

- (A) The axis of the telescope and that of the collimator must intersect the principal vertical axis of rotation of the telescope.

This adjustment is done by the manufacturer and can only be tested in the laboratory. For this purpose a pin is mounted vertically in the centre of prism table and observing its image in the telescope tube without eyepiece and for a wide slit in the collimator. If the image appears in the middle, then the adjustment is perfect otherwise the image is made in the centre by using the screws supporting the telescope and collimator till the pin appears in the middle.

- (B) Prism table should be leveled.

(i) The prism table is leveled with the help of three screws supporting the prism table. A spirit level is placed along the line joining the screws and the two screws are moved till the air bubble appears in the middle. Now place the spirit level along the line perpendicular to the previous line and adjust the third screw such that again the air bubble appears in the middle. Here one thing should be remembered that the first two screws should not be touched this time. The prism table is now leveled.

(ii) The second method which is generally used is optical leveling of the prism table. In this method the prism is placed on the prism table with its refracting edge at the centre of the prism table and one of its polished surface perpendicular to the line joining the two leveling screws P and Q as shown in figure 24.3.

Now rotate the prism table in such a way that refractive edges AB and AC face towards the collimator and light falling on the prism is usually reflected on the both side as shown in figure 24.3 (b).

The telescope is moved to one side to receive the light reflected from the face AB and the leveling screws P and Q are adjusted to obtain the image in the centre of the field of view.

Again the telescope is moved to the other side to receive the light reflected from the face AC and the remaining third screw R is adjusted till the image becomes in the central field of the view of the telescope.

The procedure is repeated till the two images from both the reflecting faces are seen in the central field of the view of the telescope. The prism table is now leveled.

(C) Telescope and collimator are adjusted for the parallel light by Schuster's Method:

First of all the prism is placed on the table and then adjusted approximately for minimum deviation position. The spectrum is now seen through the telescope. The prism table is rotated slightly away from this position towards collimator and the telescope is brought in view. The collimator is well focussed on the spectrum. Again rotate the prism table on the other side of the minimum deviation position i.e. towards telescope and focus the telescope for the best image of the spectrum. The process of focusing that collimator and telescope is

continued till the slight rotation of the prism table does not make the image to go out of focus. This means that both collimator and telescope are now individually set for parallel rays.

## **2. Measurement of The Angle of Minimum Deviation:**

(i) Place the prism so that its centre coincides with the centre of the prism table and light falls on one of the polished faces and emerges out of the other polished face, after refraction. In this position, the spectrum of light is obtained.

(ii) The spectrum is seen through the telescope and the telescope and the telescope is adjusted for minimum deviation position for a particular colour in the following way:

Set up telescope at a particular colour and rotate the prism table in one direction, of course the telescope should be moved in such way to keep the spectral line in view. By doing so a position will come where the spectral line recede in the opposite direction although the rotation of the table is continued in the same direction. The particular position where the spectral line begins to recede in opposite direction is the minimum deviation position for that colour. Note the readings of two verniers.

(iii) Remove the prism table and bring the telescope in the line of the collimator. See the slit directly through telescope and coincide the image of the vertical cross wire. Note the readings of two verniers.

(iv) The difference in minimum deviation position and direct position gives the angle of minimum deviation for that colour.

(v) The same procedure is repeated to obtain the angles of minimum deviation for other colours.

## **3. Measurement of The Angle of Prism A**

(i) Determine the least count of the spectrometer.

(ii) Place the prism on the prism table with its refracting angle  $A$  towards the collimator and with its refracting edge  $A$  at the centre as shown in the figure 24.3. in this case some of the light falling on each face will be reflected and can be received with the help of the telescope.

- (iii) The telescope is moved to one side to receive the light reflected from the face AB and the cross wires are focussed on the image of the slit. The reading of the two verniers is taken.
- (iv) The telescope is moved to other side in other side to receive the light reflected from the face AC and again the cross wires are focussed on the image of the slit. The readings of two verniers are noted.
- (v) The angle through which the telescope is moved or the difference in the two position gives twice the refracting angle A of the prism. Therefore, half of this angle gives the refracting angle of the prism.

## 24.8 Observation

Least Count of the Spectrometer:

1. Table for angle A of the prism.

S. No	Vernier	Telescope reading for reflection from first face			Telescope reading for reflection from second face			Difference $a - b$ $2A$	Mean value of $2A$	A	Mean A degree
		MS reading	VS reading	Total (a)	MS reading	VS reading	Total (b)				
1	$V_1$										
	$V_2$										
2	$V_1$										
	$V_2$										
3	$V_1$										
	$V_2$										

2. Table for the angle of minimum deviation ( $\delta_m$ ).

S. No	Colour	Vernier	Dispersed image Telescope in minimum deviation position	Telescope reading for Direct image	Difference $a - b$	Mean deviation

			MS reading	VS reading	Total (a)	MS reading	VS reading	Total (b)		$\delta_m$
1	Violet	$V_1$								
		$V_2$								
2	Blue	$V_1$								
		$V_2$								
3	Green	$V_1$								
		$V_2$								
4	Yellow	$V_1$								
		$V_2$								

## 24.9 Calculations

- Using the following formula, calculate  $\mu$  for each line

Tabulate the result like the following table.

S.NO.	Colour of line	$\delta_m$	$\mu$	$\lambda(\text{\AA})$ from the table of constants
1.	<i>Voilet</i>			4047
2.	<i>Bluish green</i>			4916
3.	<i>Green</i>			5451
4.	<i>Yellow</i>			5770
5.	<i>Red</i>			6234

$\lambda(\text{\AA})$  from the table of constants

- A graph is now plotted between  $\lambda$  (along the X axis) and  $\delta$  (along Y axis). The graph is shown below-
- Again draw a graph between  $\lambda$  (along the X axis) and  $\mu$  (along the Y axis). The graph is shown below-

From the graph of Fig.(2), we have

$$\lambda_1 = \dots \text{\AA} \text{ and } \lambda_2 = \dots \text{\AA}$$

and  $\mu_1 = \dots \dots \dots$  and  $\mu_2 = \dots \dots \dots$

$$B = \frac{\mu_1 - \mu_2}{\left(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\right)} = \dots \dots \dots \text{cm}^2$$

and

$$A = \mu_1 - \frac{B}{\lambda_1^2} = \mu_2 - \frac{B}{\lambda_2^2} = \dots \dots \dots \text{cm}^2$$

4. Using the above derived values of A and B and applying the Cauchy's formula for a particular wavelength ( say 5500 Å) we calculate the theoretical value of refractive index  $\mu$  i.e.

$$\mu = A + \frac{B}{(5500 \times 10^{-8})^2} = \dots \dots \dots$$

From the graph of given fig.(2) The value of  $\mu = \dots \dots \dots$  for wavelength 5500 Å.

**24.10 Result**

1. Fig. (1) shows the variation of angle of minimum deviation with wavelength.
2. Fig. (2) shows the variation of refractive index of the material of the prism with wavelength.
3. The value of  $\mu$  for a particular wavelength calculated by Cauchy's formula is nearly same as read on  $(\lambda - \mu)$  graph for the same wavelength. Hence cauchy's formula is verified.

**Percentage Error:**

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

**24.11 Discussion**

.....  
 .....  
 .....  
 .....



## 24.12 Precautions and Sources of Error

1. The telescope and collimator should be individually set for parallel rays.
2. Slit should be as narrow as possible.
3. While taking observations, the telescope and prism table should be clamped with the help of clamping screws.
4. Both verniers should be read.
5. The prism should be properly placed on the prism table for the measurement of angle of the prism as well as for the angle of minimum deviation.\)
6. The width of the slit should be adjusted at its optimum value.
7. The slit should be free from dust particles otherwise black horizontal lines will appear across the spectrum.
8. The mercury lamp should be placed on the axis of collimator to obtain the best intensity.

## 24.13 Self Learning Exercise -II

- Q.1** What are retarders? Give suitable diagram of wave plates.
- Q.2** How can you determine the initial phase difference between any two components of elliptical vibrations?
- Q.3** How can be obtaining the interference phenomenon.

## 24.14 Glossary

**Monochromatic light:** A light having one wavelength is known as Monochromatic light.

**Circular fringes:** Concentric circular fringes are produced with monochromatic light when the interferometer is in normal position i.e. the image  $M'_2$  of  $M_2$  in  $G_1$  is exactly parallel to  $M_1$ .

**Localized fringes:** When the mirror  $M_1$  and the virtual mirror  $M'_2$  are not exactly parallel but reflected waves are intersecting near  $M_1$ , a wedge shaped air film is formed between them. These are called as Localised fringes.

**White-light fringes:** When the surfaces  $M_1$  and  $M'_2$  are intersect then with a white light source along the line of intersection we shall get central achromatic bright fringe, known as White-light fringes.

**Optic axis:** A line passing through any one of the blunt corners and making equal angles with three faces which meet there is the direction of optic axis.

**Uniaxial Crystals:** The crystals having one direction along which the two refracted rays travel with the same velocity are called as uniaxial crystal.

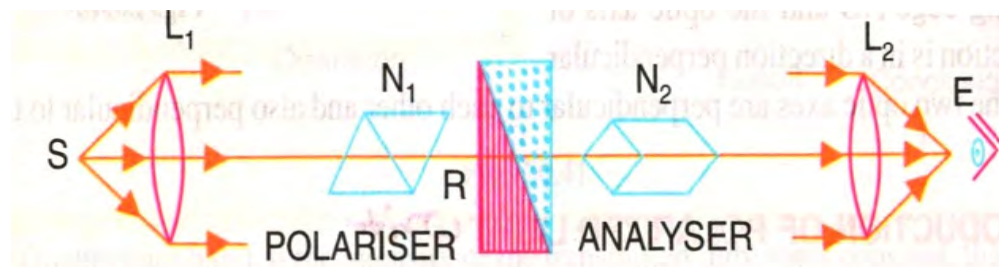
**Biaxial crystals:** In these types of crystals, there are two optic axes along which the refracted rays travel.

## 24.15 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

**Ans.1:** Analysis of Elliptically Polarizer light:

Using the compensator, we can determine the characteristics of elliptically polarized light.



Let the compensator be placed between crossed polarizer  $N_1$  and analyzer  $N_2$ , as shown in figure1. Let the transmission axis of polarizer be oriented at  $45^\circ$  with respect to the optic axis of wedge ABC of the compensator. At midpoint R the light emergent from the compensator is plane polarized in the same plane transmitted by  $N_1$  and therefore it will be extinguished by the analyzer  $N_2$ . Similarly, at distances from the midpoint for which the retardation is  $1\lambda$ ,  $2\lambda$ ,  $3\lambda$ , ..... $m\lambda$ , and the emergent light is plane polarized in the same plane as transmitted by  $N_1$  and hence will be extinguished by the analyzer. So the field of view is crossed by a series of equidistant parallel dark bands.

When we use white light source, the compensator is adjusted at that position in which the central dark band under a cross wire and the micrometer reading is noted. The micrometer screw is turned through an angle at which the compensator introduces a phase difference of  $\pi/2$  at cross wire. Then elliptically polarized light is made to be incident on the compensator. The central dark band is shifted with respect to the cross wire. The compensator is rotated through an angle  $\alpha$  in its own plane until the central dark band is on the cross wire. The axes of the incident elliptically polarized light are parallel to the optic axes of the wedge of the compensator.

**Ans.2:** Given that Shift let  $\Delta x = 0.0295\text{mm}$ , and

No. of fringes  $N=200$

We know that from the Fabry-Parot interferometer, we have

$$\lambda = \frac{2\Delta x}{N}$$

Now

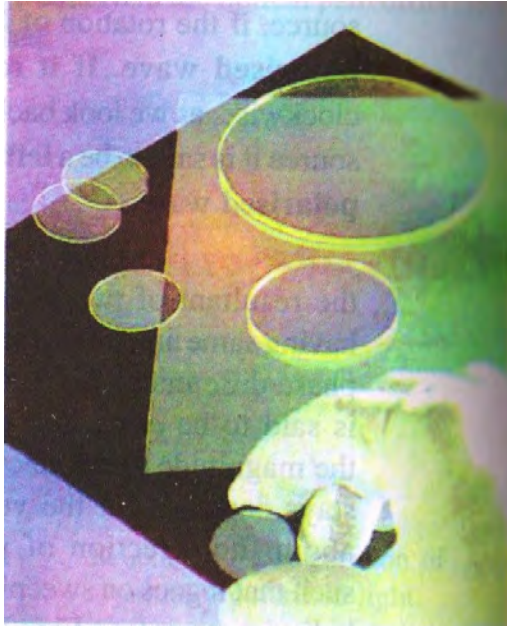
$$\lambda = \frac{2 \times 0.0295}{200}$$

or

$$\lambda = 2950\text{\AA}$$

### ***Answers to Self Learning Exercise-II***

**Ans.1:** Retarders are a class of optical elements that serve to change the state of polarization of an incident wave. The operation of a retarder is very simple. When plane polarized light is incident on a retarder, it splits the light into two plane polarized light waves and one of the wave's lags behind the other by a known amount. Upon emerging from the retarder, the two waves superpose on each other to produce a wave, which is of a different state of polarization. A quarter wave plate and a half wave plate are two important retarders. As calcite is brittle and difficult to handle in the form of thin slices, it is not generally used to make retarding plates. Retarders are frequently made from quartz but more often they are made using the biaxial crystal mica.



**Figure:** Retarders

**Ans.2:** To determine the phase difference between any two elliptical vibrations follow the points which are given below:

(1) Insert a Polaroid between the white light source and compensator obtained a system of covered band with central dark band. Rotate the compensator and analyzer to get a fine system of coloured bands. Bring the central dark band under the indicator line by working the screw. Note the reading of micrometer.

(2) Insert a quarter wave plate between the compensator and Polaroid to get elliptically polarized light. The central band shifts from the indicator line. Now, the micrometer is worked again to bring the central band under the indicator line. Note the reading find out the difference of two readings.

(3) Find the phase difference corresponding to the above difference.

**Ans.3:** The phase relation between the waves emitted by two independent light sources rapidly changes with time and therefore they can never be coherent. If the two sources are derived from a single source by some device, then any phase change occurring in one source is simultaneously accompanied by the same phase difference between the waves emerging from the two sources remains constant and the sources are coherent. The methods used for crating coherent sources of light can be divided into the following two classes:

- (a) **Amplitude splitting:** Alternately, the amplitude of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface. The two parts travel through different paths and reunite to produce interference fringes. This is known as interference due to division of amplitude.
- (b) **Wave front splitting:** one of the methods consists in dividing a light wave front, emerging from a narrow slit, by passing it through two slits closely spaced side by side. The two parts of the same wave front travel through different paths and reunite on a screen to produce fringe pattern. This is known as interference due to division of amplitude. This method is useful only with narrow sources, Young's double slit, Fresnel's double mirror, Fresnel's Biprism etc.

### 24.16 Viva Questions

- Q.1 What do you mean by refractive index?
- Q.2 Is it essential in your experiment to place the prism in the minimum deviation position? If so, why?
- Q.3 Will the angle of minimum deviation change, if the prism is immersed in water?
- Q.4 Does the angle of minimum deviation vary with the colour of light?
- Q.5 What do you mean by pure spectrum?
- Q.6 How refractive index vary with the wavelengths?
- Q.7 What is the relationship between deviation and wavelength?
- Q.8 Which source of light are you using? Is it a monochromatic source of light?
- Q.9 Can you not use a monochromatic source?
- Q.10 What is an eyepiece?
- Q.11 What is the construction of eyepiece which you used in spectrometer?
- Q.12 What is the main reason for which Ramsden's eyepiece?
- Q.13 On what factors, the dispersive power depends?
- Q.14 What is a normal spectrum?

## 24.17 Answers to Viva Questions

**Ans.1:** The ratio of the angle of incidence to the sine of angle of refraction is constant of any two media, i.e.

$$\mu = \frac{\sin i}{\sin r}$$

A constant known as refractive index.

**Ans.2:** Yes. It is essential because we obtain a bright and distinct spectrum and magnification is unity i.e. the distance of the object and image from the prism is same. The rays of different colours after refraction diverge from the same points for various colours.

**Ans.3:** Yes, the refractive index of glass in water is less than air hence angle of minimum deviation becomes less.

**Ans.4:** Yes, it is minimum for red and maximum for violet colour.

**Ans.5:** A spectrum in which there is no overlapping of colours is known as pure spectrum. Each colour occupies a separate and distinct position.

**Ans.6:** Higher is the wavelength, smaller is the refractive index.

**Ans.7:** Higher is deviation, smaller is wavelength i.e. deviation for violet colour is most but wavelength is least.

**Ans.8:** Mercury lamp. It is not a monochromatic source of light. The monochromatic source contains only one wavelength.

**Ans.9:** Yes, we can use a sodium lamp but it will give only yellow lines and not the full screen.

**Ans.10:** Eyepiece is a magnifier designed to give more perfect image than obtained by a single lens.

**Ans.11:** We use the eyepiece named as Ramsden's eyepiece, contains two plano-convex lenses each of focal length  $f$  separated by a distance equal to  $2f/3$ .

**Ans.12:** In this eyepiece, the cross wire is outside the eyepiece and hence mechanical adjustment and measurements are possible.

**Ans.13:** It depends upon

(i) Material, and

(ii) Wavelengths of colours.

**Ans.14:** A spectrum in which angular separation between two wavelengths is directly proportional to difference of the wavelengths is called a normal spectrum.

### References and Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook,
2. B. Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires.
4. Physics Practical Vol. 2 by R. K. Shukla & Anchal Srivastav.

# UNIT-25

## Electron Spin Resonance

### Structure of the Unit

- 25.1 Aim
  - 25.2 Apparatus
  - 25.3 Diagram
  - 25.4 Formula
  - 25.5 Model Graph
  - 25.6 Theory and Description
  - 25.7 Self Learning Exercise -I
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  - 25.17 Answer to Self Learning Exercises
  - 25.18 Viva Questions
  - 25.19 Answers to Viva Questions
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### 25.1 Aim

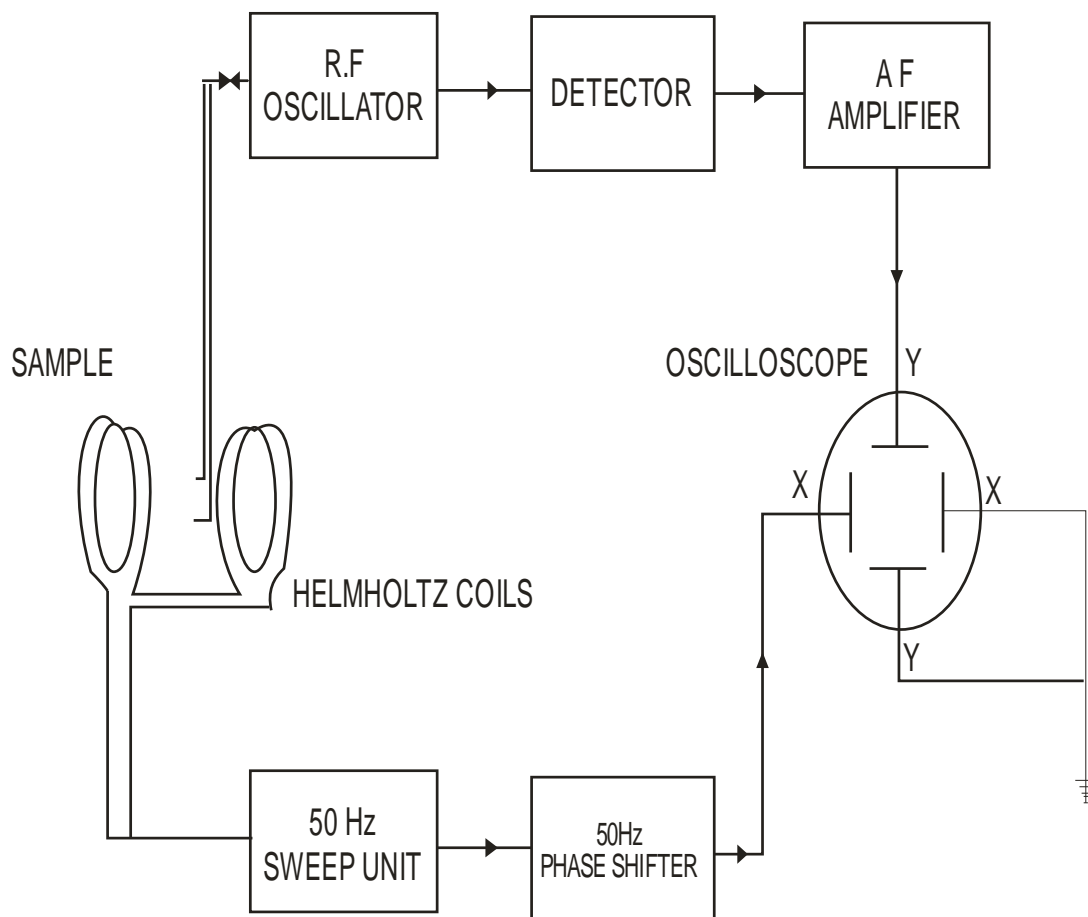
- (i) To study the ESR and determine the Lande's factor of for a given specimen.
- (ii) To determine the value of Lande's g factor using E.S.R. experiment.



## 25.2 Apparatus

ESR set-up which includes Helmholtz coil, R.F. oscillator and the test sample and in addition a cathode ray oscilloscope (CRO). DPPH connecting wire etc.

## 25.3 Diagram



**Fig. 25.1. Block Diagram of the ESR Set.**

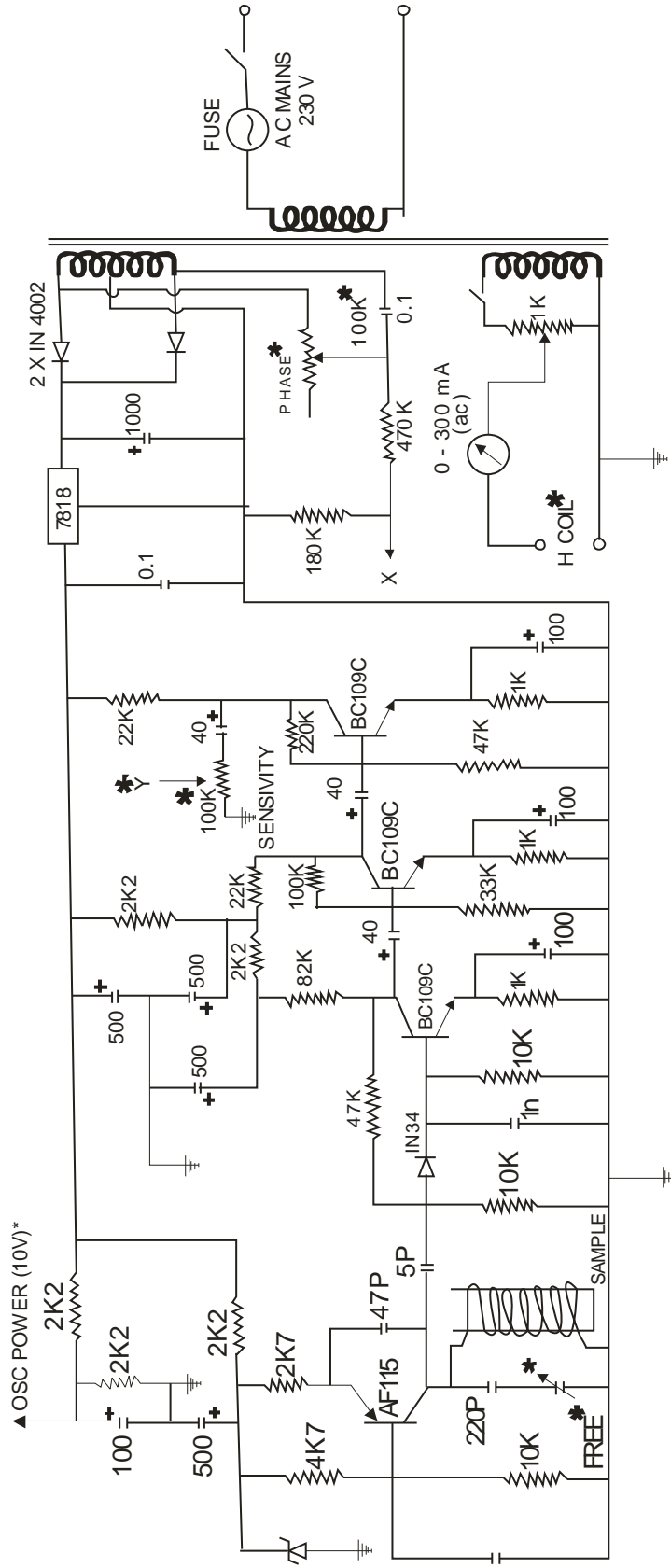


Fig. 2 : CIRCUIT DIAGRAM OF ESR SPECTROMETER

## 25.4 Formula

Magnetic field at a centre of a Helmholtz coil is

$$H_0 = \frac{32\pi n}{10\sqrt{125}a} \cdot I = KI \text{ Gauss}$$

where 'I' is the current in amperes flowing through the coils 'n' is the number of turns in each coil, 'a' is the radius of coils. Since the current measured is rms the magnetic field is also rms. The peak to peak magnetic field will be :

$$H_{pp} = 2\sqrt{2}H$$

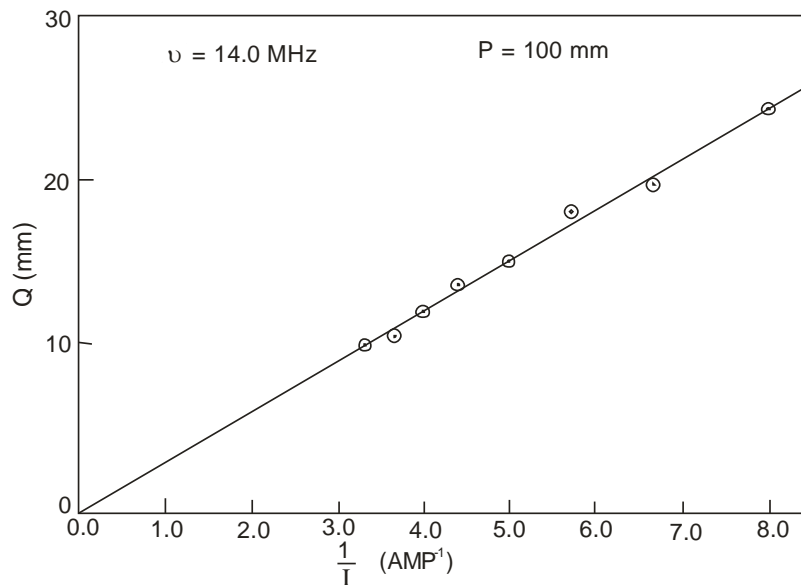
$H_{pp}$  corresponding to the total X - deflection on the oscilloscope with zero field at the centre. Hence correspond to  $H_{pp}$ , the magnetic field per mm of the X - deflection will be  $H_{pp}/P$  and hence the magnetic field for either resonance is

$$H_0 = I \times \frac{H_{pp}}{P} \times Q$$

So, for g factor,  $h\nu_1 = g\mu_0 H_0 \quad \therefore g = \frac{h\nu_1}{H_0\mu_0}$

Substitute the measured values of  $H_0$  and  $\nu_1$  universal constants  $h = 6.625 \times 10^{-27}$  erg. sec and  $\mu_0 = 9.27 \times 10^{-21}$  erg/ gauss to get the g- factor.

## 25.5 Model Graph



**Fig. 3** : Graph Between  $\frac{1}{I}$  vs. Q

## 25.6 Theory and Description

Zavoisky in 1945 performed the earliest magnetic resonance experiments in a solid. He observed strong electron spin resonance absorption in several paramagnetic salts. Applications of electron magnetic spin resonance in solid state physics are of great importance. It is a very sensitive technique and has been applied in many fields. The chief of these are:-

- (a) Paramagnetic ions in crystals,
- (b) Unpaired electron in semi-conductors and organic free radicals,
- (c) Colour centers, and radiation damage centers,
- (d) Ferro and anti-ferro magnetic materials.

### Elementary Magnetic Resonance

The fundamentals of elementary magnetic resonance may be understood in terms of simple classical concepts. Suppose a particle having a magnetic moment  $\bar{\mu}$  is placed in a uniform magnetic field of intensity  $\bar{H}_0$  (Fig. 4 a). Then the moment  $\bar{\mu}$  will precess around  $\bar{H}_0$  with an angular Larmor frequency.

$$\omega_0 = g \left( \frac{e}{2mc} \right) H_0 \quad (1)$$

$g$  being the Lande' g-factor ( $g = 1$  for pure orbital momentum and  $g = 2$  for a free electron spin). For the case of anion in a crystal, the behaviour is modified by the environment and the  $g$ - factor may differ from the Lande'  $g$ - factor. This effective  $g$ - factor is known as the spectroscopic splitting factor.

We now introduce an additional weak magnetic field oriented in the  $xy$  plane and rotating about the  $z$  axis (in the same direction as the "Larmor precessing") with an angular frequency  $\omega_1$ . If the frequency  $\omega_1$  is different from  $\omega_0$  the angle between the field  $\bar{H}_1$  and the magnetic moment  $\bar{\mu}$  will continuously change so that their interaction will average out to zero. If, however,  $\omega_1 = \omega_0$  the angle between  $\bar{\mu}$  and  $\bar{H}_1$  is maintained and net interaction is effective (Fig. 4 b). If we look at the system in a reference frame that is rotating about the  $z$  axis with the angular velocity  $\omega_0$  then the spin will appear to make an angle  $\psi = 90 - \theta$  with  $\bar{H}_1$ , and

according to the previous argument will start to precess (in the rotating frame) about  $\overline{H}_1$ . This corresponds to a "nutation" and a consequent change of the angle which implies a change in the potential energy of the particle in the magnetic field. The change in  $\theta$  is the classical analogy to a transition between sublevels with different  $m$ . We see that such transitions may take place only if the rotating field has an angular frequency  $\omega_1 = \omega_0$ .

Let us proceed to consider the quantum picture of elementary magnetic resonance. Suppose that the intrinsic angular momentum of the electron  $\overline{S}$  couples with the orbital angular momentum of electron  $\overline{L}$  to give a resultant  $\overline{J}$ . We know, that  $J + 1$  magnetic sublevels labeled by the magnetic field  $\overline{H}_0$  by equal energy difference,

$$\Delta E = g \mu_0 H_0$$

between adjacent sublevels, where  $\mu_0$  is the Bohr magneton and  $g$  is the Lande'  $g$  factor or  $g$ - factor whose correct quantum mechanical value is

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Now, if the particle is subjected to a perturbation by an alternating magnetic field with a frequency  $\nu_1$  such that the quantum  $h\nu_1$  is exactly the same as the difference between the levels.  $\Delta E$  and if the direction of the alternating field is perpendicular to the direction of the static magnetic field, then there will be induced transitions between neighbouring sublevels according to the selection rules  $\Delta m = \pm 1$  for magnetic dipolar radiation.

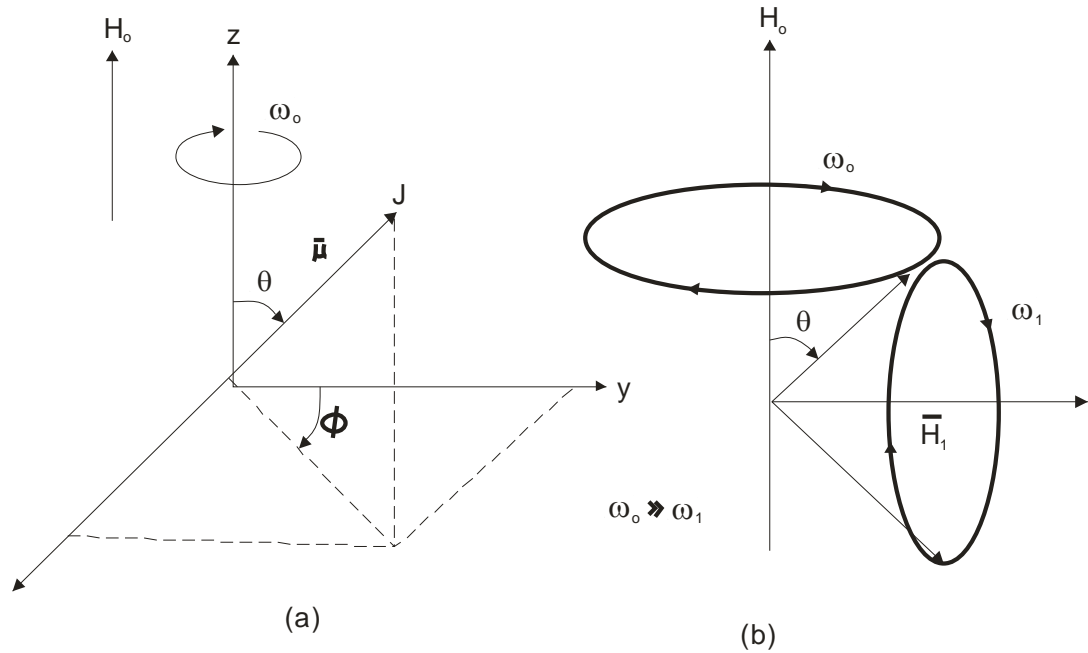
Therefore, the condition for resonance is

$$\Delta E = g \mu_0 H_0 = h\nu_0 = h\nu_1 \quad (2)$$

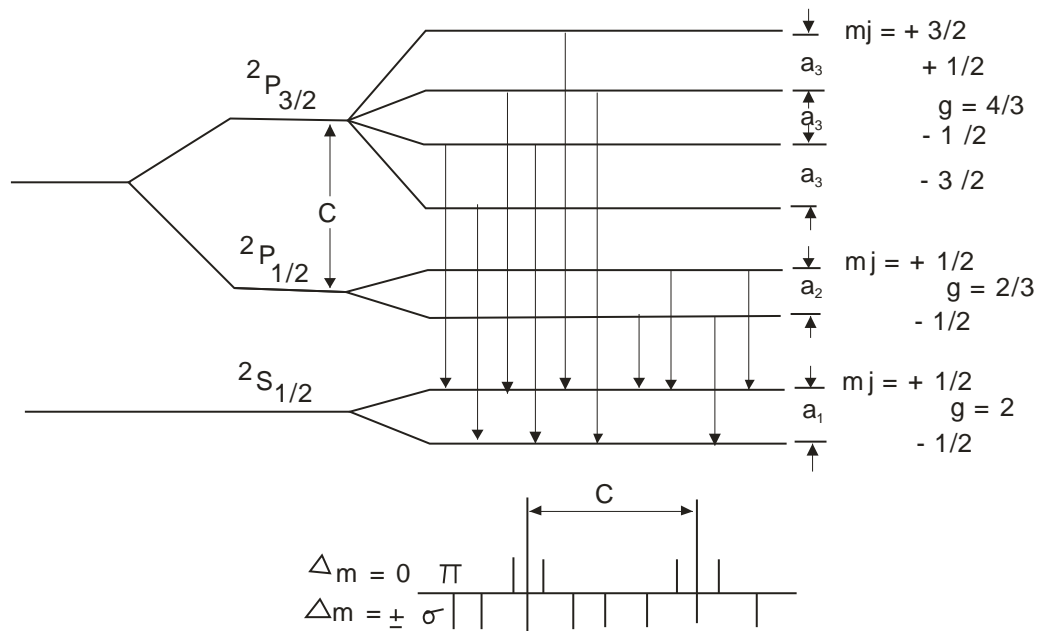
Where  $\nu_1$  is the resonance frequency in cycles/sec. This requirement is identical with the classical condition  $\omega_1 = \omega_0$ .

In atomic spectroscopy, we do not observe the transitions between sublevels with different  $m$  (labeled and selection rules  $\Delta L = \pm 1$ . Instead the splitting of a level is observed through small change in frequency of the radiation emitted in the transition between widely distant levels (Fig. 5). It is clear that, if we could directly measure the frequency corresponding to a transition between the sublevels of the

same state, a much more precise knowledge of the energy splitting would be obtained.



**Fig. 4: Precession of a magnetic moment  $\bar{\mu}$  when placed in a magnetic field  $\bar{H}_0$**



**Fig. 5 : Energy levels of a single valence electron atom showing a P state and an S state.**

Due to the fine structure, the P state is split into a doublet with  $j = 2/3$  and  $j = 1/2$ . Further, under the influence of an external magnetic field each of the three levels is split into sublevels as shown in the figure where account has been taken of the magnetic moment of the electron. The magnetic quantum number  $m_i$  for each sublevel is also shown as is the g factor for each level. The arrows indicate the allowed transitions between the initial and final states, and the structure of the line is shown in the lower part of the figure.

### Electron Spin Resonance in Solids

Let us proceed from the treatment of an isolated magnetic particle to a macroscopic body. The behavior of a paramagnetic substance in a magnetic field will depend on the interaction of the particles with one another and with the diamagnetic particles. There are mainly two types of interactions.

(a) **Spin - Spin** In which the spin interacts with a neighboring spin but the total energy of the spin system remains constant.

(b) **Spin- Lattice** in which the electron spin interact with entire solid or liquid, transforming energy from the spin system to the lattice which act as a thermal reservoir. As a matter of fact it's the spin-lattice interaction that makes possible the observation of energy absorption from the radio-frequency field when the resonance frequency is reached.

To understand this last statement, consider a paramagnetic substance in a magnetic field  $\overline{H}_0$  and say the equilibrium state has been reached. The population of individual energy levels will be determined by the Boltzmann distribution  $e^{-g\mu_0 H_0 m/kT}$  where  $m$  is the magnetic quantum number. It can be seen that the population of the lower energy levels are greater than those of the upper levels and, therefore when a periodic magnetic field with a resonance frequency is switched on; the number of induced radiation events will be more and as a result the substance will absorb energy from the radio-frequency field. Thus, two opposing processes take place in ESR. The radio frequency field tends to equalize the population of various levels and the spin lattice interaction tends to restore the Boltzmann distribution by conversion of the energy absorbed from the radio-frequency field into heat.

## Relaxation and Saturation

In the preceding para, we see the mechanism through which the electron return from an excited state to the ground state or relax back to the ground state. This process in the field of magnetic resonance's (ESR, NMR, NQR etc.) is known as relaxation and the time taken by the process is called the relaxation time. This complete process may be considered as two state processes (provided the spin-spin interactions are much stronger than the spin-lattice interaction). First, the energy is absorbed from the radio frequency magnetic field and the equilibrium is established inside the 'spin system'. The time taken by this process is known as the spin-spin relaxation time and is a measure of the rate at which magnetic energy can be distributed within the spin system though total energy is conserved. Secondly, an exchange of energy occurs between the spin system and the lattice. The time taken is known as the spin lattice relaxation time and is a measure of the rate of transfer of energy from the spin system to the lattice.

In optical spectroscopy of the relaxation time is usually very short ( $\sim 10^{-8}$  Sec) so that the relaxation time does not impede the absorption rate. In radio frequency, on the other hand, typical relaxation times are in milliseconds or longer and the spins do not have time to relax if the energy is supplied at a faster rate. This situation is called the 'Saturation State'. In other words, no additional energy is absorbed, if the radio-frequency field power is increased beyond certain level.

### Line Width and the Spin-Spin and Spin-Lattice Interactions

The effect of the spin-spin interaction is to slightly shift the exact position of energy level of any individual spin in the external field. This energy shift clearly depends on the relative orientation and distance of the spin and thus is different for each spin, resulting in apparent broadening of the energy level. Another way of thinking of the spin-spin interaction is that one electron spin produce a local magnetic field at the position of another spin. Thus, the width of absorption line due to spin-spin interaction may be estimated as  $\frac{1}{T}$ , where T' is spin-spin relaxation time.

If the spin-lattice interactions are not weak the spin lattice relaxation time T will also be introduced. Let us consider the probability of a transition of an individual paramagnetic particle from one magnetic level to another under the



influence of thermal motion. If the probability per second equal  $A$ ,  $T \sim \frac{1}{A}$  and the absorption line width would be of the order of  $\frac{1}{T}$ . In general case, however, the absorption line width may be estimated as  $\frac{1}{T} + \frac{1}{T}$ .

Thus, we see that from the width of absorption line it is possible to measure the relaxation time. In fact most of the research in this field involves the study of relaxation phenomena which in turn provide information about internal interactions in solid and liquids.

The position and number of lines of paramagnetic resonance absorption also depend on the internal interactions.

### **Electron Paramagnetic Resonance and Spectroscopy**

Paramagnetic resonance is an integral part of spectroscopy, as it provides a means for determining the position of the energy levels of magnetic particles. It is interesting to consider the peculiar features of Paramagnetic resonance in comparison with optical spectroscopy.

1. Let us first note that the frequencies used in magnetic resonance experiments range from  $10^9$  to  $10^{11}$  cps. These frequencies situated below the limits of the infrared part of the spectrum, allow highly accurate investigation of energy level splitting so small that they are inaccessible or almost inaccessible by optical methods.
2. The probability of spontaneous transition in the radio-frequency regions is very small, since this probability is proportional to  $\nu^3$ . Therefore, in paramagnetic resonance studies one is forced to deal only with induced absorption and emission.
3. While in the great majority of cases optical spectra arise from electric dipole transitions between energy levels, the lines of paramagnetic resonance absorption arise exclusively from magnetic dipole transitions. Consequently, the Einstein Coefficients for induced absorption and emission will, in the case of paramagnetic resonance, be smaller by roughly four orders of magnitude.
4. As a result, the paramagnetic resonance effect is exceedingly small; if it can be observed at all is due to the high sensitivity of electronic methods of detection

and the enormous number of photons coming into play (1m W corresponds to  $n \cong 10^{20}$  photons per sec at a frequency of  $10^{10}$  cps).

5. In the optical frequency region the line width is always very small in comparison with the fundamental frequency. In paramagnetic resonance the relation between these quantities becomes quite different, since the interactions causing a broadening of the lines can be of the same order of magnitude as the energy splitting which determines the resonance frequency. Because of this the width of paramagnetic resonance lines is often comparable to the fundamental frequency and can be measured with great accuracy. This opens up wide possibilities for investigation of different types of interactions in paramagnetic substances by means of analysis of the shape and width of a paramagnetic resonance line and of the character of its dependence upon various factors.
6. The most important factors determining the line width are magnetic dipole interactions, exchange forces, local electrical fields created by neighboring magnetic particles and finally, thermal motion; the natural line widths of radio-frequency spectra are completely negligible.
7. In contrast with optical experiments, in radio-frequency spectroscopy it is customary to use radiation which is so monochromatic that the generated band of frequencies is incomparably narrower than the absorption line width.
8. Paramagnetic resonance spectra are not studied by varying the frequency of the incident radiation, but by varying the characteristic frequencies of the absorbing systems. This is achieved by varying the static magnetic field.

### **Description of ESR Spectrometer:-**

A block diagram of the ESR spectrometer is given in fig. 1.

**1. Basic Circuit:** - The first stage of the ESR circuit consists of a critically radio frequency oscillator having a frequency range of approximately 10-17 MHz. A marginal oscillator here so that the slightly increase in the load decreases the amplified of oscillation to an appreciable extent. The sample is kept inside the tank coil of the oscillator, which in turn, is placed in the 50HZ magnetic field generated by Helmholtz coils. At resonance i.e. when the frequency of oscillation equal to the Larmor's frequency of the sample, the oscillator amplitude registers a dip due to

the absorption of power by the sample.

**2. Phase shifter:** - In order to make it possible to use an ordinary displaying type oscilloscope instead of measuring oscilloscope which preserves the phase between X & Y plate signals, a phase shifter is provided. This can compensate the phase difference which is introduced in the amplification stage of the ordinary oscilloscope.

**3. 50 Hz sweep unit:-** For modulation with a low frequency magnetic field, a 50 Hz current flows through the Helmholtz coils. As the resonance in this frequency range occurs at low magnetic field, no static D.C. magnetic field is required.

**4. Power Supplies:-**

(a) **D. C. Power Supply:** - The E.S.R. circuit requires a highly stabilized almost ripple free voltage, it is obtained by using an integrated circuit regulator.

(b) **Helmholtz coils power supply:-** The Helmholtz coil power supply consists of a step down transformer separate winding on the mains transformer, potentiometer (12-15W) and a moving coil rectifier type meter. The output is taken from the two terminals provided on the panel.

**5. Helmholtz coils:-** There are two coils exactly and parallel to each other so connected that current passes through them in the same direction. The two coils increase the uniformity of the field near the centre.

In the centre of the coils, an attachment is provided to keep the sample in place and to minimize shocks and vibrations.

Number of turns = 500 in each coil

Diameter of the windings = 15.4 cm

Separation of the coil = 7.7 cm.

**6. Test Sample:** - A test sample, Di phenyl picryl hydrazyl (DPPH) is placed in a plastic tube, which itself is in the induction coils. This increases the filling factor to the maximum. DPPH is a free radical and widely used as a standard for ESR measurements.

**7. R. F. Oscillator:** - It is a transistorized radiofrequency oscillator suitable for the determination of resonance frequency. The power to it is provided from the ESR spectrometer through a cable.

Frequency range = 9.5 MHz - 18.0 MHz

Accuracy = Better than 0.5%

8. **Oscilloscope:** - An oscilloscope is not supplied with the spectrometer it is presumed that the worker is already familiar with the control knobs and functioning of the oscilloscope in question.

Screen diameter = 12.5 cm

Vertical amplifier sensitivity = 50 mV/cm

### 25.7 Self Learning Exercise -I

- Q.1 What is the mass of electron?
- Q.2 What is the charge of electron?
- Q.3 State Pauli exclusive Principle?
- Q.4 What do you mean by resonant frequency?
- Q.5 What is the value of spin quantum number?

### 25.8 Procedure

1. Increase the horizontal sensitivity of the oscilloscope to the maximum within the linear range.
2. Obtain the best possible resonance peaks by varying the frequency, detection level and vertical sensitivity of the oscilloscope keeping the current at 150 mA (say).
3. Keeping the frequency fixed but varies the current flowing through the coils and measures the corresponding horizontal separation between the peaks ( $2Q$ ) after adjusting the phase. Take five to six sets of observation.
4. Draw a graph in  $1/T$  Vs  $Q$  which should be straight line. Calculate the  $g$ -factor using the  $QI$  value from the graph.
5. Repeat the experiment with different frequency.

### 25.9 Observation

Maximum deflection on X-plate  $P = \dots\dots\dots$

**Observation Table:-**

The frequency of RF oscillator  $\nu_0 = \dots\dots\dots$  MHz

S.No.	Current (mA)	$\frac{1}{I} \times \text{Amp}^{-1}$	Distance between peaks $2\theta$ (mm)	$\theta$ (mm)

**Observation table - 2 :-**

The frequency of RF oscillator  $\nu_0 = \dots\dots\dots$  MHz

S.No.	Current (mA)	$\frac{1}{I} \times \text{Amp}^{-1}$	Distance between peaks $2\theta$ (mm)	$\theta$ (mm)


**Observation Table 3 :-**

The frequency of RF oscillator  $\nu_0 = \dots\dots\dots$  MHz

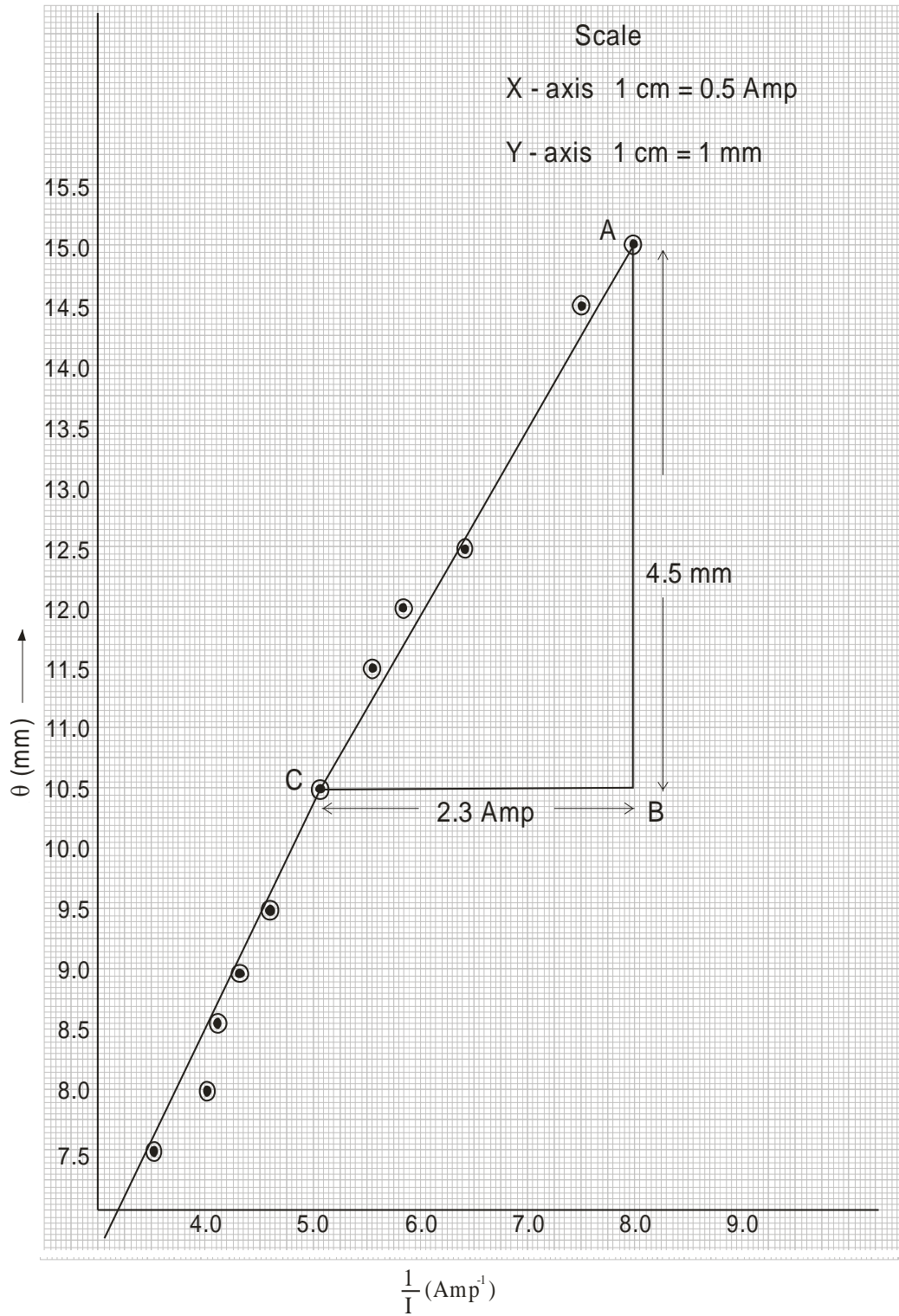
S.No.	Current (mA)	$\frac{1}{I} \times \text{Amp}^{-1}$	Distance between peaks $2\theta$ (mm)	$\theta$ (mm)

**25.10 Graph**

$\frac{1}{I} \text{ Amp}^{-1}$										
$\theta$ (mm)										

Scale On x - axis 1 cm = ..... Amp-1

On Y - axis 1 cm = ..... mm



## 25.11 Calculations

For  $\nu_0 = \dots\dots\dots$  MHz

$P = \dots\dots\dots$  mm

$$H_0 = 4.126 \left( \frac{\Delta\theta}{\Delta\left(\frac{1}{I}\right)} \right)$$

From graph of Q with 1/I slope of the above curve

$$\frac{\Delta\theta}{\Delta\left(\frac{1}{I}\right)} = \frac{dy}{dx} = \dots\dots\dots$$

$H_0 = \dots\dots\dots$

So g factor,

$$g = \frac{h\nu_0}{H_0\mu_0} \text{ where } h = 6.62 \times 10^{-27} \text{ erg. sec.}, \mu_0 = 9.27 \times 10^{-21}$$

$g = \dots\dots\dots$

Hence the mean value of g is

$$g = \frac{g_1 + g_2 + g_3}{3} = \dots\dots\dots$$

## 25.12 Result

Using ESR spectrometer set-up we obtain the value of

Lande's g factor =  $\dots\dots\dots$

## 25.13 Discussion

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## 25.14 Precautions and Sources of Error

1. Experiment should be set up at a quite place free from mechanical and electrical disturbances.
2. Y - output from the ESR - spectrometer should be taken through shielded cable to minimize external pick-ups.
3. X and Y plate's sensitivities of the Oscilloscope should be adjusted such that they should be in the linear range only.
4. Since the Helmholtz current is unstabilised, care should be taken that it should remain constant during the observation.
5. High currents (~200 mA) should not be allowed to flow through the Helmholtz coils for an extended period. This will unnecessary heat the coils which may get damaged after some time.
6. If the peaks do not coincide on the x-scale, check the sinusoidal wave form of the mains voltage which may be distorted due to over loading of the main line due to other heavy gadgets working on the same line.

## 25.15 Self Learning Exercise-II

- Q.1 What do you mean by resonance?
- Q.2 What is resonant frequency?
- Q.3 On what factors does the resonance in a circuit depend?
- Q.4 Why is the value of current in a series resonance maximum?
- Q.5 What do you understand by the resonance curve?

## 25.16 Glossary

**Spin:** to rotate rapidly.

**Electron spins resonance:** A technique for investigating paramagnetic substances by subjecting them to high frequency radiation in a strong magnetic field. Changes in the spin of unpaired electrons cause radiation to be absorbed at certain frequencies.

## 25.17 Answer to Self Learning Exercises

### *Answer to Self Learning Exercise-I*

**Ans.1:**  $9.1 \times 10^{-31}$  Kg.

**Ans.2:**  $1.6 \times 10^{-19}$  coulomb.

**Ans.3:** No two electrons have same quantum number i.e. First three quantum numbers are same but fourth quantum number (spin quantum number) are not same because the value of spin quantum number is  $\pm 1/2$ .

**Ans.4:** If the frequency of applied e.m.f. becomes equal to the natural frequency of the circuit then the current of large amplitude flows. Such a circuit is known as a resonant circuit.

**Ans.5:**  $\pm 1/2$

### *Answer to Self Learning Exercise-II*

**Ans.1:** When the impedance of a. c. circuit becomes purely resistive, i.e. the current and the voltage become in the same phase then this state of the circuit is known as the state of resonance.

**Ans.2:** The frequency of applied e.m.f. or the natural frequency of the circuit at resonance is known as resonant frequency.

**Ans.3:** It depends upon the inductance  $L$ , capacitance  $C$  and applied frequency  $\omega$ .

**Ans.4:** In the state of series resonance the impedance of the circuit becomes minimum, hence the current becomes maximum.

**Ans.5:** The curves representing the frequency of applied e.m.f. and the variation in corresponding current is known as the resonance curve.

## 25.18 Viva Questions

**Q.1** What is meant by the sharpness of resonance?

**Q.2** How does the sharpness of resonance depend upon the resistance?

**Q.3** What is magnetic effect of current?

**Q.4** What is induced magnetic field?

**Q.5** What is the unit of intensity of induced magnetic field?

- Q.6** Which law is used for finding intensity of induced magnetic field?
- Q.7** What is the value of magnetic moment of a current carrying coil?
- Q.8** How does a current - carrying small coil behave?
- Q.9** How is uniform magnetic field produced?
- Q.10** What is the direction of the magnetic field at the centre of this coil?
- Q.11** What is electron spin resonance?
- Q.12** What is cathode ray oscilloscope?
- Q.13** What is the basis component of a CRO?
- Q.14** What is the function of probe in CRO?
- Q.15** How many types of probe used in CRO?
- Q.16** Which device is used for the source of emission of electrons in a CRT?
- Q.17** What is the function of electron gun assembly used in CRT?
- Q.18** What is the function of electron gun used in CRT?
- Q.19** What is the function of aqua age in a CRO?
- Q.20** In the study of this experiment, where do you expect the magnetic field to be maximum?

### 25.19 Answers to Viva Questions

**Ans.1:** In the state of resonance if a large change in the value of current takes place for a small change in frequency then it is known as sharp resonance.

**Ans.2:** Increase in resistance decreases the sharpness of resonance.

**Ans.3:** When current is passed through any conductor magnetic field is produced around it. This effect is called as magnetic effect of current.

**Ans.4:** Magnetic field produced around the current carrying conductor is called as induced magnetic field.

**Ans.5:** Unit of  $\overline{B}$

MKS system - Weber/m<sup>2</sup> or Tesla.

CGS system - Maxwell/ cm<sup>2</sup> or Gauss

**Ans.6:** Intensity of induced magnetic field is given by Biot-Savart's law and Ampere's law.

**Ans.7:** Magnetic moment of a current carrying coil is equal to the product of amount of current passing through the coil and effective area of the coil.

$$M = IA = I \pi r^2$$

where r is the radius of the coil.

If the coil has n turns then

$$M = I n \pi r^2$$

**Ans.8:** Current - carrying small coil behaves like a short bar magnet.

**Ans.9:** Uniform magnetic field is produced with the help of a pair of identical coils called Helmholtz's coil.

**Ans.10:** The direction of the magnetic field at the centre of the coil is normal to its plane.

**Ans.11:** Electron spin resonance is the flipping back and forth between two spin directions of electrons in a magnetic field when electromagnetic radiation of the proper frequency is applied.

**Ans.12:** Cathode ray oscilloscope is a instrument used for display, measurement and analysis of waveforms and other phenomenon in electrical and electronic circuit.

**Ans.13:** CRO Circuit consists of following components:

1. Vertical deflection system
2. Horizontal deflection system
3. Synchronization system
4. Blanking circuit
5. Intensity modulation
6. Positioning control
7. Focus control
8. Cathode ray tube
9. Calibration circuit

**Ans.14:** The probe performs the very important function of connecting the test circuit to oscilloscope without altering, loading or otherwise disturbing the test circuit.

**Ans.15:** There are three types of probe used in CRO:

1. Direct Probe
2. Isolation Probe
3. Detector Probe

**Ans.16:** A barium and strontium oxide coated cathode is used for the source of emission of electrons in a CRT.

**Ans.17:** The electron gun assembly produces a sharply focused beam of electrons which are accelerated to high velocity. This focused beam of electrons strikes the fluorescent screen with sufficient energy to cause a luminous spot on the screen.

**Ans.18:** The source of focused and accelerated electron beam is the electron gun. The electron gun emits electrons and forms them into a beam consist of a heater, cathode, a grids, a pre - accelerating anode, a focusing anode and an accelerating anode.

**Ans.19:** An aqudage is used in a CRO to collect secondary emission electron.

**Ans.20:** The field due to the current is maximum, at the centre of the coil.

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# UNIT-26

## Heat Capacity

### Structure of the Unit

- 26.1 Aim
  - 26.2 Apparatus
  - 26.3 Diagram
  - 26.4 Formula
  - 26.5 Theory And Description
  - 26.6 Self Learning Exercise - 1
  - 26.7 Procedure
  - 26.8 Observation
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  - 26.12 Precautions And Sources Of Error
  - 26.13 Self Learning Exercise-2
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  - 26.15 Answer To Self Learning Exercises
  - 26.16 Viva Questions
  - 26.17 Answers To Viva Questions
- References And Suggested Readings

### 26.1 Aim

- (i) Determine the heat capacity of a given specimen at temperature  $T$  using the value of  $W$ . Calculate by standard sample at two different temperatures.
- (ii) To determine the heat capacity of test sample.

## 26.2 Apparatus

Sample in cylindrical form, Thermo couple, and standard sample (Aluminum), Test Sample (Brass), Stopwatch, power supply, connecting wires.

## 26.3 Diagram

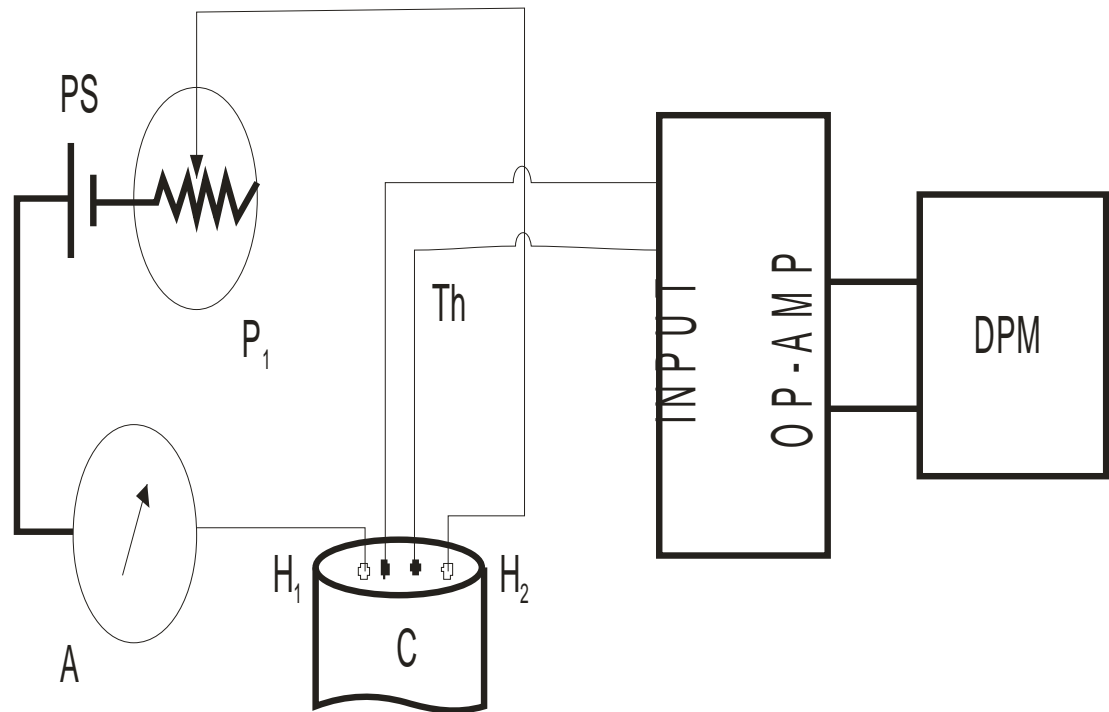


Fig. 26.1: Experimental Setup of Heat capacity.

## 26.4 Formula

If  $I$  is the current flowing through the heater of resistance  $R$  for a time  $t$  sec. then

$$I^2 R t = (W + mS) (T_2 - T_1)$$

Where  $I$  = Current flowing through the calorimeter heater.

$R$  = Resistance of the heater

$W$  = Water equivalent of Calorimeter & its contents

$m$  = Mass of the sample.

$S$  = Specific heat

$$(T_2 - T_1) = \text{Temperature Change}$$

## 26.5 Theory and Description

Heat capacity is one of the important thermal properties of solids. The kit is designed to determine the heat capacity of solids like metals, alloys and polymeric materials taken in the form of cylindrical rods of size (Length = 3 to 4 cm and diameter  $\approx$  6 mm). The experimental set up is most elegant and accurate. The important feature of the set up is that the temperature change of the specimen kept inside the silver calorimeter is detected using an iron-constant a uni-junction thermocouple. The resulting thermo-emf is amplified and displayed on a digital panel meter using suitable electronic circuitry. A built in power supply (for calorimeter heater) in series with the current meter and potentiometer with provision for connecting the calorimeter heater leads is provided in the kit. This set up will be useful for students of PG course in Physics Chemistry, Polymer Science, Materials Science and Engineering Physics.

The specimen is taken in the form of a cylindrical rod (Length 3.4 cm. diameter 6 mm) with 1 mm diameter and 3 mm depth hole at one end along the axis. The specimen is inserted into the calorimeter. Al-Cr thermocouple (Uni-junction thermocouple) is inserted into the specimen hole provided to detect the temperature of the sample.

By connecting the heater leads of the silver calorimeter to the terminals  $H_1$  &  $H_2$  provided on the front panel of the Heat Capacity Kit, adjust the current control potentiometer  $P_1$ , so that a current of 300 mA (or any desired value) may be passed through the heater. The thermocouple output leads are connected to the INPUT terminals provided on the front panel of the Heat Capacity Kit.

If  $I$  is the current flowing through the heater of resistance  $R$  for a time  $t$  sec. then,

$$I^2 R t = (W + mS) (T_2 - T_1)$$

Where  $I$  is the current flowing through the calorimeter heater,  $R$  is the resistance of the heater, where  $W$  is the water equivalent of calorimeter and its contents,  $m$  is the mass of the sample and  $S$  is its specific heat  $(T_2 - T_1)$  is the temperature change.

### Experimental Details:-

The set up consists of the following parts:



- (a) Amplifier with Digital display assembly and power supply (For calorie meter heater) in series with current meter (A) and potentiometer  $P_1$  with provision for connecting calorimeter heater leads.
- (b) Silver Calorimeter (C) Fig. and sample (standard sample - silver. Test Sample- Aluminum, Brass)
- (c) 500 mA current meter (A) - Provided in the Heat capacity kit.
- (d) Stop watch 0.1 sec accuracy
- (e) Al-Cr uni- Junction thermo couple.

## 26.6 Self Learning Exercise-1

- Q.1 What is the thermodynamics?
- Q.2 What is Heat?
- Q.3 What is Temperature?
- Q.4 What is unit of Heat?
- Q.5 What is Kilo Calorie?

## 26.7 Procedure

1. Insert the thermocouple junction into the hole of the specimen kept tightly fixed into the silver calorimeter (Silver calorimeter is attached to the lid of the tumbler flask). Connect thermocouple ends to the main unit.
2. Connect the Calorimeter heater leads to the terminals provided on the front panel of the Kit (Heater).
3. Suppose that we have to measure the heat capacity of the material at  $T^0$  C (say  $40^0\text{C}$ ). Choose Temperature  $T_1$  as  $(T-2.5^0\text{C})$  ( $37.5^0\text{C}$ ) and  $T_2$  as  $(T+2.5^0\text{C})$  ( $42.5^0\text{C}$ )
3. Adjust the heater current control potentiometer  $P_1$  so that a current  $I$  of 300 mA pass through the heater as can be read on the current meter. Due to the current flow in the calorimeter heater, the temperature of the specimen increases and as a result the thermo-emf of the thermocouple increases and hence the display reading. When the display reading reaches  $T_1$ , start the stop watch and note the time  $t$  for the temperature to build to  $T_2$ .

4. Using the relation

$$I^2 Rt = (W + m S) (T_2 - T_1)$$

Determine  $W$  employing a standard specimen.

5. The specific heat of the test sample at temperature  $T$  can be determined using the value of  $W$  calculated above by determining the time required to heat the specimen from  $T_1$  to  $T_2$  when the same current  $I$  pass through the calorimeter heater.

## 26.8 Observation

(i) **For Aluminum as standard Sample :-**

Current ( $I$ ) = 304 mA

Mass ( $M$ ) = 2.306 gm

Specific heat ( $S$ ) = 913 J kg<sup>-1</sup> k<sup>-1</sup>

Calorimeter heater Resistance ( $R$ ) = 6.6  $\Omega$

**Observation Table 1:-**

S.No.	$T_1$ °C	$T_2$ °C	t (sec)


**2. For Test Sample (Silver):-**

Current (I) = 304 mA

Mass (M) = 7.068 gm

Specific at (S) = 913 J kg<sup>-1</sup> K<sup>-1</sup>

Calorimeter heater Resistance (R) = 6.6 Ω.

**Observation Table 2:-**

S.No.	T <sub>1</sub> °C	T <sub>2</sub> °C	t (sec)


**3. For test sample (Brass) :-**

Current (I) = 304 mA

Mass (M) = 8.213 gm

Specific at (S) =  $913 \text{ J kg}^{-1} \text{ K}^{-1}$

Calorimeter heater Resistance (R) =  $6.6 \Omega$ .

**Observation Table 3 :-**

S.No.	$T_1$ °C	$T_2$ °C	t (sec)

## 26.9 Calculations

### 1. For standard Sample (Aluminum)

(i) For  $W_1$

$$T_1 = 273 + \dots = \dots \text{ K}$$

$$T_2 = 273 + \dots = \dots \text{ K}$$

$$I^2Rt = (W_1 + MS) (T_2 - T_1)$$

$$W_1 = \dots$$

(ii) For  $W_2$

$$T_1 = \dots \text{ K}$$

$$T_2 = \dots \text{ K}$$

$$I^2Rt = (W_2 + MS) (T_2 - T_1)$$

$$W_2 = \dots$$

(iii) For  $W_3$

$$T_1 = \dots \text{ K}$$

$$T_2 = \dots \text{ K}$$

$$I^2Rt = (W_3 + MS) (T_2 - T_1)$$

$$W_3 = \dots$$

So We can write

$$W = \frac{W_1 + W_2 + W_3}{3}$$

$$W = \dots$$

### 2. For Test Sample (Silver)

(i) For  $S_1$

$$T_1 = \dots \text{ K}$$

$$T_2 = \dots \text{ K}$$

$$I^2Rt = (W + MS_1) (T_2 - T_1)$$

$$S_1 = \dots$$

(ii) For  $S_2$

$$T_1 = \dots\dots\dots K$$

$$T_2 = \dots\dots\dots K$$

$$S_2 = \dots\dots\dots$$

(iii) For  $S_3$

$$T_1 = \dots\dots\dots K$$

$$T_2 = \dots\dots\dots K$$

$$S_3 = \dots\dots\dots$$

So, We can write

$$S = \frac{S_1 + S_2 + S_3}{3}$$

$$S = \dots\dots\dots$$

### 3. For Test Sample (Brass)

(i) For  $S_1$

$$T_1 = \dots\dots\dots K$$

$$T_2 = \dots\dots\dots K$$

$$I^2Rt = (W + MS_1)(T_2 - T_1)$$

$$S_1 = \dots\dots\dots$$

(ii) For  $S_2$

$$T_1 = \dots\dots\dots K$$

$$T_2 = \dots\dots\dots K$$

$$S_2 = \dots\dots\dots$$

(iii) For  $S_3$

$$T_1 = \dots\dots\dots K$$

$$T_2 = \dots\dots\dots K$$

$$S_3 = \dots\dots\dots$$

So, We can write

$$S = \frac{S_1 + S_2 + S_3}{3}$$

$$S = \dots\dots\dots$$

## 26.10 Result

The specific heat of the test sample (Brass) is .....  $\text{J K}^{-1} \text{gm}^{-1}$

The specific heat of the test sample (Silver) is .....  $\text{J K}^{-1} \text{gm}^{-1}$

## 26.11 Discussion

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## 26.12 Precautions and Sources of Error

1. While switching the stopwatch switching off the stop watch to determine the time required to heat the sample from temperature.
2.  $T_1$  &  $T_2$   $^{\circ}\text{C}$  it is desirable to see that the condition are indicate, when the panel reading is just displaced  $T_1$  start the stop watch and stop the watch when the display is just  $T_2$ .
3. The heater inside the calorimeter must be coiled so that together with temperature increases there may occur auto stirring.
4. The current in the coil must be switched on and off while water is flowing otherwise the coil may burn.
5. Sensitive thermometers must be used to determine the temperature accurately.
6. Loss of heat may be more by radiation.

## 26.13 Self Learning Exercise-2

- Q.1** State Zeroth law of thermodynamics.
- Q.2** What is system?
- Q.3** What do you know about environment?

**Q.4** What do you mean by open system?

**Q.5** What is closed system?

## 26.14 Glossary

**Heat Capacity:** - The heat required raising the temperature of a substance one degree.

**Specific heat:** - The number of calories required raising the temperature of 1 gm of substance  $1^{\circ}\text{C}$

**Temperature:** - a measure of the warmth or coldness of an object or substance with reference to some standard value. The temperature of two systems is the same when the systems are in thermal equilibrium.

**System:** - an assemblage or combination of things or parts forming a complex or unitary whole.

**Heat:** - The state of a body perceived as having or generating a relatively high degree of warmth.

**Mass:** - A body of coherent matter, usually of indefinite shape and often of considerable size.

## 26.15 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise- 1*

**Ans.1:** It is the branch of physics which deals with the concepts of heat and temperature.

**Ans.2:** Heat is a form of energy which flows due to the temperature difference.

**Ans.3:** Temperature is a degree of hotness or coldness of the body. It is the property of the system which determines the direction of flow of heat.

**Ans.4:** M. K. S. Unit of Heat is kilo calorie.

**Ans.5:** It is the amount of heat required to raise the temperature of 1 kg of pure water by  $1^{\circ}\text{C}$ .

### *Answers To Self Learning Exercise - 2*

**Ans.1:** If two systems are in thermal equilibrium with a third system then they must be in thermal equilibrium with each other.



**Ans.2:** A particle or group of particles bounded by some closed boundary is known as the system.

**Ans.3:** The region around the system in which the system can transform energy is known as the environment.

**Ans.4:** A system which can transform energy as well as matter with its surroundings.

**Ans.5:** A system which can transform only energy with its surrounding but it cannot transform matter to its surroundings.

## 26.16 Viva Questions

- Q.1 What do you mean by Isothermal Process?
- Q.2 What is Adiabatic Process?
- Q.3 What is Isobaric Process?
- Q.4 What do know about isochoric Process?
- Q.5 What do you mean by specific heat?
- Q.6 What is the unit of specific heat?
- Q.7 Write the formula of Heat capacity?
- Q.8 What is specific heat at constant volume?
- Q.9 What is specific heat at constant pressure?
- Q.10 Write the Mayer's formula?
- Q.11 How much work is required to be done to produce one calorie?
- Q.12 What is the material of the heating coil?
- Q.13 Why the heating wire is coiled?
- Q.14 What do you mean by the mechanical equivalent of heat J?
- Q.15 Which current is passed in the heating coil?
- Q.16 How much potential difference should be applied across the heater?
- Q.17 What will happen if a large potential difference is applied?
- Q.18 How radiation losses are minimized?
- Q.19 Why is it not necessary to determine radiation losses?

**Q.20** What do you mean by isolated system?

**Q.21** What is Heat Capacity?

**Q.22** What is an Environment?

### 26.17 Answers to Viva Questions

**Ans.1:** If a thermodynamic system is perfectly conducting to the surroundings and undergoes a physical process in such a way that its temperature remains constant throughout, the process is said to be isothermal process.

**Ans.2:** If a thermodynamic system is perfectly insulated from the surroundings and undergoes a process in such a way that no exchange of heat takes place between it and the surroundings, the process is said to be adiabatic process.

**Ans.3:** If the working substance is taken in an expanding chamber in which the pressure is constant, the process is called isobaric process.

**Ans.4:** If a substance undergoes a process in which the volume remains unchanged, the process is called an isochoric process.

**Ans.5:** Specific heat of a material is the quantity of heat required to raise the temperature of unit mass of the material through 1 degree.

**Ans.6:** In M. K. S. System - Joule per kg per °C.

In C. G.S. System - Calories per gm per °C

**Ans.7:** Heat Capacity =  $\frac{Q}{\Delta T}$

**Ans.8:** The specific heat at constant volume which is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1°C when its volume is kept constant. It is denoted by  $C_v$  and it is given by

$$C_v = \left( \frac{\Delta Q}{\Delta T} \right)$$

**Ans.9:** The specific heat at constant pressure which is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1°C when its pressure is kept constant. It is denoted by  $C_p$  and given by.

$$\left( \frac{\Delta Q}{\Delta T} \right)_p$$

**Ans.10:**  $C_p - C_v = R$ .

**Ans.11:** 4.18 joules of work is required to be done to produce 1 calorie of heat.

**Ans.12:** Nichrome.

**Ans.13:** The heating wire if coiled also acts as a stirrer.

**Ans.14:** The ratio of work done by a system and the heat produced is known as the mechanical equivalent heat, it is denoted by J.

**Ans.15:** Alternating Current.

**Ans.16:** Approximately 8-10 Volts.

**Ans.17:** If a large potential difference is applied across the heater, some current may also flow through water which will lead to electrolysis of water.

**Ans.18:** Radiation loss may be minimized by decreasing the difference of temperature of two thermometers.

**Ans.19:** Two sets of electric current and potential difference are taken and radiation loss is minimized.

**Ans.20:** If a system is not influenced in any way by its surroundings, it is said to be isolated. Thus an isolated system exchanges neither matter nor energy with its surroundings.

**Ans.21:** It is the amount of heat required by the substance to increase its temperature through  $1^{\circ}\text{C}$ .

**Ans.22:** The region around the system in which the system can transform energy is known as an environment.

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# UNIT-27

## The Dissociation Energy of Iodine

### Structure of the unit

- 27.1 Aim
- 27.2 Apparatus
- 27.3 Diagram
- 27.4 Formula
- 27.5 Theory and Description
- 27.6 Self Learning Exercise -I
- 27.7 Procedure
- 27.8 Observation
- 27.9 Calculations
- 27.10 Result
- 27.11 Discussion
- 27.12 Precautions and Sources of Error
- 27.13 Self Learning Exercise -II
- 27.14 Glossary
- 27.15 Answers to Self Learning Exercises
- 27.16 Viva Questions
- 27.17 Answers to Viva Questions

References and Suggested Readings

### **27.1 Aim**

To determine the dissociation energy of Iodine molecule.

### **27.2 Apparatus**

Spectrometer, Iodine molecule, energy measurement and important tables for calculation.

# 27.3 Diagram

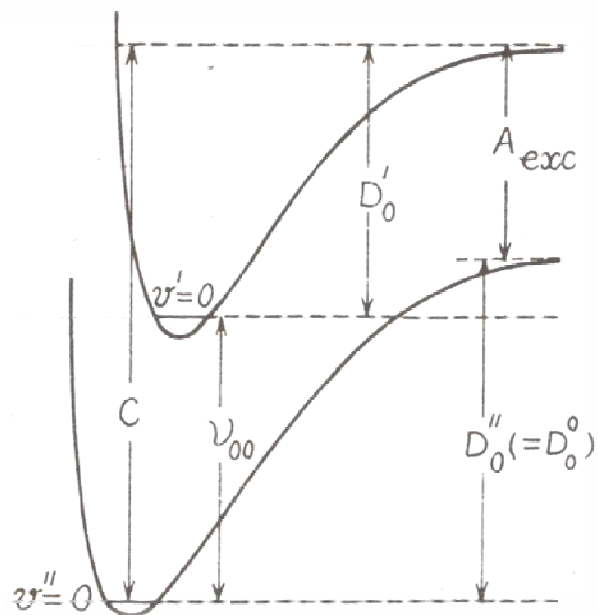


Figure 27.1

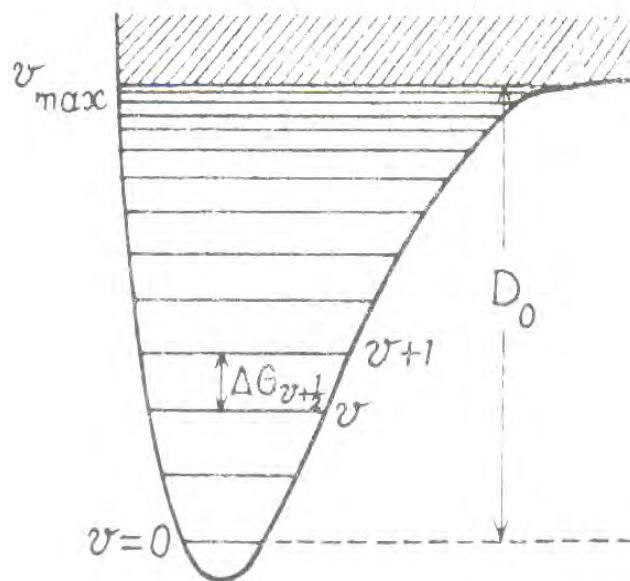


Figure 27.2

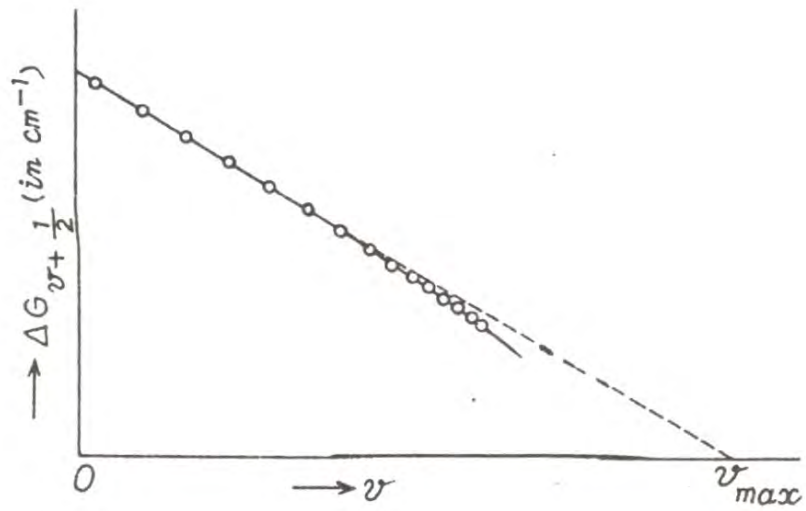


Figure 27.3

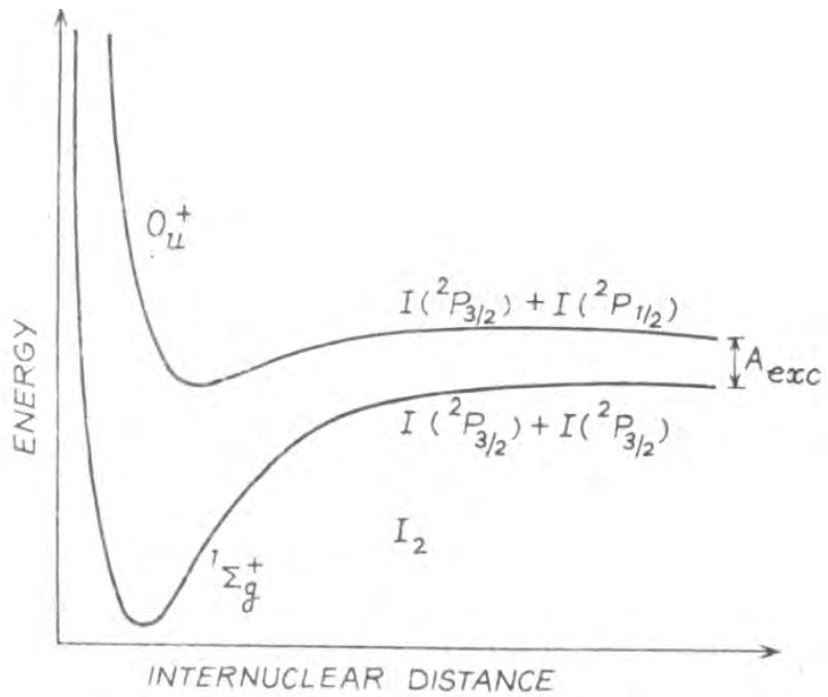


Figure 27.4

### 27.4 Formula

The dissociation energy is simply obtained by the formula:

$$D_0^0 = D_0'' = C - A_{exc}.$$

where

$D_0^0$  is spectroscopic determination of dissociation energy of a molecule and  $C$  is the energy of dissociation limit above ground state and  $A_{exc.}$  is the atomic excitation energy.

## 27.5 Theory and Description

The dissociation energy  $D_0^0$  of a diatomic molecule  $AB$  is defined as the work required to dissociate the molecule from the lowest level ( $v = 0, J = 0$ ) of the electronic ground state into normal atoms,  $A+B$ .

In each case of its stable electronic states, the molecule has a certain dissociation energy  $D_0$  which corresponds to a dissociation from the lowest level of that state. When two electronic states are under consideration, the dissociation energies are denoted by  $D_0'$  and  $D_0''$  for the upper and lower states respectively (figure 27.1). One or both of the atoms produced by dissociation may be in an excited state. If the lower is the ground state, then  $D_0''$  is the energy of dissociation as ordinarily defined ( $D_0'' = D_0^0$ ), provided the ground state dissociates into normal atoms. However, there are a few cases such as the  $BeO$  molecule in which the ground state does not dissociate into normal atoms. Then  $D_0^0$  is smaller than  $D_0''$ .

The spectroscopic determination of the dissociation energy  $D_0^0$  of a molecule always involves two steps: (1) determination of the energy of a dissociation limit  $C$ , above the ground state, and (2) determination of the products of dissociation at this limit, that is, atomic excitation energy  $A_{exc.}$ . When these two are known, the dissociation energy is immediately obtained by the simple expression

$$D_0^0 = D_0'' = C - A_{exc.}$$

For the dissociation energy of Iodine molecule is determined from its most readily observed band system in absorption. The spectrum shows a well-marked convergence of bands to a continuum. The extrapolated limit,  $C$ , of this convergence is found to be at  $4995 \text{ \AA}$  which corresponds to  $2.484 \text{ eV}$ .

The electronic states of Iodine approach Hund's coupling case (c), and the absorption transition is from a ground state to an excited state (figure 27.4). The ground state can be derived from normal iodine atoms  $2P_{3/2} + 2P_{3/2}'$ , while the



upper state must arise from one normal and one excited atom  $2P_{3/2} + 2P_{1/2}$ , as derived from correlation rules. The atomic excitation energy of  $2P_{1/2}$  is  $0.942 \text{ eV}$  as obtained from an analysis of atomic spectra. The dissociation energy to normal atoms would be given by the energy, of the convergence limit less this amount, that is

$$\begin{aligned} D_0^0 &= D_0'' \\ &= C - A_{exc.} \\ &= 2.484 - 0.942 \\ &= 1.542 \text{ eV}. \end{aligned}$$

## 27.6 Self Learning Exercise-I

- Q.1** Give an brief account of dissociation energy of any other molecule.
- Q.2** The value of  $\omega_e''$  and  $\omega_e'' x_e''$  are  $691.75$  and  $4.720 \text{ /cm}$  respectively for the molecule  $NBr^{79}$ . From Birge-Sponer method, Estimate the dissociation energy and also the highest value of  $\nu''$  before dissociation energy.

## 27.7 Procedure

For the dissociation energy of iodine molecule, we should have two values which are determine by following processes

### 27.7.1 Determination of dissociation Limit

The methods of determination of dissociation limit are the following:

#### (i) Band Convergence:

A very accurate value for the dissociation limit is obtained when a  $\nu'$ -progression ( $\nu'' = \text{constant}$ ) of bands is observed up to the convergence point, with its adjoining continuum, in absorption.

The position of the convergence point, that is, the beginning of the continuum, corresponds to the energy of the dissociation limit  $C$  of the excited electronic state. This can be obtained by direct measurement. The dissociation energy  $D_0^0$  can be evaluated by the above expression after  $A_{exc.}$  is known.

If the wave number of the (0, 0) band,  $\nu_{00}$ , of the observed progression is known, then the dissociation energy for the excited state can be evaluated:

$$D'_0 = C - \nu_{00}.$$

**(ii) Extrapolation to Convergence Limit: Birge-Sponer Method:**

For those cases in which no band convergence is observed, Birge and Sponer have suggested an extrapolation to the convergence limit from the observed bands. We know that in a vibrational energy-level diagram for a given electronic state, the dissociation energy for the state is distance from the  $\nu = 0$  level to the (unobserved) limiting level  $\nu = \nu_{max}$ . at which the intervals between successive levels would vanish Figure 27.2. That is,  $D_0$  is equal to the sum of all the vibrational quanta:

$$D_0 = \sum_{\nu} \Delta G_{\nu+\frac{1}{2}}$$

Now, the vibrational energy levels can be expressed as a power series in terms of the vibrational quantum number  $\nu$ ;

$$G(\nu) = \omega_e \left( \nu + \frac{1}{2} \right) - \omega_e x_e \left( \nu + \frac{1}{2} \right)^2 + \dots \dots \dots$$

The higher term being neglected as the series is rapidly convergent. The separation between successive levels  $\nu$  and  $\nu + 1$  is thus

$$\begin{aligned} G_{\nu+\frac{1}{2}} &= G(\nu + 1) - G(\nu) \\ &= \left\{ \omega_e \left( \nu + \frac{3}{2} \right) - \omega_e x_e \left( \nu + \frac{3}{2} \right)^2 \right\} - \left\{ \omega_e \left( \nu + \frac{1}{2} \right) - \omega_e x_e \left( \nu + \frac{1}{2} \right)^2 \right\} \\ &= \omega_e - 2\omega_e x_e - 2\omega_e x_e \nu \\ &= -2\omega_e x_e \nu + (\omega_e - 2\omega_e x_e) \end{aligned}$$

This equation is of the same form as the general equation of a straight line. Hence a graphical plot of the observed  $\Delta G_{\nu+\frac{1}{2}}$  against  $\nu$  will be straight line with a negative slope of  $-2\omega_e x_e$  (Figure 27.3). The area under this straight extrapolated

to  $v_{max}$  (where  $\Delta G = 0$ ) is the sum of all the  $\Delta G_{v+\frac{1}{2}}$ 's and gives the dissociation energy  $D_0$ .

Analytically, the linearly extrapolated dissociation energy is given by

$$\begin{aligned}
 D_0 &= \sum_{v=0}^{v_{max}} \Delta G_{v+\frac{1}{2}} \\
 &= (\omega_e - 2\omega_e x_e) + (\omega_e - 4\omega_e x_e) + (\omega_e - 6\omega_e x_e) + \dots \text{to zero} \\
 &= \frac{\omega_e^2}{4\omega_e x_e} - \frac{1}{2}\omega_e \\
 &= \frac{1}{2}\omega_e \left( \frac{\omega_e}{2\omega_e x_e} - 1 \right) \\
 &= \frac{1}{2}\omega_e v_{max}
 \end{aligned}$$

Thus, on determining  $\omega_e$  and  $v_{max}$  (by extrapolation), the dissociation energy  $D_0$  can be calculated.

By this method we obtain the dissociation energy of the electronic state whose vibrational quanta are used. The advantage of the extrapolation method is that it can be applied to any electronic state for which a few vibrational levels are known. It enables us to estimate  $v$  at the dissociation limit and emphasises the importance of the anharmonicity of molecular vibrations near the dissociation limit.

### **(iii) Predissociation Limits:**

Whenever predissociation (including the induced predissociation and accidental predissociation) is observed, the predissociated energy level must lie above the dissociation energy. Thus, the energy of predissociation gives an upper limiting value for the dissociation energy state causing the predissociation. However, only in special cases, predissociation can lead to an exact numerical value for a dissociation limit.

### **27.7.2 Determination of dissociation Products:**

The products of dissociation and their state of excitation is also required to be

known in order to find the dissociation energy of the molecule in its ground state into normal atoms. There are several ways for it:

**(i) Energy difference of dissociation Limits:**

If two or more different dissociation limits of a molecule have been determined, their differences must be equal to possible energy differences of the separate atoms.

**(ii) Application of Wigner-Witmer Correlation Rules:**

By applying these rules we can find out from which atomic states (of the dissociated atoms) the molecular state in question (whose dissociation limit is being considered) can actually result.

**(iii) Application of Non-crossing Rule:**

Except in rare circumstances according to non-crossing rule, the electronic ground state must always dissociate into normal atoms. Therefore, of the various possible dissociation products at a given dissociation limit, those may be excluded that would lead to a dissociation of the ground state into excited atoms.

**(iv) Observation of Atomic Fluorescence:**

When a molecular gas is irradiated with light of frequency higher than dissociation limit, it shows an atomic fluorescence spectrum indicating that the molecules have actually been dissociated into atoms. The atomic lines so obtained can lead to the determination of the state of the excitation of atoms.

**(v) Use of Thermo-chemical data:**

An approximate value of the dissociation energy can be obtained from thermochemical measurements. Such values always refer to dissociation limit is known, the dissociation products at this limit are obtained.

## 27.8 Observation

Table for the Dissociation energy of Iodine Molecule:

S.NO.	Internuclear Distance (in cm)	Energy Corresponding Distance (in eV)
1		

2		
3		
4		
5		
6		
7		

### 27.9 Calculations

By the value of above table, we can calculate and find the value of dissociation energy of Iodine Molecule.

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### 27.10 Result

The dissociation Energy of Iodine Molecule corresponding distance:.....

$$\% \text{ error} = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

### 27.11 Discussion

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## 27.12 Precautions and Sources of Error

1. We should observe that there is no error in spectrometer.
2. We should observe that there is no dust particle on the spectrum.
3. Calculation should be done very clearly unless value can be vary.
4. We should use a lens to read the reading of spectrometer.
5. Graph is very important to describe the result of experiment, so it draw very clearly.

## 27.13 Self Learning Exercise -II

- Q.1** How is a four-level laser better than a three-level laser? Give examples of each.
- Q.2** The Schumann-Runge bands of oxygen in the ultraviolet converge to a limit followed by a continuum. The converge limit is  $1750 \text{ \AA}$  and the (0, 0) band of the system corresponds to  $\nu_{00} = 49410 \text{ cm}^{-1}$ . Calculate the heat of dissociation of the upper state in eV.

## 27.14 Glossary

**Dissociation Energy:** it is defined as the work required to dissociate the molecule from lowest level of the electronic ground state into normal atoms.

**Predissociation:** It is establish either by the absorption bands becoming diffuse, or by the emission bands breaking off at a certain point.

**Excited State:** When any electron gets energy from outside source, then a state of electron is defined is known as excited state.

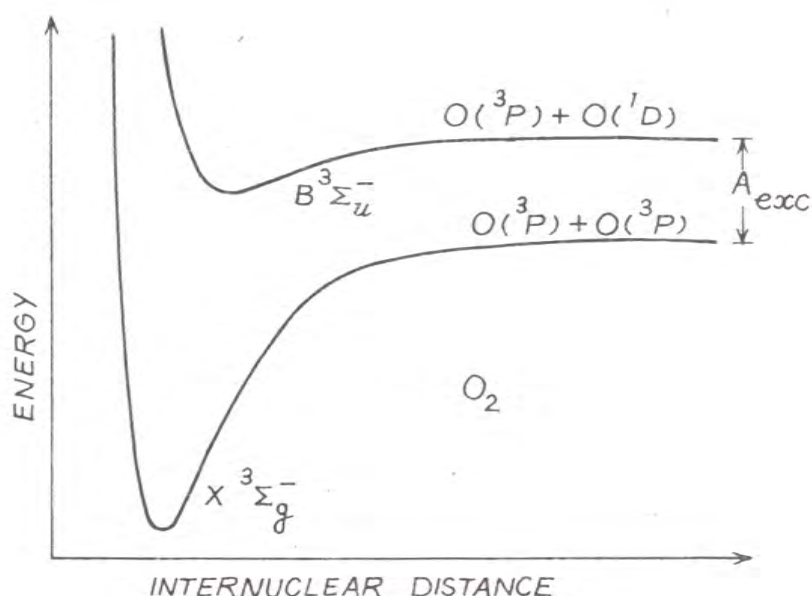
**Ground State:** Normal state of an electron in which it revolves, is known as Ground state.

## 27.15 Answers to Self Learning Exercises

### *Answers to Self Learning Exercise-I*

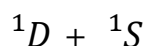
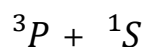
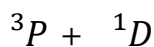
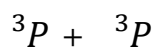
**Ans.1: Dissociation Energy of Oxygen Molecule:** Oxygen is one of the molecules for which spectroscopic data gives the most certain and accurate

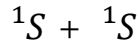
value for the dissociation energy. The strongest system of  $O_2$  is the Schumann-Runge progression bands which converges to a well defined limit, at which a continuum states in. the slightly extrapolated convergence limit is found to be at  $1759 \text{ \AA}$ , which is corresponds to  $7.047 \text{ eV}$ . This is the energy required to go from the lowest vibrational level of the ground electronic state to a pair of oxygen atoms at least one of which is in excited state. We know now to find the products of dissociation at the convergence limit in the electronic state, before the energy of dissociation to normal atoms can be obtained.



**Figure 1**

Now oxygen atom has three-lying states:  $3P$  ground state, the metastable states  $1D$  and  $1S$ . The following combinations are therefore possible as dissociation products:





The last two three combinations can be eliminated immediately because two singlet atoms ( $S=0$ ) cannot combine to give a triplet molecule. ( $S=1$ )

The first combination  ${}^3P + {}^3P$  cannot give an electronic state, according to Wigner-Witmer rules, so it is also to be eliminated. Thus, there remains only two possibilities,  ${}^3P + {}^1D$  and  ${}^3P + {}^1S$  for the dissociation products, which have energies of  $1.967 \text{ eV}$  and  $4.188 \text{ eV}$  respectively, above the  ${}^3P + {}^3P$  ground state. Thus, the dissociation energy for the ground state of oxygen assuming the above two possibilities, are

$$= 7.047 - 1.967 = 5.080 \text{ eV}$$

$$= 7.047 - 4.188 = 2.859 \text{ eV}.$$

The latter value is smaller than the energy of the highest observed vibrational level in the ground state, which lies  $3.4 \text{ eV}$  above the lowest level  $v'' = 0$ , since dissociation energy must be greater than this, the combination  ${}^3P + {}^1S$  is ruled out. Thus we, are left with the combination  ${}^3P + {}^1D$ , giving a dissociation energy of  $5.080 \text{ eV}$  for Oxygen.

**Ans.2:** The dissociation energy in a given electronic state is the sum of all the vibrational quanta is given by:

$$D_0 = \sum_{v=0}^{v_{max}} \Delta G_{v+\frac{1}{2}}$$

Now,

$$\Delta G_{v+\frac{1}{2}} = G(v+1) - G(v)$$

$$= \left\{ \omega_e \left( v + \frac{3}{2} \right) - \omega_e x_e \left( v + \frac{3}{2} \right)^2 \right\} \\ - \left\{ \omega_e \left( v + \frac{1}{2} \right) - \omega_e x_e \left( v + \frac{1}{2} \right)^2 \right\}$$

$$= \omega_e - 2\omega_e x_e - 2\omega_e x_e v$$

$$= \omega_e - 2\omega_e x_e (v+1)$$

So



$$\begin{aligned}
D_0 &= \sum_{v=0}^{v_{max}} \Delta G_{v+\frac{1}{2}} \\
&= (\omega_e - 2\omega_e x_e) + (\omega_e - 4\omega_e x_e) + (\omega_e - 6\omega_e x_e) + \dots \text{to zero} \\
&= \frac{\omega_e^2}{4\omega_e x_e} - \frac{1}{2} \omega_e
\end{aligned}$$

Substituting the values we get

$$\begin{aligned}
D_0 &= \frac{(691.75)^2}{4 \times 4.720} - \frac{1}{2} \times 691.75 \\
&= 24999 / cm
\end{aligned}$$

Or

$$\begin{aligned}
D_0 &= 24999 \times 2.858 \times 10^{-3} \\
&= 71.5 \text{ kcal/mole}
\end{aligned}$$

The highest value of  $v$  before dissociation occurs is finite and corresponds to  $\Delta G_{v+\frac{1}{2}} = 0$ , that is

$$\omega_e - 2\omega_e x_e (v_{max} + 1) = 0$$

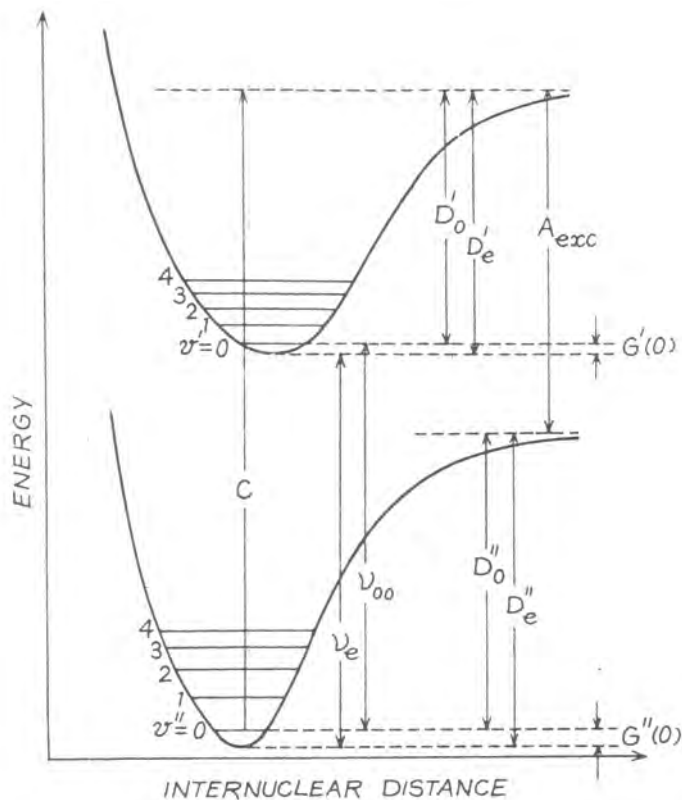
$$\begin{aligned}
v_{max} &= \frac{\omega_e}{2\omega_e x_e} - 1 \\
&= \frac{691.75}{4 \times 4.720} - 1 \\
&= 73 - 1 = 72.
\end{aligned}$$

### ***Answers to Self Learning Exercise-II***

**Ans.1:** Ruby laser is a three-level solid state laser whereas He-Ne is a four-level gas laser. Ruby laser has a pulsed output whereas He-Ne laser has a continuous output.

**Ans.2:** From below given figure 1, the maximum wavelength of light corresponding to the onset of absorption continuum is given to be

$$\lambda_{disso.} = 1750 \text{ \AA}$$



**Figure 1**

The corresponding energy absorbed is

$$\begin{aligned}
 C &= \frac{1}{\lambda_{disso.}} \\
 &= \frac{1}{1750 \times 10^{-8}} \\
 &= 57143 / cm
 \end{aligned}$$

Also

$$\begin{aligned}
 \nu_{00} &= 49410 / cm \\
 D_0' &= C - \nu_{00} \\
 &= 57143 - 49410 \\
 &= 7733 / cm
 \end{aligned}$$

Now  $1eV=8066/cm$

$$D_0' = \frac{7733}{8066} = 0.96 eV.$$

## 27.16 Viva Questions

- Q.1 What is a normal spectrum?
- Q.2 What do you mean by pure spectrum?
- Q.3 What is LASER?
- Q.4 What is an Excimer Laser?
- Q.5 How are spectral lines formed?
- Q.6 When I look at a star, why do I see dark absorption lines rather than bright emission lines?
- Q.7 Why are the hydrogen lines weaker in cooler stars?
- Q.8 State Heisenberg's uncertainty principle.
- Q.9 What is Principle Quantum Number?
- Q.10 Differentiate between electromagnetic radiation and Spectrum.

## 27.17 Answers to Viva Questions

- Ans.1:** A spectrum in which angular separation between two wavelengths is directly proportional to difference of the wavelengths is called a normal spectrum.
- Ans.2:** A spectrum in which there is no overlapping of colours is known as pure spectrum. Each colour occupies a separate and distinct position.
- Ans.3:** It is a device by which an intense, monochromatic, collimated and highly-coherent light beam can be obtained.
- Ans.4:** Lasing can take place on the transition between upper bound state and the lower unbound level. Such a laser is called an *excimer* laser. The *excimer* laser has unique characteristics indicating the condition, that the ground state is repelling in nature.
- Ans.5:** By electrons jumping between different energy levels in the atoms in the star's outer layers. Bound electrons can absorb and emit energy only in certain discrete amounts. When an electron absorbs a photon of light with just the right amount of energy, it jumps to a higher energy level. When the electron spontaneously jumps back to a lower energy level, a photon is

emitted. Enough electrons jumping between any two given energy levels of a given element will result in a spectral emission or absorption line at a characteristic wavelength.

**Ans.6:** Gas under high pressure produces a continuous spectrum, a rainbow of colors. Continuous radiation viewed through a low density gas results in an absorption-line spectrum. What's happening here is that radiation emitted by gas under high pressure deep within the star is being absorbed by low density gas in the star's outer layers.

**Ans.7:** At cooler temperatures most electrons are in the ground state, at the lowest energy level, because of the relatively few atomic collisions jumps to and from the ground state of neutral hydrogen produce spectral lines in the Lyman series, the strongest of which fall in the ultraviolet part of the spectrum.

**Ans.8:** This principle states that it is impossible to measure simultaneously the position and momentum of a small particle with absolute accuracy or certainty.

If an attempt is made to measure any one of these two quantities with higher accuracy, then the other becomes less accurate. The product of uncertainty in the position and uncertainty in momentum is always constant and is equal to or greater than  $h/4\pi$  i.e.

$$\Delta x \cdot \Delta p = h/4\pi$$

Where,  $h$  is the Planks constant,  $\Delta x$  is the uncertainty in position and  $\Delta p$  is the uncertainty in momentum.

**Ans.9:** It is denoted by  $n$ . It tells us about the energy level or shell in which the electron is present. The value of  $n$  can be 1,2,3,4.....etc. but it cannot be zero. It gives us the information about the average distance of electrons from the nucleus.

**Ans.10:** Electromagnetic radiation is a transverse wave with magnetic and electric components that oscillate perpendicular to each other. The **electromagnetic spectrum** is the range of all possible wavelengths and frequencies of electromagnetic radiation including visible light.

## References and Suggested Readings

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# **UNIT-28**

## **Dielectric Constant of Turpentine Oil by Lecher Wire System**

### **Structure of the Unit**

- 28.1 Aim
- 28.2 Apparatus
- 28.3 Diagram
- 28.4 Formula
- 28.5 Theory and Description
- 28.6 Self Learning Exercise -I
- 28.7 Procedure
- 28.8 Observation
- 28.9 Calculations
- 28.10 Result
- 28.11 Discussion
- 28.12 Precautions and Sources of Error
- 28.13 Self Learning Exercise -II
- 28.14 Glossary
- 28.15 Answers to Self Learning Exercises
- 28.16 Viva Questions
- 28.17 Answers to Viva Questions

References and Suggested Readings

### **28.1 Aim**

To determine the dielectric constant of Turpentine oil by Lecher wire system.

## 28.2 Apparatus

Shorting bar, incandescent lamp, RF voltmeter, a rigid bar, UHF oscillator neon bulb, a wavemeter, glass vessels, etc.

## 28.3 Diagram

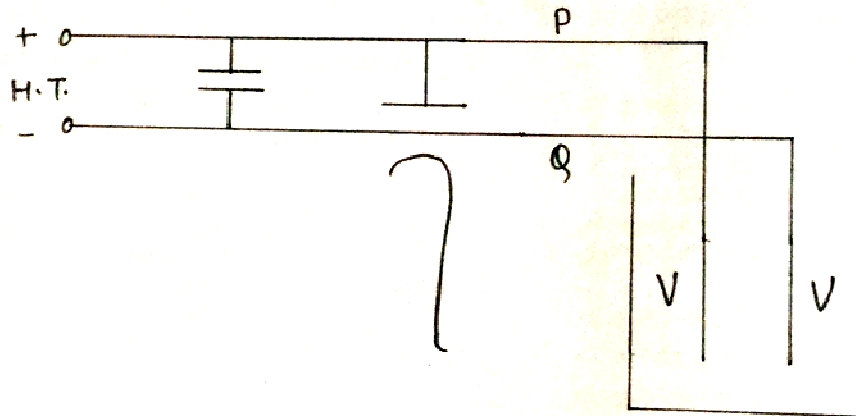


Figure 28.1

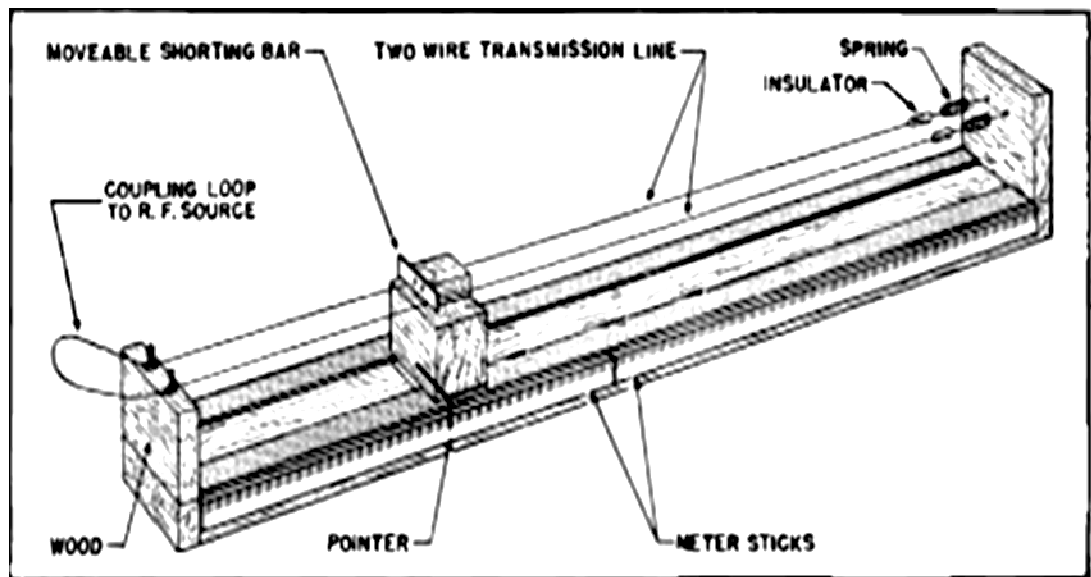


Figure 28.2: Lecher Wire System.

## 28.4 Formula

Dielectric Constant  $\delta$  can be calculated by the below given formula:

$$\frac{\lambda d \lambda}{2\pi^2 L} = (\delta - 1) \frac{0.109l}{\log_{10} \left( \frac{D}{d} \right)}$$

where  $L$  = inductance of the oscillatory circuit

and  $l$  = length of the wire dipped in the liquid,

$D$  = distance between the two dipping wires,

$d$  = diameter of the wire,

$\lambda$  = Wavelength.

## 28.5 Theory and Description

The method of ultraviolet waves was also applied to the measurement of dielectric constant. For this purpose figure 28.1 was used. The wires PQ on the right side are bent at right angles as shown in the fig. the vertical portions of these wires which are equal in length are dipped in turpentine oil which is put in glass vessel by which we calculate the dielectric constant.

The principle of the method is to measure the change in wave-length produced by the change in capacity of the vertical portions of the wires when they are dipped in different liquids. From the change in wavelength thus produced, the dielectric constants of liquids can be determined.

Let  $k$  be the capacity of the two parallel vertical portions of the wires in air. Then the wave-length of the oscillatory circuit is given by:

$$\lambda = 2\pi\sqrt{L(C + k)} \quad (28.1)$$

where  $L$  = inductance of the oscillatory circuit,

and  $C$  = capacity of the oscillatory circuit.

Or

$$\lambda = 2\pi\sqrt{L(C + k)}$$



$$\lambda^2 = 4\pi^2 L(C + k)$$

Differentiating it, we get

$$2\lambda d\lambda = 4\pi^2 L dk$$

or

$$dk = \frac{\lambda d\lambda}{2\pi^2 L} \quad (28.2)$$

All the quantities in the right hand, expression of above equation being known,  $dk$  can be calculated.

Now, if  $l$  = length of the wire dipped in the liquid,

$D$  = distance between the two dipping wires,

$d$  = diameter of the wire,

Then the capacity formed by the two vertical portions of the wires is given by:

$$k = \frac{0.109l}{\log_{10} \left( \frac{2D}{d} \right)} \quad (28.3)$$

With air as the dielectric.

When the wires are dipped in a liquid of dielectric constant  $\delta$ , the capacity is given by:

$$k = \frac{\delta \times 0.109l}{\log_{10} \left( \frac{D}{d} \right)}$$

So now change in capacity

$$dk = (\delta - 1) \frac{0.109l}{\log_{10} \left( \frac{D}{d} \right)} \quad (28.4)$$

Substituting the value of  $dk$  obtained from above equation (28.2),  $\delta$  can be calculated as:

$$\frac{\lambda d \lambda}{2\pi^2 L} = (\delta - 1) \frac{.109l}{\log_{10} \left( \frac{D}{d} \right)} \quad (28.5)$$

- Length of wires of the oscillatory circuit = 25 cm ,  
 Distance between the two wires of the oscillator circuit = 8.5 cm,  
 Diameter of the wire = 0.123 cm  
 Distance between the two wires dipping = 1.0 cm.  
 Valves used: .....

## 28.6 Self Learning Exercise-I

- Q.1** What is dielectric? Give information about the stored electric charge.  
**Q.2** Give the theory of Series Inductance filter.

## 28.7 Procedure

The experimental procedure can be varied for the different types of oil. The R F milliammeter is kept fixed on the lecher wires and a graph is plotted between the lengths of the wires VV and the current in the milliammeter. The wires VV are then dipped in the liquid and from the change of current in the milliammeter which is kept fixed on the lecher wires, the change in the capacity of the wires VV is noted by means of the graph already plotted. This change in capacity gives immediately the dielectric constant. This procedure gives more accurate results and is better adapted to the study of the temperature variation of the dielectric constant which is to form the subject of a subsequent communication.

## 28.8 Observation

Table for the Dielectric constant of the oil which is used:

Dielectric	Length dipping	Wave-length (in cm)	Dielectric Constant
Air			

Turpentine Oil			
Kerosene Oil			

## 28.9 Calculations

On the basis of the formula given we can calculate particular value of dielectric constant of oil which is used.

$$\frac{\lambda d \lambda}{2\pi^2 L} = (\delta - 1) \frac{0.109l}{\log_{10}\left(\frac{D}{d}\right)}$$

.....

.....

.....

## 28.10 Result

On the basis of the formula given calculated value of dielectric constant =

.....

.....

.....

**Percentage Error:**

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

## 28.11 Discussion

.....

.....

.....

.....

.....

## 28.12 Precautions and Sources of Error

1. Precaution should be taken to read the reading of milliammeter.
2. There should be use high tension battery for the whole process.
3. Wire should not be touch to each other.
4. Whole system should be on a plane surface.
5. Before practical first read and understand the lecher wire system.

## 28.13 Self Learning Exercise -II

- Q.1** A full-wave rectifier uses shunt capacitor  $C = 12\mu F$ . The supply frequency is  $f = 50 Hz$ . The angle of conduction  $\theta_C = \theta_2 - \theta_1$  has value  $27^\circ$  for a load resistance of  $R = 10 k\Omega$ . Find
- (a) the ripple factor at the output of the filter;
  - (b) ratio of this ripple factor to that for a rectifier with the filter capacitor.
- Q.2** Give one phenomenon which is related to dielectric constant.
- Q.3** Give the Limitations of LC and RC Oscillators.
- Q.4** Give the brief description about the dielectric breakdown.

## 28.14 Glossary

**Impedance:** In LCR circuit the ratio of total voltage with the total current, is known as Impedance.

**Inductance:** The property of an electric conductor or circuit that causes an electromotive force to be generated by a change in the current flowing.

**Capacitance:** The ability of a system to store an electric charge.

**Resistance:** A measure of the degree to which conductor opposes an electric current through that conductor.

## 28.15 Answers to Self Learning Exercises

### ***Answers to Self Learning Exercise-I***

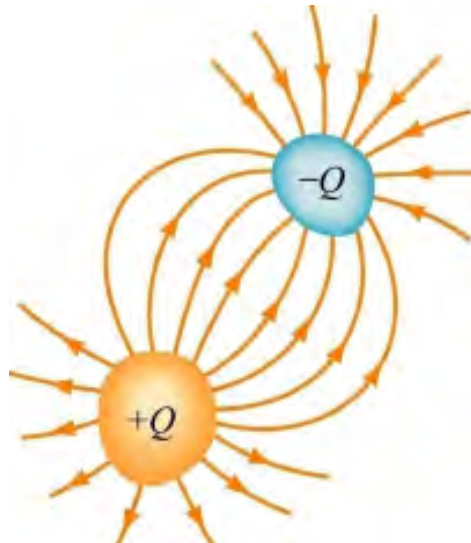
**Ans.1:** A parallel plate capacitor is charged to a total charge  $Q$  and the battery

removed. A slab of material is inserted between the plates which is known as dielectric plate and constant is known as dielectric constant.

Every insulation material has the capacity to store charge when placed in between two conducting plates as in capacitors. Relative permittivity or dielectric constant, it is the ratio of the capacitance of a capacitor with a specified dielectric material placed between the plates, to the capacitance of the same capacitor with free space i.e. air between the plates.

Capacitor: Two isolated conductors Equal and opposite charges  $+Q$  and Potential difference  $V$  between them, then stored charge is

$$Q = \frac{C}{V}$$



Units: Coulombs/Volt or Farads

**Ans.2:** The operation of the inductor filter depends on the fundamental property of an inductor to oppose any change of current. As a result, any sudden changes that might occur in a circuit without an inductor are smoothed out by the presence of an inductor in the circuit.

Suppose that an inductor input filter is applied to the output of a full-wave rectifier. The circuit with load current waveform is shown in below given figure 1.

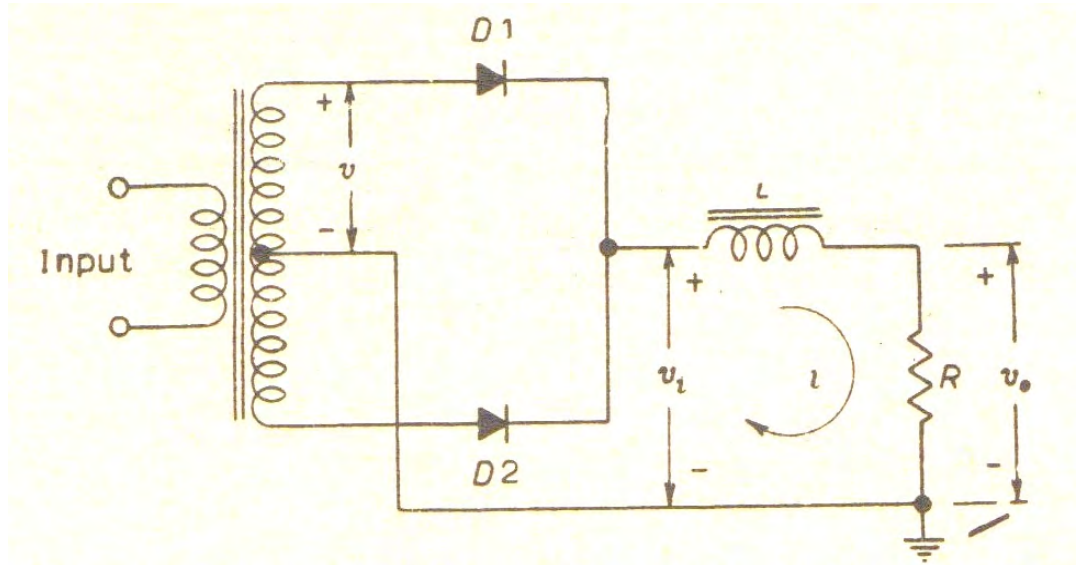


Figure 1(a)

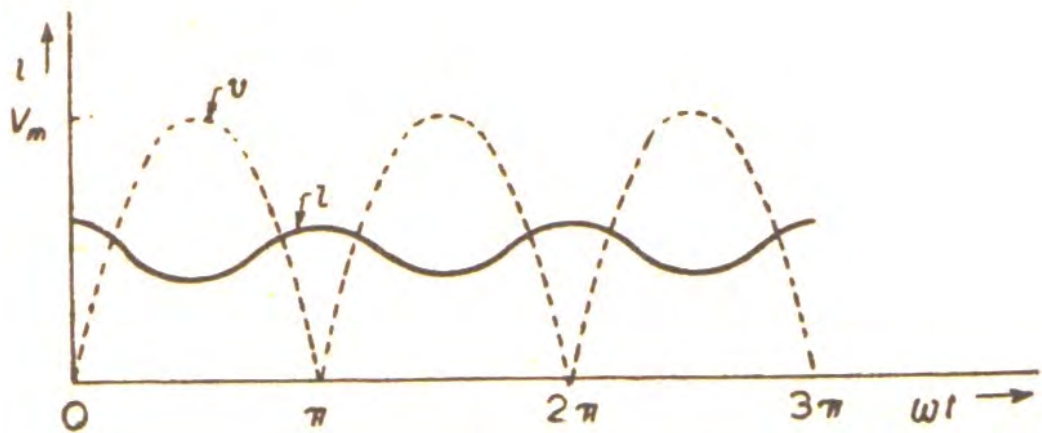


Figure 1(b)

The voltage applied to the circuit comprising the load resistor and the inductor filter is that given in equation

$$i_L = i_{D1} + i_{D2} = I_m \left\{ \frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t \right\}$$

With the current replaced by the voltage, since the impedance of the inductor increases with the frequency, better filtering action for the higher-harmonic terms results. It is, therefore, expected that the waveform in the output will be principally of second-harmonic frequency, and we may neglect all harmonics

except the first ac term. Under these circumstances, the equivalent circuit of the rectifier is shown in below given figure 2.

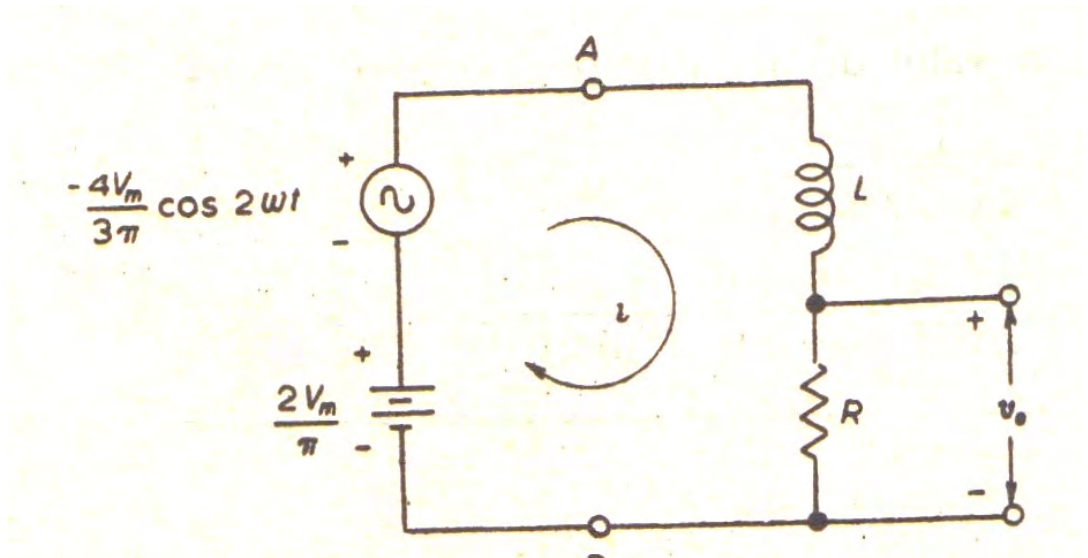


Figure 2

### Answers to Self Learning Exercise-II

**Ans.1:** (a) The ripple factor is given by the equation is

$$r = \frac{\pi - (\theta_2 - \theta_1)}{2\sqrt{3} \omega RC}$$

$$= \frac{\pi - (\theta_2 - \theta_1)}{4\sqrt{3} \pi f RC}$$

Here  $\theta_c = \theta_2 - \theta_1$

$$= 27^\circ = \frac{27\pi}{180} \text{ radian}$$

Now  $r = \frac{\pi - \frac{27\pi}{180}}{2\sqrt{3} \times \pi \times 50 \times (10 \times 10^3) \times (12 \times 10^{-6})}$

$$r = \frac{1.67}{130.6} = \mathbf{0.02 \text{ or } 2\%}$$

(b) Ripple factor without filter is 0.482. Therefore, the ratio of ripple factor with filter to that without filter is

$$\frac{0.02}{0.482} = \mathbf{0.04}$$

**Ans.2:** Consider a slab of material in which there is a current density  $J$  resulting from an applied electric field  $E_x$  in the  $x$ - direction. The electrons will drift with an average velocity  $x V$  in the  $x$  direction, when a magnetic field of flux density  $B_z$  ( $\text{Wb/m}^2$ ) is superposed on the applied electric field in the  $Z$  direction the electrons will experience a Lorentz force perpendicular to  $x V$  and to  $B_z$  ; the magnitude of this force will be given by  $B_z (\mu x)e$ .

Thus the electrons are driven towards one face in the sample resulting in an excess of electrons near one face and a deficiency of electrons near the other face. These charges will in turn create a counteracting electric field  $E_y$  in the  $y$ -direction.  $E_y$  would build up, until it is of sufficient magnitude to compensate the Lorentz force exerted on the electrons due to the magnetic field we may therefore write

$$e E_y = B_z e (\times V)$$

In the steady state , a Hall voltage,  $V_H$ , is thereby established in the  $y$ -direction given by

$$V_{H'} = E_y \cdot a = B_z (V_x) \cdot a.$$

The current density in the sample is given by

$$J_x = NeV_x$$

where  $N$ =no. of conduction electrons/ $\text{m}^3$

The current density can be calculated from the total current and the cross section ( $axb$ ) of the sample.

$$\begin{aligned} \text{Thus,} \quad J_x &= \frac{I}{axb} = NeV_x \\ I &= NeV_x axb \\ V_{H'} &= B_z (V_x) \cdot a \end{aligned}$$

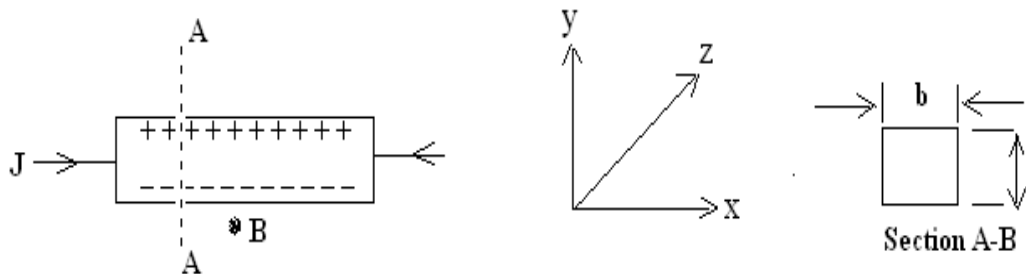
Eliminating ( $V_x$ ). from equating above equation, we have.



$$V_H = \left( \frac{1}{n_e} \frac{B_z I}{B} \right)$$

The ratio of  $\pm \frac{1}{n_e} = \frac{E_y}{J_x B_z}$  must be constant.

It is called the **hall coefficient** and is denoted by RH.



**Diagram for the Hall Coefficient**

**Ans.3:** The LC and RC oscillators discussed so far have their own limitations. The major problem in such circuits is that their operating frequency does not remain strictly constant. There are two principal reasons for it

(i) As the circuit operates, it will warm up. Consequently, the values of resistors and inductors, which are the frequency determining factors in these circuits, will change with temperature. This causes the change in frequency of the oscillator.

(ii) If any component in the feedback network is changed, it will shift the operating frequency of the oscillator.

However, in many applications, it is desirable and necessary to maintain the frequency constant with extreme low tolerances. For example, the frequency tolerance for broadcasting station should not exceed 0.002% i.e. change in frequency due to any reason should not be more than 0.002% of the specified frequency. The broadcasting stations have frequencies which are quite close to each other. In fact, the frequency difference between two broadcasting stations is less than 1%. It is apparent that if we employ LC and RC circuits, a change of temperature may cause the frequencies of adjacent broadcasting stations to overlap.

**Ans.4:** The electric strength at breakdown is defined as the minimum electric stress usually expressed in kV/cm which cause rupture or breakdown of the material

under specified conditions of temperature, duration, waveform, frequency and type of electrodes. The electric strength of a material depends on its composition, thickness, temperature, and moisture content and to some extent on the time of application of the applied voltage. The shape of the waveform and steepness of the wave front of the applied voltage also affect it. There is no definite relationship between these variables, but in general for sheet materials, the electric strength is an inverse function of the thickness and time and decreases with increasing temperature and moisture content. At breakdown the high electric stress is assumed to cause an inter-atomic displacement of the orbital electrons, which alters the atomic structure causing heating and a conducting path in the material. The breakdown mechanisms of gaseous, liquid and solid dielectrics are different in nature.

### **28.16 Viva Questions**

- Q.1** What are the bright and dark fringes?
- Q.2** Give the conditions of interference.
- Q.3** What are the applications of thin film interference?
- Q.4** What is etalon?
- Q.5** Give some electric properties of copper.
- Q.6** Define Dissipation factor.
- Q.7.** Give application of dielectric constant.
- Q.8** Give the definition of Radio frequency.
- Q.9** Give the definition of Audio frequency.
- Q.10** What is differentiating Circuit?
- Q.11** What is integrating circuit?
- Q.12** What is clipping circuit?
- Q.13** What are Polymers?

### **28.17 Answers to Viva Questions**

**Ans.1:** When the two or more waves of light are superposed, the resultant effect is brightness in certain regions and darkness in certain regions. The regions of

brightness and darkness alternate and may take the form of straight bands, or circular rings or any other complex shape. The alternate bright and dark bands are called interference fringes.

The fringes which appear in bright regions are called as **Bright fringes** and which appears in dark regions are called as **Dark fringes**.

**Ans.2:** The conditions of interference are given as:

- (1) The waves from the two sources must be of the same frequency.
- (2) The two light waves must be coherent.
- (3) The path difference between the overlapping waves must be less than the coherence length of the waves.
- (4) If the two sets of waves are plane polarized, their planes of polarization must be the same.
- (5) The two coherent sources must lie close to each other in order to discern the fringe pattern.
- (6) The distance of the screen from the two sources must be large.
- (7) The vector sum of the overlapping electric field vectors should be zero in the dark regions.

**Ans.3:** There is various type of application of thin film interference, which are:

- (1) Measurement of small displacement,
- (2) Testing of surface finish,
- (3) Testing of a lens surface,
- (4) Thickness of a thin film coating.

**Ans.4:** An etalon is a substandard for length. It consists of two mirrors, which are plane-parallel and semi silvered on their front faces. The mirrors can be made perfectly parallel by means of screws attached to them.

**Ans.5:** (i). The conductivity of copper is decreased when it is hard drawn into wires for use in machines. Annealing is therefore necessary before the material can be used in machine.

(ii) Copper has the second highest electrical conductivity. The conductivity of copper is only few percent less than that of silver.

**Ans.6:** It is the reciprocal of the 'Quality'  $Q$ , i.e. the dissipation factor is the ratio of the resistance to reactance. Hence

$$D = \frac{1}{Q} = \frac{R}{X}$$

Here  $X$  is the reactance.

**Ans.7:** For different applications different properties of dielectric materials are required such as: electrical, mechanical, thermal and chemical. Electrically the insulating material should have high resistivity to reduce the leakage current and high dielectric strength to enable it to withstand higher voltage without being punctured. Further, the insulator should have small dielectric loss. Since the insulators are used on the basis of volume and not weight, a low density is preferred.

**Ans.8:** Radio frequency is the rate of oscillation in the range of oscillation in the range of around 3 kHz to 300 GHz, which corresponds to the frequency of radio waves, and the alternating current carries the radio signals. RF usually refers to electrical rather than mechanical oscillations.

**Ans.9:** An audio frequency is defined as a periodic vibration whose frequency is audible to the average human. It is the property of sound that most determines pitch.

The range of audio frequency generally accepted is 20 to 20,000 Hz.

**Ans.10:** A circuit in which output voltage is directly proportional to the derivative of the input is known as a differentiating circuit.

$$Output \propto \frac{d}{dt} (Input)$$

**Ans.11:** A circuit in which output voltage is directly proportional to the integral of the input is known as an integrating circuit.

$$Output \propto \int Input$$

**Ans.12:** The circuit with which the waveform is shaped by removing a portion of the applied wave is known as a clipping circuit.

**Ans.13:** Polymeric materials or plastics comprise a large group of organic or organo-metallic high molecules compound.

The common properties of these materials are their ability to soften and even melt, ability to pass into a liquid state, insolubility in water and solubility in one or more organic solvents.

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5. Practical Physics by G. L. Squires.

# **UNIT-29**

## **Determination of Unknown Impedance using Slotted Line Section of K Band**

### **Structure of the Unit**

- 29.1 Aim
  - 29.2 Apparatus
  - 29.3 Diagram
  - 29.4 Formula
  - 29.5 Theory and Description
  - 29.6 Self Learning Exercise -I
  - 29.7 Procedure
  - 29.8 Observation
  - 29.9 Calculations
  - 29.10 Result
  - 29.11 Discussion
  - 29.12 Precautions and Sources of Error
  - 29.13 Self Learning Exercise -II
  - 29.14 Glossary
  - 29.15 Answers to Self Learning Exercises
  - 29.16 Viva Questions
  - 29.17 Answers to Viva Questions
- References and Suggested Readings

### **29.1 Aim**

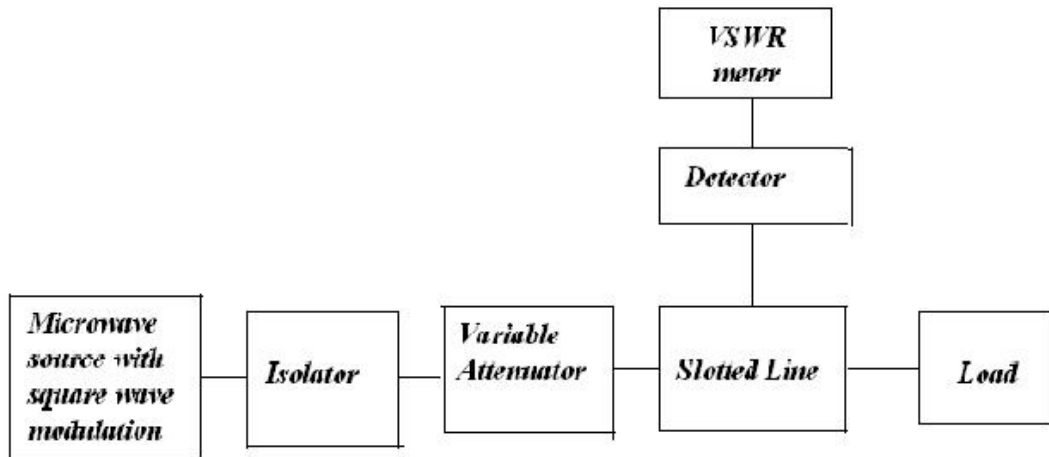
To determine the unknown impedance using slotted line section of K band.

### **29.2 Apparatus**

Microwave source with square wave modulation, Isolator, Variable attenuator

Detector, VSWR meter, Slotted line, Matched Termination, Slide screw Tuner.

### 29.3 Diagram



**Figure 29.1: Diagram for determine unknown impedance**

### 29.4 Formula

The unknown terminating impedance can be determined by measuring standing wave ratio & distance of a convenient maxima or minima from the load. Normally for distance measurement minima is used because it is more sharply defined. The unknown load admittances is given by the transmission equation as

$$Y = \frac{S - j \tan\left(\frac{2\pi d_{min.}}{\lambda_g}\right)}{1 - j S \tan\left(\frac{2\pi d_{min.}}{\lambda_g}\right)}$$

where,  $S =$  VSWR (Voltage Standing Wave Ratio).

and  $d_{min.} =$  distance of first minima from the load.

A screw projecting into the waveguide offers variation in the admittance with the insertion of the screw. The depth of screw, changes only the reactive part of the admittance, so if the line is matched, the load offered by the screw for a certain depth is

$$Y = 1 + j b$$

where  $b$  is the susceptance due to the screw. The unknown impedance can also be determined by using chart, once the VSWR and position of minima is known with the load.

## 29.5 Theory and Description

**Slotted lines** are used for microwave measurements and consist of a movable probe inserted into a slot in a transmission line. They are used in conjunction with a microwave power source and usually, in keeping with their low-cost application, a low cost Schottky diode detector and VSWR meter rather than an expensive microwave power meter.

Slotted lines can measure standing waves, wavelength, and, with some calculation or plotting on Smith charts, a number of other parameters including reflection coefficient and electrical impedance. A precision variable attenuator is often incorporated in the test setup to improve accuracy. This is used to make level measurements, while the detector and VSWR meter are retained only to mark a reference point for the attenuator to be set to, thus eliminating entirely the detector and meter measurement errors. The parameter most commonly measured by a slotted line is SWR. This serves as a measure of the accuracy of the impedance match to the item under test. This is especially important for transmitting antennas and their feed lines; high standing wave ratio on a radio or TV antenna can distort the signal, increase transmission line loss and potentially damage components in the transmission path, possibly even the transmitter.

Slotted lines are no longer widely used, but can still be found in budget applications. Their main drawback is that they are labour-intensive to use and require calculation, tables, or plotting to make use of the results. They need to be made with mechanical precision and the probe and its detector need to be adjusted with care, but they can give very accurate results.

The slotted line is one of the basic instruments used in radio frequency test and measurement at microwave frequencies. It consists of a precision transmission line, usually co-axial but waveguide implementations are also used, with a movable insulated probe inserted into a longitudinal slot cut into the line. In a co-axial slotted line, the slot is cut into the outer conductor of the line. The probe is inserted past the outer conductor, but not so far that it touches the inner conductor. In a rectangular waveguide, the slot is usually cut along the centre of the broad wall of the waveguide. Circular waveguide slotted lines are also possible.

Slotted lines are relatively cheap and can perform many of the measurements



done by more expensive equipment such as network analysers. However, slotted line measurement techniques are more labour-intensive and often do not directly output the desired parameter; some calculation or plotting is frequently required. In particular, they can only carry out a measurement at one spot frequency at a time so producing a plot of a parameter versus frequency is very time consuming. This is to be compared to modern instruments like network and spectrum analysers which are intrinsically frequency swept and produce a plot instantly. Slotted lines have now largely been superseded, but are still found where capital costs are an issue. Their remaining uses are mostly in the millimetre band, where modern test apparatus is either prohibitively expensive or not available at all, and with academic laboratories and hobbyists. They are also useful as a teaching aid as the user is more directly exposed to basic line phenomena than with more sophisticated instruments.

### **Operation:**

The slotted line works by sampling the electric field inside the transmission line with the probe. For accuracy, it is important that the probe disturbs the field as little as possible. For this reason the probe diameter and slot width are kept small (usually around 1 mm) and the probe is inserted in no further than necessary. It is also necessary in waveguide slotted lines to place the slot at a position where the current in the waveguide walls is parallel to the slot. The current will then not be disturbed by the presence of the slot as long as it is not too wide. For the dominant mode this is on the centre-line of the broad face of the waveguide, but for some other modes it may need to be off-centre. This is not an issue for the co-axial line because this operates in the TEM (transverse electromagnetic) mode and hence the current is everywhere parallel to the slot. The slot may be tapered at its ends to avoid discontinuities causing reflections.

The disturbance to the field inside the line caused by the insertion of the probe is minimised as far as possible. There are two parts to this disturbance. The first part is due to the power the probe has extracted from the line and manifests as a lumped equivalent circuit of a resistor. This is minimised by limiting the distance the probe is inserted into the line so that only enough power is extracted for the detector to operate effectively. The second part of the disturbance is due to energy stored in the field around the probe and manifests as a lumped equivalent of a

capacitor. This capacitance can be cancelled out with an inductance of equal and opposite impedance. Lumped inductors are not practical at microwave frequencies; instead, an adjustable stub with an inductive equivalent circuit is used to "tune out" the probe capacitance. The result is an equivalent circuit of high impedance in shunt across the line which has little effect on the transmitted power in the line. The probe is more sensitive as a result of this tuning and the distance it is inserted can be further limited as a result.

## 29.6 Self Learning Exercise -I

**Q.1** What is an amplifier? Show by giving example that how can you use an amplifier in large electronic systems.

**Q.2** Give the General classification of Amplifier.

## 29.7 Procedure

1. Set up the circuit as shown in the fig 29.1. Fix the frequency and amplitude of the source with the help of slider and knob. Fix the modulator frequency using slider. Select "**Forward direction**" in isolator and adjust the attenuation using slider.
2. Four types of loads i.e. matched load, short circuit, open circuit and unmatched load are used here. Realizing a screw whose reactance vary with insertion is not feasible. Hence we are using distance of minima to find the impedance.
3. Choose "**Unmatched Load**" to perform the experiment from the Loads given in Loads window. We can vary the resistance and reactance of load by using sliders in case of "**Unmatched Load**".
4. Use button to select between "**Short Circuit**" and other "**load**". First select "**Short Circuit**".
5. Find first and second minima by using slider. Fix the first minima by clicking "**First minima from load (Short Circuited)**" and second minima by clicking "**Second minima from load (Short Circuited)**".
6. Use toggle button to switch to "**Load**" mode.
7. Again find the first minima for load condition by using slider and fix it by

pressing the button “**First minima from load (Unknown Impedance)**”.

8. Fix the value of VSWR by pressing “**VSWR value of unknown impedance**”. Now press the “**Value of Unknown Impedance**” button.
9. The resistance and reactance for the unknown impedance can be seen in the visible tabs. That can be matched with the chosen one.

## 29.8 Observation

S. No.	Load	VSWR Value	Value of Reactance
1.			
2.			
3.			

## 29.9 Calculations

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## 29.10 Result

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### Percentage Error

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

## 29.11 Discussion

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## 29.12 Precautions and Sources of Error

1. Precaution should be taken to read the value of reactance.
2. All Calculation should be done in the same unit.
3. Connections should be checked by teacher and then on the button.
4. Voltage standing wave ratio should always be calculated.

## 29.13 Self Learning Exercise -II

- Q.1** Give brief account of theory on Impedance.
- Q.2** Give a brief account of theory on Reactance.
- Q.3** Give a brief account theory on Low pass Filter.

## 29.14 Glossary

**SWR:** In a radio frequency (RF) electrical transmission system, the standing wave ratio is a measure of how efficiently RF power is transmitted from the power source, through the transmission line.

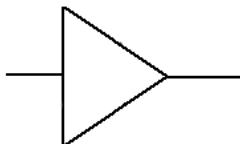
**VSWR:** It is defined as the ratio of the maximum voltage to the minimum voltage in the standing wave.

**Reactance:** It is the opposition of a circuit element to a *change* in current or voltage, due to that element's inductance or capacitance.

## 29.15 Answers to Self Learning Exercises

### **Answers to Self Learning Exercise-I**

**Ans.1:** An amplifier is used to increase the amplitude of a signal waveform, without changing other parameters of the waveform such as frequency or wave shape. They are one of the most commonly used circuits in electronics and perform a variety of functions in a great many electronic systems.



*Fig 29.2: Amplifier general symbol, used in system diagrams*

The general symbol for an amplifier is shown in above given Fig 29.2. The symbol gives no detail of the type of amplifier described, but the direction of signal flow can be assumed (as flowing from left to right of the diagram). Amplifiers of different types are also often described in system or block diagrams by name.

### Amplifiers as Parts of Large Electronic Systems:

For example look at the block diagram of an analogue TV receiver in below given figure and see how many of the individual stages (shaded green) that make up the TV are amplifiers. Also notice that the names indicate the type of amplifier used. In some cases the blocks shown are true amplifiers and in others, the amplifier has extra components to modify the basic amplifier design for a special purpose. This method of using relatively simple, individual electronic circuits as "building blocks" to create large complex circuits is common to all electronic systems; even computers and microprocessors are made up of millions of logic gates, which are simply specialised types of amplifiers. Therefore to recognise and understand basic circuits such as amplifiers is an essential step in learning about electronics.

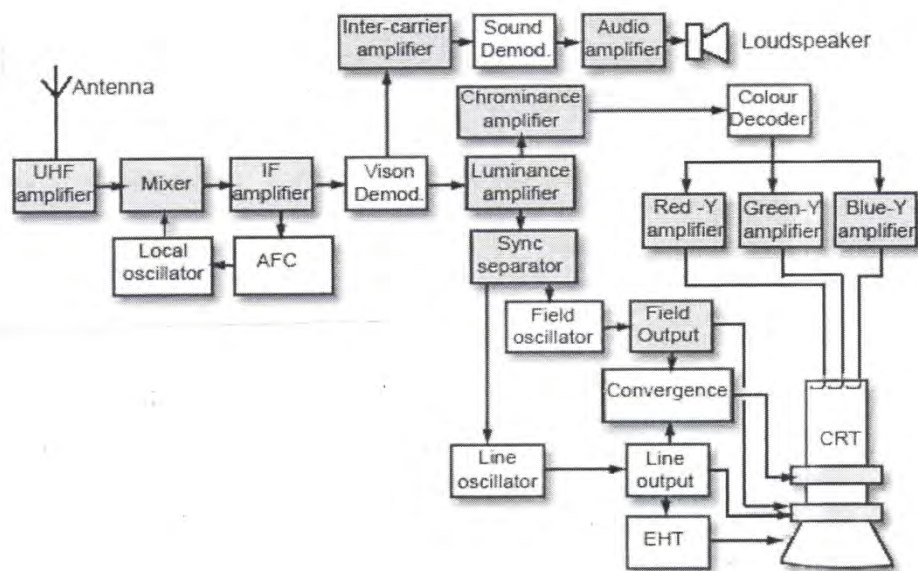


Figure 29.3: Block diagram of TV Receiver.

**Ans.2:** General amplifiers are given below:

## A.F. Amplifiers:

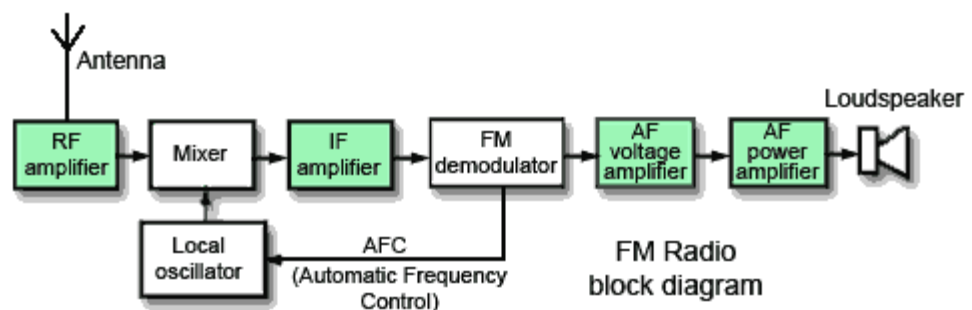
Audio frequency amplifiers are used to amplify signals in the range of human hearing, approximately 20Hz to 20kHz, although some Hi-Fi audio amplifiers extend this range up to around 100kHz, whilst other audio amplifiers may restrict the high frequency limit to 15kHz or less.

Audio voltage amplifiers are used to amplify the low level signals from microphones, tape and disk pickups etc. With extra circuitry they also perform functions such as tone correction equalisation of signal levels and mixing from different inputs, they generally have high voltage gain and medium to high output resistance.

Audio power amplifiers are used to receive the amplified input from a series of voltage amplifiers, and then provide sufficient power to drive loudspeakers.

## I.F. Amplifiers:

Intermediate Frequency amplifiers are tuned amplifiers used in radio, TV and radar. Their purpose is to provide the majority of the voltage amplification of a radio, TV or radar signal, before the audio or video information carried by the signal is separated (demodulated) from the radio signal. They operate at a frequency lower than that of the received radio signal, but higher than the audio or video signals eventually produced by the system. The frequency at which I.F. amplifiers operate and the bandwidth of the amplifier depends on the type of equipment. For example, in AM radio receivers the I.F. amplifiers operate at around 470 kHz and their bandwidth is normally 10kHz (465 kHz to 475kHz), while TV commonly uses 6MHz bandwidth for the I.F. signal at around 30 to 40MHz, and in radar a band width of 10 MHz may be used.



*Fig: FM Radio using AF, IF and RF amplifiers.*

### **R.F. Amplifiers:**

Radio Frequency amplifiers are tuned amplifiers in which the frequency of

Radio Frequency amplifiers are tuned amplifiers in which the frequency of operation is governed by a tuned circuit. This circuit may or may not, be adjustable depending on the purpose of the amplifier. Bandwidth also depends on use and may be relatively wide, or narrow. Input resistance is generally low, as is gain. (Some RF amplifiers have little or no gain at all but are primarily a buffer between a receiving antenna and later circuitry to prevent any high level unwanted signals from the receiver circuits reaching the antenna, where it could be re-transmitted as interference). A special feature of RF amplifiers where they are used in the earliest stages of a receiver is low noise performance. It is important that background noise generally produced by any electronic device, is kept to a minimum because the amplifier will be handling very low amplitude signals from the antenna ( $\mu\text{V}$  or smaller). For this reason it is common to see low noise FET transistors used in these stages.

### **Ultrasonic Amplifiers:**

Ultrasonic amplifiers are a type of audio amplifier handling frequencies from around 20 kHz up to about 100kHz; they are usually designed for specific purposes such as ultrasonic cleaning, metal fatigue detection, ultrasound scanning, remote control systems etc. Each type will operate over a fairly narrow band of frequencies within the ultrasonic range.

### **Wideband Amplifiers:**

Wideband amplifiers must have a constant gain from DC to several tens of MHz. They are used in measuring equipment such as oscilloscopes etc. where there is a need to accurately measure signals over a wide range of frequencies. Because of their extremely wide bandwidth, gain is low.

### **DC Amplifiers:**

DC amplifiers are used to amplify DC (0Hz) voltages or very low frequency signals where the DC level of the signal is important. They are common in many electrical control systems and measuring instruments.

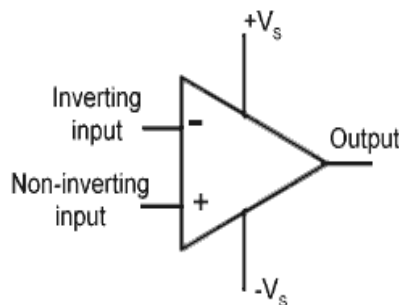
## Video Amplifiers:

Video amplifiers are a special type of wide band amplifier that also preserve the DC level of the signal and are used specifically for signals that are to be applied to CRTs or other video equipment. The video signal carries all the picture information in TV, video and radar systems. The bandwidth of video amplifiers depends on use. In TV receivers it extends from 0Hz (DC) to 6MHz and is wider still in radar.

## Buffer Amplifiers:

Buffer amplifiers are a commonly encountered, specialised amplifier type that can be found within any of the above categories, they are placed between two other circuits to prevent the operation of one circuit affecting the operation of the other. (They ISOLATE the circuits from each other). Often buffer amplifiers have a gain of one, i.e. they do not actually amplify, so that their output is the same amplitude as their input, but buffer amplifiers have a very high input impedance and a low output impedance and can therefore be used as an impedance matching device. This ensures that signals are not attenuated between circuits, as happens when a circuit with a high output impedance feeds a signal directly to another circuit having a low input impedance.

## Operational Amplifiers



Operational amplifiers (Op-amps) have developed from circuits designed for the early analogue computers where they were used for mathematical operations such as adding and subtracting. Today they are widely used in integrated circuit form where they are available in single or multiple amplifier packages and often incorporated into complex integrated circuits for specific applications.

The design is based on a differential amplifier, which has two inputs instead of one, and produces an output that is proportional to the difference between the



two inputs. Without negative feedback, op amps have an extremely high gain, typically in the hundreds of thousands. Applying negative feedback increases the op amp's bandwidth so they can operate as wideband amplifiers with a bandwidth in the MHz range, but reduces their gain. A simple resistor network can apply such feedback externally and other external networks can vary the function of op-amps.

### ***Answers to Self Learning Exercise-II***

**Ans.1:** The term Impedance is familiar to anyone who has connected a loudspeaker to an audio system, it is generally a number of Ohms, often printed next to many input or output sockets. However the property of Impedance is perhaps less understood.

The word impedance is used in many engineering disciplines to refer to an opposition to work done, so this article refers specifically to electrical impedance, which describes the combined effect of resistance ( $R$ ), inductive reactance ( $X_L$ ) and capacitive reactance ( $X_C$ ) in an AC circuit, whether it occurs in a single component, or in a whole circuit.

Because electrical impedance (here called simply "impedance") is affected by reactance ( $X$ ), as well as resistance ( $R$ ), it is also affected by frequency ( $f$ ) and the value of impedance will change at different frequencies. To describe the impedance of a component or circuit in terms of so many Ohms is therefore inadequate (but commonly done). To be accurate, the impedance needs to be expressed as so many Ohms at a particular frequency. What frequency is used will depend on the type of circuit being described and what range of frequencies the circuit is handling. The frequency chosen for the measurement of impedance is usually around the equipment's mid frequency range.

For example, in audio circuits which may have a frequency range from a few Hz to around 20kHz a frequency of 1kHz is commonly specified. Although at first this does seem like the centre of the frequency range, if this range is considered on a logarithmic scale (as frequency scales commonly are) then 1kHz is at the centre of the audio range.

**Ans.2:** Resistance is unaffected by the rate of change of voltage or current and remains the same whether DC, or AC of any frequency is applied. In a purely

resistive circuit Ohms law applies and  $V = IR$ . Calculations of resistance in AC circuits are the same as for DC. The only extra thing to remember when calculating AC values, is that the same type of value, e.g. the RMS, Average value, or the peak-to-peak value must be used for all AC voltages or currents throughout the equation. Inductance and capacitance however, have an effect on current which depends on frequency, and when either component is used with AC voltages and currents, then the frequency of the waveform must be taken into account.

Inductive and capacitive reactance has opposite effects in relation to frequency. Inductive reactance rises with an increase in frequency, and capacitive reactance falls. Both are similar to resistance and are measured in Ohms, but resistance remains the same, whatever the frequency. The interplay between resistance and reactance will form the basis of many effects in the later modules.

When the current in an Inductor changes, a back emf is created that opposes the change in current, and the faster the initial change in current the greater the back emf. So it is not surprising that, the faster rates of change of current that occur as the frequency of the wave increases, produce a greater back emf effect that in turn, reduces current flow more than it does at lower frequencies.

This variable opposition to current flow in an inductor, is related to the amount of the inductance, because the larger the value of inductance the greater the back emf effect produced. The opposition to current flow through an inductor is proportional to both the amount of inductance and to the frequency of the current in the inductor. This opposition to current flow is called **INDUCTIVE REACTANCE** ( $X_L$ ). The formula for Inductive Reactance multiplies the angular velocity of the AC wave by the value of Inductance:

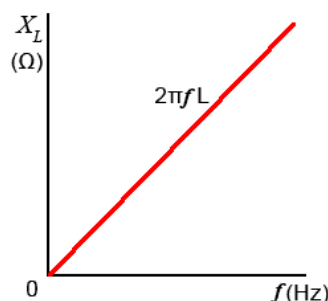


Fig: Inductive Reactance.

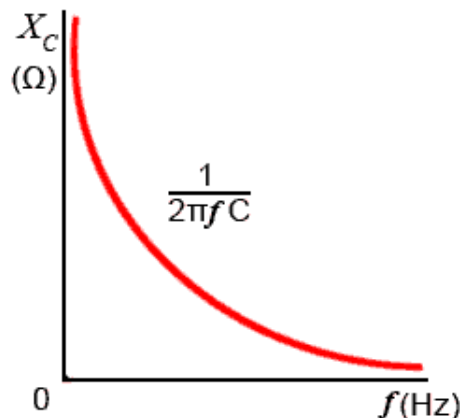
$$X_L = 2\pi fL = \omega L$$

where  $2\pi f$  or  $\omega$  is the angular velocity and  $L$  is the inductance

In a capacitor with direct voltage applied that the current flow falls to zero after the initial transient period. When an alternating voltage of any appreciable frequency is applied however, current flows first in one direction, and then the other. The capacitor is first charging, and then discharging, so provided that the frequency of the applied AC is high enough, the capacitor never reaches its fully charged, zero current state in either polarity, and current continues to flow all the time. The amount of current flowing will depend on the angular velocity of the applied voltage, and on the capacitance of the capacitor.

The opposition to current flow in any capacitor of a given size therefore reduces as frequency increases. This frequency dependent opposition to current flow in a capacitor is called CAPACITIVE REACTANCE ( $X_C$ ). The formula for capacitive reactance is:

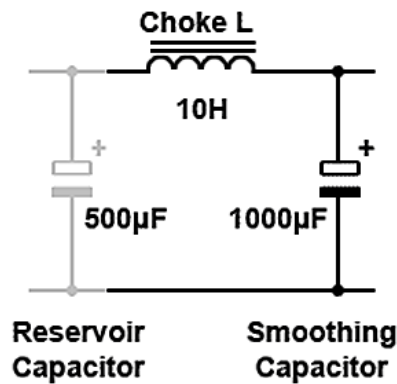
$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$



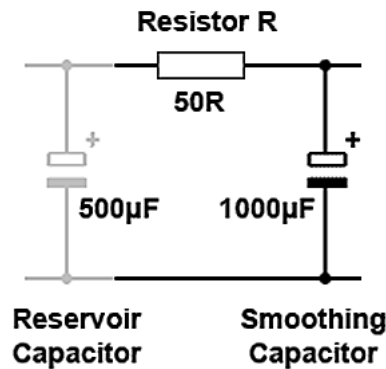
Above given fig. shows a graph of capacitive reactance against frequency for a given value of capacitor, with capacitive reactance ( $X_C$ ) inversely proportional to frequency, ( $X_C$  reducing as frequency increases).

**Ans.3:** Although a use able power supply can be made using only a reservoir capacitor to remove AC ripple, it is usually necessary to also include a low pass filter and/or a regulator stage after the reservoir capacitor to remove any

remaining AC ripple and improve the stabilisation of the DC output voltage under variable load conditions.



**Fig: 1: LC Filter**



**Figure 2: RC Filter**

Either LC or RC low pass filters can be used to remove the ripple remaining after the reservoir capacitor. The LC filter shown in Fig. 1 is more efficient and gives better results than the RC filter shown in Fig.2 but for basic power supplies, LC designs are less popular than RC, as the inductors needed for the filter to work efficiently at 50 to 120Hz need to be large and expensive laminated or toroidal core types. However modern designs using switch mode supplies, where any AC ripple is at much higher frequencies, much smaller ferrite core inductors can be used.

The low pass filter passes low frequency, in this case DC (0Hz) and blocks higher frequencies, whether 50Hz or 120Hz in basic circuits or tens of kHz in switch mode designs.

The reactance ( $X_C$ ) of the capacitor in either of the filters is very low compared with the resistance of resistor  $R$  or the reactance of the choke  $X_L$  at the ripple frequency. In RC designs the resistance of  $R$  must be a fairly low value as the entire load current, maybe several amperes, must pass through it, generating a considerable amount of heat. A typical value would therefore be 50 ohms or less, and even at this value, a large wire wound resistor would normally need to be used. This limits the efficiency of the filter as the ratio between the resistance of  $R$  and the capacitor reactance will not be greater than about 25:1. This then would be the typical reduction ratio of the ripple amplitude. By including the low pass filter some voltage is lost across the resistor, but this disadvantage is offset by the better ripple performance than by using the reservoir capacitor alone.

The LC filter performs much better than the RC filter because it is possible to make the ratio between  $X_C$  and  $X_L$  much bigger than the ratio between  $X_C$  and  $R$ . Typically the ratio in a LC filter could be 1:4000 giving much better ripple rejection than the RC filter. Also, since the DC resistance of the inductor in the LC filter is much less than the resistance of  $R$  in the RC filter, the problem of heat being generated by the large DC current is very much reduced in LC filters.

With a combined reservoir capacitor and low pass filter it is possible to remove 95% or more of the AC ripple and obtain an output voltage of about the peak voltage of the input wave. A simple power supply consisting of only transformer, rectifier, reservoir and low pass filter however, does have some drawbacks.

### 29.16 Viva Questions

- Q.1** Give the definition of Direct current.
- Q.2** What are passive Filters?
- Q.3** What are high pass filter?
- Q.4** What are band pass filter?
- Q.5** What are band stop filter?
- Q.6** What are conductors?
- Q.7** What are Insulators?

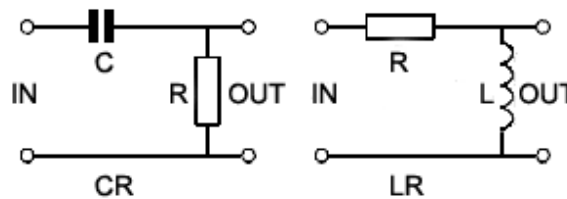
- Q.8 What is resistivity?  
 Q.9 What is Ohm's Law?  
 Q.10 What are semiconductors?

### 29.17 Answers to Viva Questions

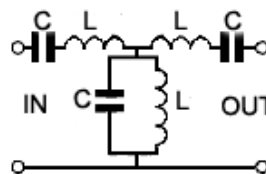
**Ans.1:** It is the unidirectional flow of electric charge. Direct current is produced by sources such as batteries, thermocouples, solar cells. It may flow in conductor such as a wire, but can also flow through in semiconductors. A term also use for this type of the current is *Galvanic current*.

**Ans.2:** Passive filters are circuit sub units often consisting of only two or three components, which reduce (ATTENUATE) the amplitude of signals.

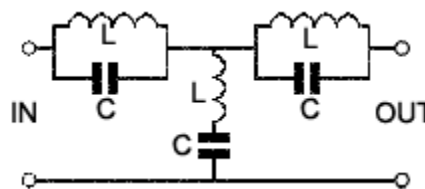
**Ans.3:** High pass filters are used to remove or attenuate the lower frequencies in amplifiers, especially audio amplifiers where it may be called a "BASS CUT" circuit.



**Ans.4:** Band pass filters allow only a required band of frequencies to pass, while rejecting signals at all frequencies above and below this band.



**Ans.5:** These filters have the opposite effect to band pass filters, there are two parallel LC circuits in the signal path to form a high impedance at the unwanted signal frequency, and a series circuit forming a low impedance path to ground at the same frequency, to add to the rejection.



**Ans.6:** These are materials that allow electric current to flow through them because their atomic structure lets the outermost (free) electrons move easily from one atom to another, and because the electrons carry a negative electric charge they are easily repelled by an applied negative electric charge and attracted by a positive charge. Therefore applying a voltage between the two ends of a conductor causes an 'electron drift' from negative to positive giving rise to an electric current.

**Ans.7:** Materials that prevent the flow of electric current are called INSULATORS. These are mostly solid materials in which the outer electrons of each atom are tightly bonded to the nucleus of the atom, preventing any electron movement within a 'normal range' of applied voltage.

**Ans.8:** This is the amount of resistance present in a piece of the material of STANDARD DIMENSIONS.

**Ans.9:** Ohm's Law states that: "In metallic conductors at a constant temperature and in a zero magnetic field, the current flowing is proportional to the voltage across the ends of the conductor, and is inversely proportional to the resistance of the conductor."

**Ans.10:** Semiconductors fall between group conductor and insulator. They do not normally pass current easily at room temperature, having a resistivity higher than conductors, but lower than insulators. They have properties however which make them very useful for the manufacture of electronic devices such as transistors and diodes.

## References and Suggested Readings

1. E.L. Ginzton: Microwave Measurements.
2. Annapurna Das, Sisir K Das, Microwave Engineering.
3. C. G. Montgomery, Techniques of Microwave Measurements.

# **UNIT-30**

## **Magnetic Susceptibility “Gouy’s Method”**

### **Structure of the Unit**

- 30.1 Aim
  - 30.2 Apparatus
  - 30.3 Diagram
  - 30.4 Formula
  - 30.5 Theory And Description
  - 30.6 Self Learning Exercise-I
  - 30.7 Procedure
  - 30.8 Observation
  - 30.9 Calculations
  - 30.10 Result
  - 30.11 Percentage Error
  - 30.12 Discussion
  - 30.13 Precautions And Sources Of Error
  - 30.14 Self Learning Exercise-II
  - 30.15 Glossary
  - 30.16 Answers To Self Learning Exercises
  - 30.17 Viva Questions
  - 30.18 Answers To Viva Questions
- References and Suggested Readings

### **30.1 Aim**

To determine the magnetic susceptibility of a solid by Gouy’s method.



## 30.2 Apparatus

An electromagnet with power supply, a sensitive balance (preferably a digital weighing machine), solid object in the shape of a long cylinder, a gauss-meter (or a flux-meter) and fractional weights.

## 30.3 Diagram

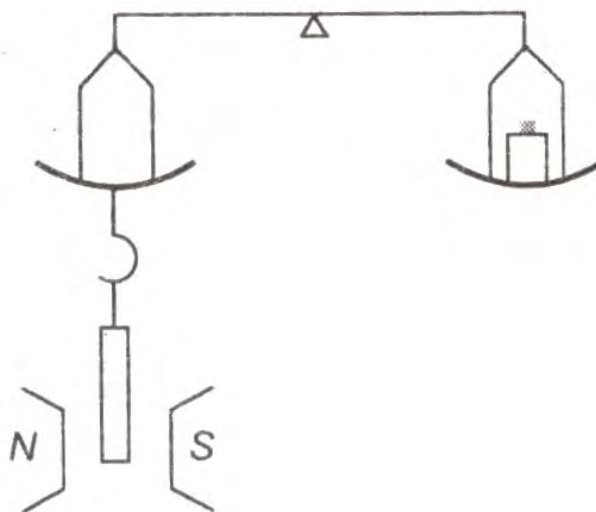


Figure 30.1: Diagram for the determination of magnetic susceptibility.

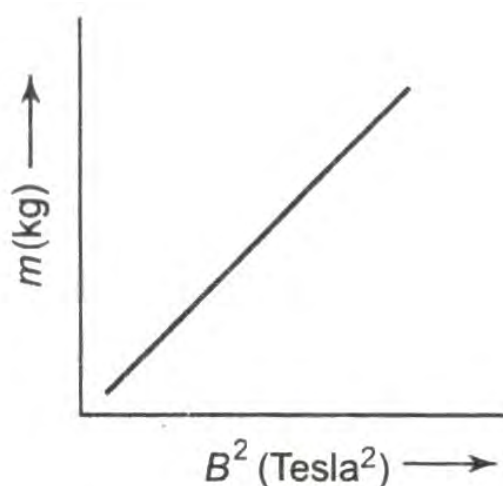


Figure 30.2: Graph between mass ( $m$ ) and Magnetic field ( $B$ ).

## 30.4 Formula

The formula for the determination of the magnetic susceptibility of solid is given by

$$\chi = \mu_0 \frac{2 mg}{AB^2}$$

Where  $A$  is the area of cross section of the substance (*in meter<sup>2</sup>*)

$g$  is the acceleration due to gravity ( $9.8 \text{ m/s}^2$ )

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A.m.}$$

$B$ = Homogeneous magnetic field.

Thus by the measurement of the  $m$ ,  $A$ ,  $B$ , the susceptibility  $\chi$  of the sample can be determined.

## 30.5 Theory and Description

This method name is based on the name of the scientist Gouy. This method depends on the force exerted on a body placed in a non-homogeneous magnetic field. The variable magnetic field is provided by an electromagnet with wedge shaped pole pieces. Figure 30.1 illustrates the experiment set-up. The field of the magnet varies rapidly along the vertical direction, due to the wedging of the pole pieces. Thus, the force on the specimen is vertical. In this experiment the influence of the earth's field is neglected. The specimen whose magnetic susceptibility is to be determined is connected to one arm of a sensitive balance or a digital weighing machine. The balance is enclosed in a box so that the air does not affect the weighing.

Magnetic susceptibility is a dimensionless proportionality constant that indicates the degree of magnetization of a material in response to an applied magnetic field. A related term is **magnetizability**, the proportion between magnetic moment and magnetic flux density. A closely related parameter is the permeability, which expresses the total magnetization of material and volume.

The Gouy balance measures the apparent change in the mass of the sample as it is repelled or attracted by the region of high magnetic field between the poles. Some commercially available balances have a port at their base for this application. In use, a long, cylindrical sample to be tested is suspended from a balance, partially

entering between the poles of a magnet. The sample can be in solid or liquid form, and is often placed in a cylindrical container such as a test tube. Solid compounds are generally ground into a fine powder to allow for uniformity amongst the sample. The sample is suspended between the magnetic poles through an attached thread or string. The experimental procedure requires two separate readings to be performed. An initial balance reading is performed on the sample of interest without a magnetic field ( $m_a$ ). A subsequent balance reading is taken with an applied magnetic field ( $m_b$ )

The apparent change in mass from the two balance readings is a result of magnetic force on the sample. The magnetic force is applied across the gradient of a strong and weak magnetic field. A sample with a paramagnetic compound will be pulled down towards the magnetic, and provide a positive difference in apparent mass  $m_b - m_a$ . Diamagnetic compounds can either exhibit no apparent change in weight or a negative change as the sample is slightly repelled by the applied magnetic field. With a paramagnetic sample, the magnetic induction is stronger than the applied field and magnetic susceptibility is positive. A diamagnetic sample has a magnetic induction much weaker than the applied field, and a respective negative magnetic susceptibility. The following mathematical equation relates the apparent change in mass to the volume susceptibility of the sample:

$$F = (m_b - m_a)g = 1/2 (K_2 - K_1)AH^2$$

where

$m_b - m_a$  = apparent difference in mass

$g$  = gravitational acceleration

$K_1$  = volume susceptibility of sample

$K_2$  = volume susceptibility of medium, usually air and of negligible value

$H$  = applied magnetic field

$A$  = area of the sample tube.

There is a hole in the box through which the specimen hangs out. It is hung in the field between the pole pieces of the magnet in such a way that its lower end is in a homogeneous magnetic field  $B$  and the upper end is in a much weaker field. The force resulting from is counter-balanced by altering the weights in the other

pan of the balance. If the difference is denoted by  $mg$ , this is equal to the force arising from the magnetic field.

$$mg = \frac{1}{2} \frac{\chi AB^2}{\mu_0}$$

Or magnetic susceptibility

$$\chi = \mu_0 \frac{2mg}{AB^2}$$

Where  $A$  is the area of cross section of the substance in  $\text{metre}^2$  and  $g$  is the acceleration due to gravity =  $(9.8 \text{ m/s}^2)$

By measurement of  $m$ ,  $A$ , and  $B$ , the susceptibility  $\chi$  of the sample can be determined.

### 30.6 Self Learning Exercise -I

- Q.1 What is Earth magnetism ?
- Q.2 Give the Causes of Earth Magnetism ?
- Q.3 What is ferromagnetism ?
- Q.4 Give the causes of Susceptibility ?

### 30.7 Procedure

For the readings, we should follow below given method:

- (1) First of all, connect the electromagnet with power supply.
- (2) The specimen in the shape of a rod is connected to one arm of a sensitive balance and hung vertically such that its lower end is in between the two pole pieces and the upper end is outside the pole pieces.
- (3) Put the weight in the other arm of the balance so as to bring its needle in the centre. In case of a digital weighing machine a knob is provided for the purpose and the mass is directly given by the machine. Note down the mass  $M$  of the rod.
- (4) Switch on the field  $B$  and add weights in the pan to bring the balance needle at the centre. Note down the mass  $M'$  of the rod in the presence of the

magnetic field. Then  $(M' - M)$  gives the mass  $m$  required to counter balance the effect of the magnetic field. Measure the field using a gauss-meter. Convert it to tesla by multiplying with  $10^{-4}$ .

- (5) Change the field by varying the current in the power-supply of the electromagnet. Note down the magnetic field  $B$  and the corresponding mass  $M'$  of the rod.
- (6) Repeat above process and take 5-6 readings.
- (7) Measure the area of cross-section of the sample with suitable instrument.
- (8) Plot a graph between  $B^2$  along  $x$  - axis and  $m (= M' - M)$  along  $y$ -axis.

### 30.8 Observation

Mass of the rod,  $M = \dots\dots\dots gm = \dots\dots\dots kg$

Cross-section area of the specimen,  $A = \dots\dots\dots cm^2 = \dots\dots\dots m^2$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A.m}$$

$$g = 9.8 \text{ m/s}^2$$

S.No.	Magnetic field B (Tesla)	$B^2$ (Tesla) <sup>2</sup>	Mass of the rod in Mag. Field $M'$ (kg)	Mass to counter balance the field $m =$ $M' - M$
1				
2				
3				
4				
5				
6				
7				

### 30.9 Calculation

$$\chi = \dots \dots \mu_0 \frac{2mg}{AB^2}$$

$$\chi = \mu_0 \frac{2g}{A} \times slope$$

$$\chi = \dots \dots \dots$$

### 30.10 Result

The magnetic susceptibility of given specimen =.....

#### Percentage Error

$$\%error = \frac{Experimental\ value - Standard\ value}{standard\ value} \times 100$$

### 30.12 Discussion

### 30.13 Precautions and Sources of Errors

- (1) The pole pieces of the electromagnetic should be wedge shaped with enough spacing so that the field may be uniform.
- (2) The current in the electromagnet should not be passed for long time.
- (3) The balance should be very sensitive preferably a digital balance and should be enclosed in a box so that the weighing is not affected by the air.
- (4) The specimen should be freely suspended with no mechanical restraint from the balance.
- (5) Care should be taken in reading the position of equipment.

### 30.14 Self Learning Exercise-II

- Q.1** The vertical component of earth's magnetic field at a place is  $\sqrt{3}$  times the horizontal component. What is the value of angle of dip at this place?
- Q.2** The magnetic induction and magnetizing field in a sample of magnetic material are  $1.0 \text{ Wb/m}^2$  and  $2 \times 10^3 \text{ A/m}$  respectively. Find
- magnetic permeability
  - relative permeability of the material
  - magnetic susceptibility and
  - intensity of magnetization.
- Q.3** Give the definitions of Diamagnetic, paramagnetic and ferromagnetic material.

### 30.14 Glossary

**Magnetic Field** : The magnetic field is defined as the space around a magnet in which its magnetic influences extends.

**Magnetic Susceptibility** : The magnetic susceptibility of a magnetic material is defined as the ratio of the intensity of magnetization induced in the material to the magnetic field intensity. It measures the ease with which a specimen can be magnetized.

**Magnetic Permeability** : Magnetic permeability is defined as the measure of the degree of penetration of magnetic field through the substance and is the ratio  $B$  and  $H$ .

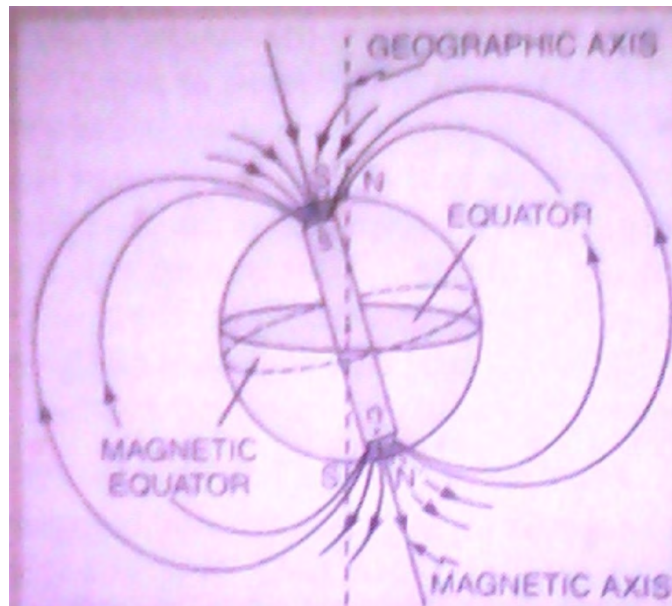
### 30.15 Answers To Self Learning Exercises

#### **Answers To Self Learning Exercise-I**

**Ans.1:** Whenever a bar magnet is suspended freely (near the equator), its N-pole points towards the geographical north and S-pole points towards the geographical south of the earth. William Gilbert suggested that earth itself behaves as a huge magnet, it was assumed that a huge magnet is lying towards the south of the earth. It, then, easily explains as to why the N-pole of a freely suspended bar magnet always points towards the geographical north of the earth. It may be pointed out

that the line joining the geographic north and south of the earth, called geographic axis or geographical N-S line makes a small angle ( $\approx 20^\circ$ ) with the magnetic axis. Figure 30.2 represents the magnetic field lines around the earth.

The axis of rotation of earth is called *geographic axis* and the points, where it passes through the surface of earth, are called geographic poles. The points N and S represent the geographic north and south poles. The great circle on the earth's surface perpendicular to the geographic axis is called *equator*.



**Figure30.2: Earth Magnetism and its axis and equator.**

A vertical plane passing through the geographic axis is called the *geographic meridian*.

The axis of the huge magnet assumed to be lying inside the earth is called *magnetic axis* of the earth and the points, where the magnetic axis cuts the earth's surface are called *earth's magnetic poles*. The points *N'* and *S'* indicate the magnetic north and south of the earth. The great circle on the earth's surface perpendicular to the magnetic axis is called *magnetic equator*.

A vertical plane passing through the magnetic axis of the earth is called *magnetic meridian*.



As said earlier, the geographic and magnetic axis of the earth do not coincide with each other. The magnetic axis of the earth makes an angle of about  $20^\circ$  with the geographic axis. It follows that when a magnet is freely suspended, its north pole will point towards magnetic south ( $S'$ ) and South Pole towards magnetic north ( $N'$ ). In other words, the poles of a freely suspended magnet only approximately point towards geographic north and south. The magnetic north and south poles are in fact at a quite large distance from the geographic poles. At present, the magnetic South pole is located in Northern Canada and is about 1,600 km from the geographic north at  $73^\circ$  north latitude and  $96^\circ$  west longitude. The north magnetic pole is at the diametrically opposite point i.e. at  $75^\circ$  south latitude and  $84^\circ$  east longitude.

**Ans.2: Cause of earth's magnetism:** It may be pointed out that the source of earth's magnetism is not a large solid mass of magnetic material located inside the earth. It is guessed that earth's magnetism is due to molten charged metallic fluid. It gives rise to a current flowing inside the core of the earth. This core has a radius of about 3500 km. Up to a distance of about  $3 \times 10^4$  km, the magnetic field is entirely due to earth's magnetism. At larger distances, the motion of the ions influences the earth's magnetic field. The motion of the ions is affected by a thin hot gas expelled by the sun, called *solar wind*.

Professor Blackett suggested that since every substance is made up of charged particles, hence a substance rotating about an axis will become magnetized. The earth's magnetism is also due to the rotation of the earth about its own axis.

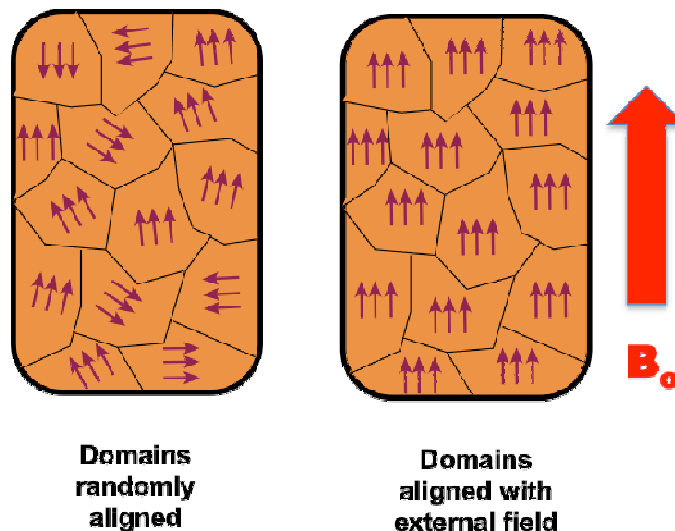
There is another theory about the earth's magnetism. The gases in the atmosphere are in ionized states. The high energy radiation coming from the sun ionizes the atoms in the upper part of the atmosphere. The radioactivity of the atmosphere and cosmic rays also ionize the gases in the atmosphere. Hence, due to rotation of the earth, strong electric currents flow. Due to action of these currents, the earth is magnetized.

The third view regarding earth's magnetism is that the core of the earth is made of iron and nickel. Due to rotation, the earth behaves as a dynamo and becomes magnetized. The earth's magnetism cannot be due to the magnetization of

iron deposits inside the earth. It is because iron cannot retain magnetism at such a high temperature inside the earth.

**Ans.3:** A few elemental solids and alloys, especially those containing iron, nickel, and cobalt, exhibit extremely large positive susceptibilities and are termed **ferromagnetic**. Whereas paramagnetism and diamagnetism are properties of individual atoms or molecules, ferromagnetism is a property of a group of atoms or molecules in a solid crystal or lattice. All ferromagnetic substances have unpaired electron spins that are strongly entwined by a quantum mechanical force, **exchange interaction**. Large groups of atoms in a ferromagnetic substance form **magnetic domains** in which arrays of electron spins become locked together in alignment.

Exchange interaction, also called exchange coupling, is a quantum mechanical phenomenon arising from the balancing of kinetic energies and repulsive forces among outer shell electrons in various spin orientations. Because it is primarily electrostatic (rather than magnetic in nature), the exchange interaction is localized to just a few neighboring atoms. Magnetic domains are small, randomly oriented and spontaneously magnetized regions in a ferromagnetic material whose spins are locked into orderly arrays. Application of small external fields moves the walls between them to form larger domains. Higher external fields reorient the domains to face the same direction and increase net magnetization.



**Figure 30.3: Figure for domain of ferromagnetism.**

The formation of magnetic domains accounts for the extremely large values of  $\chi$  observed in ferromagnetic materials. It also gives them a "magnetic memory," or *remanence*, in which the actual value of  $\chi$  depends on the past magnetization history of the material. Iron, for example, can retain appreciable residual magnetization even after the external magnetic field has been removed. This magnetic memory is one reason iron and iron alloys are frequently chosen for making permanent magnets.

**Ans.4: Susceptibility is caused by interactions of electrons and nuclei with the externally applied magnetic field.**

Nuclei and electrons each possess *spin*, a quantum mechanical property with no exact analogue in classical physics. Don't worry if you don't understand this. Not many people do. For now, it's OK just to think of nuclei and electrons as tiny spinning magnets. When nuclear and electron spins in a material orient in the same direction as the magnetic field, their individual magnetic moments locally augment the field. This augmentation of the external field is called *paramagnetism*. Subatomic particles may also create magnetic effects that oppose the applied field; this is called *diamagnetism*.

Electrons are much smaller than nuclei, so we can think of their spins being as being "concentrated" into smaller volumes. As a consequence of this spin/size discrepancy, electron-field interactions are thousands of times stronger than nuclear ones.

**Electrons, not nuclei, primarily determine overall magnetic susceptibility of a material.**

In addition to having spin, electrons "orbit" the nucleus and possess a second quantum property - *orbital angular momentum (L)*. How electrons are distributed among orbitals is critical in determining magnetic susceptibility. **Orbitals containing paired electrons contribute to diamagnetism; orbitals with unpaired electrons contribute to paramagnetism.**

In many substances there are several competing diamagnetic and paramagnetic effects whose net sum determines bulk susceptibility. The major mechanisms contributing to susceptibility are briefly described as:

**a. Langevin (Larmor) diamagnetism:**

This phenomenon results from the angular momentum ( $L$ ) of electrons in filled orbitals. Through a quantum process similar to Lenz' Law, the orbital motion of paired electrons generates an internal field that opposes the externally applied one. The effect is relatively weak, but because all molecules contain at least some filled orbitals, Langevin diamagnetism is present in all materials. Langevin diamagnetism dictates the overall susceptibility of a material unless it is overridden by a more powerful mechanism.

Water, most biological materials, nonmetallic atoms, stable salts, and covalently bonded molecules have no unpaired electrons, so their overall susceptibilities are dominated by Langevin diamagnetism. Their susceptibilities ( $\chi$ ) are weak and negative, typically with values close to  $-0.00001$ .

**b. Van Vleck paramagnetism:**

This can be considered a minor counter-effect to Langevin diamagnetism, becoming important only for a few polyatomic molecules and certain atoms/ions with shells exactly one electron short of being half-filled. In water, for example, Van Vleck paramagnetism is only 10% the strength of Langevin diamagnetism.

**c. Nuclear paramagnetism:**

Certain nuclei with non-zero spins ( $^1\text{H}$ ,  $^{13}\text{C}$ ,  $^{23}\text{Na}$ , etc) - the same nuclei that undergo NMR - make a minimal positive contribution to the susceptibility of materials that contain them. As stated above, this effect is extremely small (typically  $< 0.01\%$  of the already weak Langevin diamagnetism) so it can be largely disregarded except in the immediate vicinity of the nucleus.

**d. Curie Paramagnetism:**

This important mechanism occurs in atoms and molecules with one or more unpaired electrons. It is moderately powerful -- a thousand times greater than Langevin diamagnetism. In such materials the unpaired electrons experience net "alignment" with the external field (similar to the net alignment of nuclei in NMR). This alignment augments the applied field and is therefore paramagnetic. Curie paramagnetism is important in:

- Atoms, molecules, and lattice defects with odd numbers of electrons (ions, NO, organic free radicals, charged molecules);
- Free atoms and ions with partly filled inner shells (transition metals, rare earth elements, actinides and lanthanides -- especially gadolinium);
- A few compounds with an even number of electrons but with unpaired spins in bonding or antibonding orbitals.

Among the elements, ***gadolinium*** has one of the strongest Curie paramagnetic effects because it possesses 7 unpaired electrons in its 4f shells. For ions like gadolinium with partially filled shells whose magnetism comes purely from electron spin, the degree of expected paramagnetism is proportional to  $N(N + 2)$ , where  $N$  = the number of unpaired electrons.

**e. Mechanisms in simple metals (Pauli Paramagnetism and Landau Diamagnetism):**

Metals can be viewed as closed-shell cores surrounded by a "sea" of free conduction (valence) electrons. The cores have no unpaired electrons and thus exhibit only mild Langevin diamagnetism. The conduction electrons, however, interact more strongly with the external magnetic field. The usual dominant effect is to augment the applied field (***Pauli paramagnetism***), but they also generate an opposing field (***Landau diamagnetism***), which is usually smaller than the Pauli effect, but not always. Thus some metals with filled inner shells may demonstrate mild paramagnetism (Al, Ti), while others are weakly diamagnetic (Cu, Pb, Ag).

**f. Exchange Coupling and Magnetic Domains:**

A few elemental and compound solids (containing transition metal and/or rare-earth atoms) demonstrate enormous positive susceptibilities that may even remain after the external magnetic field is removed. The resultant state is called **ferromagnetism**, whose origin is based on a quantum mechanical effect known as ***exchange coupling***. These materials form ***magnetic domains*** - small regions whose spins become locked into orderly arrays and become spontaneously magnetized. Such phenomena are responsible for permanent magnetism of iron, nickel, cobalt and their alloys, and may manifest in different forms including ferromagnetism, ferrimagnetism, anti-ferromagnetism, and ***superparamagnetism***.

## Answers to Self Learning Exercise-II

**Ans.5:** Here given

$$B_V = \sqrt{3}B_H$$

Now,  $\tan \theta = \frac{B_V}{B_H}$

$$\tan \theta = \frac{\sqrt{3}B_H}{B_H}$$

$$\tan \theta = \tan 60^\circ$$

Hence

$$\theta = 60^\circ$$

**Ans. 6:** Here  $B = 1.0 \text{ wb/m}^2$  and  $H = 2 \times 10^3 \text{ A/m}$

(a) Magnetic permeability

$$\mu = \frac{B}{H} = \frac{1.0}{2 \times 10^3} = 5 \times 10^{-4} \text{ T} - \text{m/A}$$

(b) Relative permeability

$$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 397.9$$

(c) Magnetic susceptibility

$$\chi_m = \mu_r - 1 = 397.9 - 1 = 396.9$$

(d) Intensity of magnetization

$$\begin{aligned} I &= \chi_m H \\ &= 396.9 \times 2 \times 10^3 \\ &= 7.94 \times 10^5 \text{ A/m} \end{aligned}$$

**Ans.7: Diamagnetic:** Those substance, which when placed in a magnetic field are freely magnetized in a direction opposite to that of the magnetizing field, are called diamagnetic substance.

When a diamagnetic substance is placed inside an external magnetic field, the magnetic field inside the diamagnetic is found to be slightly less than the external magnetic field. It is observed that when a diamagnetic sample is placed inside a

non-uniform magnetic field, it tends to move from stronger part to the weaker part of the magnetic field. The diamagnetic substance is independent of temperature.

**Paramagnetic:** Those substance, which when placed in a magnetic field are freely magnetized in the direction of the magnetizing field, are called paramagnetic substances.

When paramagnetic substance is placed inside an external magnetic field, the magnetic field inside the paramagnetic is found to be a slightly greater than the external magnetic field. In contrast to the behavior of diamagnetic, a paramagnetic substance tends to move from weaker part of the magnetic field to stronger part when placed in a non-uniform magnetic field. The paramagnetic substance is dependent of temperature.

**Ferromagnetic:** Those substance, which when placed in a magnetic field are strongly magnetized in the direction of the magnetizing field, are called ferromagnetic substances.

When a ferromagnetic substance is placed inside a magnetic field, the field inside the ferromagnetic substance gets greatly enhanced. As a result, when a ferromagnetic is placed in a non-uniform magnetic field, it quickly moves from weaker part to stronger part of magnetic field. The ferromagnetic behavior of a substance is temperature dependent.

### 30.16 Viva Questions

- Q.1 What is magnetic field?
- Q.2 What is magnetic susceptibility?
- Q.3 Give any weakness of the Gouy's method.
- Q.4 Why balance is enclosed in a box?
- Q.5 What are the dimensions of Magnetic susceptibility?
- Q.6 What is the relation between permeability and susceptibility of any substance?
- Q.7 Give the relation between relative permeability with magnetic susceptibility.
- Q.8 Define magnetic permeability.

- Q.9** How does the magnetic susceptibility of para, dia and ferro-magnetic materials vary with temperature?
- Q.10** Define horizontal component of earth's magnetic field.
- Q.11** What is magnetic pole?
- Q.12** What is power supply?

### 30.17 Answers To Viva Questions

**Ans.1:** The magnetic field is defined as the space around a magnet in which its magnetic influences extends, is known as the *magnetic field*.

**Ans.2:** The magnetic susceptibility of a magnetic material is defined as the ratio of the intensity of magnetization induced in the material to the magnetic field intensity. It measures the ease with which a specimen can be magnetized.

**Ans.3:** Gouy's method is a simple but not sufficiently accurate. This is because the force is weak even in strong fields because of the low values of susceptibilities of force equivalent to 1 *gwt*, the field required will be of the order of  $10^5$  gauss.

**Ans.4:** The balance is enclosed in a box so that the air does not affect the weighing.

**Ans.5:** The magnetic susceptibility is unit less and dimension less quantity.

**Ans.6:** The relation between permeability and susceptibility is given below:

$$\mu = \mu_0(1 + \chi)$$

**Ans.7:** Relation between relative permeability and magnetic susceptibility is given below:

$$\mu_r = (1 + \chi)$$

**Ans.8:** Magnetic permeability is defined as the measure of the degree of penetration of magnetic field through the substance and is the ratio *B and H*.

$$\mu = \frac{B}{H}$$

**Ans.9:** For paramagnetic materials  $\chi \propto \frac{1}{T}$  where T is the absolute temperature.



For diamagnetic materials  $\chi$  is independent of temperature.

For ferromagnetic materials  $\chi \propto \frac{1}{T-T_C}$  where  $T_C$  is Curie temperature and  $T > T_C$ .

**Ans.10:** The horizontal component of earth's magnetic field at a place is the component of the total magnetic field of the earth in the horizontal direction in magnetic meridian.

Its value at a place on the surface of earth is of the order of  $0.32 \times 10^{-4} \text{ Wb/m}^2$  (tesla).

**Ans.11:** A magnetic pole is said to have unit pole strength if it repels a like pole of same strength kept at a distance of 1 metre with a force of  $10^{-7}$  Newton.

**Ans.12:** A power supply is an electronic device that supplies electric energy to an electrical circuit or load. The main aim of a power supply is to convert one form of electrical energy to another form of electrical energy.

### References and Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook,
2. B. Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires.
4. Physics Practical Vol. 2 by R. K. Shukla & Anchal Srivastav.

# UNIT-31

## Magnetic Susceptibility by Quincke's Method

### Structure of the Unit

- 31.1 Aim
  - 31.2 Apparatus
  - 31.3 Diagram
  - 31.4 Formula
  - 31.5 Theory and Description
  - 31.6 Self Learning Exercise-I
  - 31.7 Procedure
  - 31.8 Observation
  - 31.9 Graph
  - 31.10 Calculations
  - 31.11 Result
  - 31.12 Discussion
  - 31.13 Precautions and Sources of Error
  - 31.14 Self Learning Exercise-II
  - 31.15 Glossary
  - 31.16 Answers to Self Learning Exercises
  - 31.17 Viva Questions
  - 31.18 Answers to Viva Questions
- References and Suggested Readings

### 31.1 Aim

To find magnetic susceptibility by Quincke's method.

### 31.2 Apparatus

Quincke's tube with stand, Sample:  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$ , R.D. Bottle, Travelling Microscope, Digital Gauss meter, Constant Current Power Supply, Electromagnet

### 31.3 Diagram

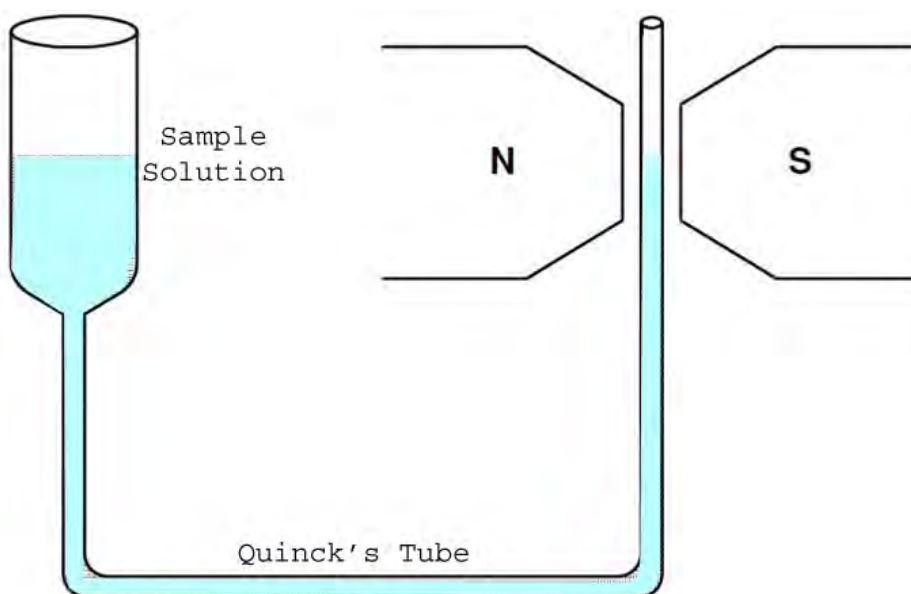


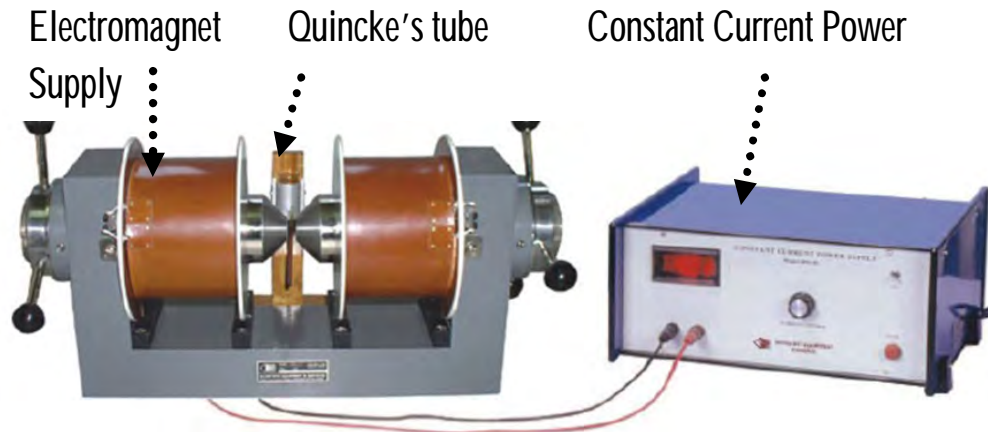
Figure 1 : Schematic diagram for Quincke's method[1].



Travelling Microscope

Digital Gaussmeter





**Figure 2:** *Experimental arrangement of Quincke's method Experiment[1].*

### 31.4 Formula

#### (a) Mass Susceptibility of a Salt:

The resultant susceptibility of a solution is due to water and dissolved salt

$$\chi_{\text{solution}} = \chi_s + \chi_w$$

If we have water of mass  $M_w$  and dissolved mass of salt  $M_s$ , then mass susceptibility of the solution

$$\chi_{\text{Solution}}^m = \frac{M_w}{M_s + M_w} \frac{\chi_w}{\rho_{\text{water}}} + \frac{M_s}{M_s + M_w} \frac{\chi_s}{\rho_{\text{salt}}}$$

#### (b) Susceptibility by Quinck's Method:

The magnetic force on liquid in U-tube is balanced by pressure acting on surface area of  $A$  after a height difference ( $h$ ) between the level of the liquid surfaces in the two arms of the U-tube. Then the susceptibility is given as following

$$\chi = \chi_a + \frac{2h(\rho - \rho_a)g}{\mu_0(H^2 - H_0^2)}$$

Where :

$\chi_a$  = a susceptibility of the air

$\rho_a$  = a density of the air

$\mu_0 = 4\pi \times 10^{-7}$  H/m is the permeability of free space

$g$ =Acceleration due to gravity

$\rho$ = density of the salt solution

$H_0$ = is the field at the other surface, away from the magnet.

Because in practice ,  $H_0$  , air density and air susceptibility are negligible hence:

$$\chi = \frac{4gh\rho}{\mu_0 H^2}$$

The slope of the straight line graph between  $h$  and  $H^2$  allows us to calculate the susceptibility.

## 31.5 Theory and Description

### i Magnetic Moment:

The most fundamental term in magnetism is the magnetic moment. The magnetic moment of a current loop of area  $A$  is given by  $\mathbf{m} = I\mathbf{A}$ . Similarly, a circular orbit around the nucleus of an electron inside the atom (say H atom) is equivalent to current carrying loop ( $i = -ve/2\pi r$ ), having magnetic moment [3]

$$\mu = i \pi r^2 = -(\pi r^2)ve/2\pi r = -(e/2m)(mvr)$$

$v$  is the speed of the electron in circular orbit of radius  $r$ .

According to Bohr theory, the magnitude of the angular momentum  $L = mvr$  of the electron, in the ground state is equal to  $\hbar$ . Hence

$$\mu = -e \hbar / 2m = -\mu_B$$

where  $\mu_B = e \hbar / 2m = 9.274 \times 10^{-24}$  Am<sup>2</sup> ( $e$  and  $m$  are the electronic charge and mass and  $\hbar$  is Planck's constant) is the Bohr magneton, which is unit used for atomic magnetic moments. Here we consider only the orbital motion of the electron, but generally the total magnetic moment of an atom or ion due to both the orbital motion and the spin of the electrons.

### ii Magnetization and field

The bulk magnetic material made of a large number of atoms/ions with magnetic moments. And magnetic properties of such materials generally described in terms of the magnetization  $\mathbf{M}$ , which is defined as the net magnetic dipole moment per unit volume. In free space (vacuum), there is no material hence no magnetization ( $\mathbf{M}=\mathbf{0}$ ). So for free space the magnetic field ( $\mathbf{B}$ ) is linearly related by to magnetic intensity ( $\mathbf{H}$ ) or [4]

$$\mathbf{B}=\mu_0\mathbf{H}$$

where,  $\mu_0 = 4\pi \times 10^{-7}$  H/m is the permeability of free space.

In a magnetic material, due to magnetization the relation between  $\mathbf{B}$  and  $\mathbf{H}$  may be more complicated. These two vector fields may be different in magnitude as well as in direction. The general vector relationship is written as:

$$\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$$

### iii Magnetic susceptibility

The magnetic properties of a material not only characterized by the magnitude and direction of magnetization, but also depend on how the  $\mathbf{M}$  varies with applied magnetic field  $\mathbf{H}$ . In the linear material, the magnetization  $\mathbf{M}$  is linearly depend to the magnetic field  $\mathbf{H}$ ,

$$\mathbf{M}=\chi\mathbf{H}$$

where  $\chi$  is a dimensionless quantity called the magnetic susceptibility. In terms of the susceptibility  $\chi$ , magnetic materials also classified as diamagnetic, paramagnetic and ferromagnetic. A material is diamagnetic when  $\chi$  is negative, paramagnetic if  $\chi$  is small and positive, and ferromagnetic if  $\chi$  is large and positive. The relationship between  $\mathbf{B}$  and  $\mathbf{H}$  is still a linear for the linear materials (like paramagnetic material)

$$\mathbf{B}=\mu_0(1+\chi)\mathbf{H}=\mu_0\mu_r\mathbf{H}$$

where  $\mu_r = 1 + \chi$  is the relative permeability of the material.

Since  $\mathbf{M}$  is defined as the magnetic moment per unit volume, hence  $\chi$  also called the volume susceptibility.

#### iv Magnetic susceptibility of paramagnetic salts

For a solution of paramagnetic salt, the volume susceptibility depends on the density of dissolved salt. As a result the resultant susceptibility of same material will vary for different concentration of salt. To avoid such dependency we can define the molar and mass susceptibility. Mass magnetic susceptibility is the ratio of the volume susceptibility to density of the material.

$$\chi_m = \chi_v / \rho \text{ (m}^3/\text{kg)}$$

where the Molar magnetic susceptibility is the product of mass magnetic susceptibility  $\chi_m$  and molar mass  $M$  (kg/mol)

$$\chi_M = M\chi_m / \rho \text{ (m}^3/\text{mol)}$$

Generally in the solution of salt ( $\text{MnCl}_2 \cdot 4\text{H}_2\text{O}$  or  $\text{MnCl}_2$  or  $\text{FeCl}_3$ ,  $\text{FeCl}_3 \cdot 6\text{H}_2\text{O}$ ) contains ions of  $\text{Fe}^{3+}$  or  $\text{Mn}^{2+}$  having permanent magnetic dipole moment  $m$ . In such solutions (at low concentration) the magnetic interaction between magnetic dipoles is negligible, such non-interacting magnetic dipoles consider as a paramagnetic.

**Table 1:** Magnetic Molar susceptibility of paramagnetic salts.

Name	Formula	$\chi_m \times 10^{-6}$ ( $\text{cm}^3/\text{mol}$ )
Iron(III) chloride	$\text{FeCl}_3$	+13450
Iron(III) chloride	$\text{FeCl}_3 \cdot 6\text{H}_2\text{O}$	+15250
Manganese(II) sulfate	$\text{MnSO}_4$	13660

Manganese(II) sulfate monohydrate	MnSO <sub>4</sub> ·H <sub>2</sub> O	+14200
Manganese(II) sulfate tetrahydrate	MnSO <sub>4</sub> ·4H <sub>2</sub> O	+14600

When we applied a magnetic field, the dipoles tend to align parallel to field consequently we have a net magnetization. Due to thermal effects try to randomize the dipole, hence susceptibility of a paramagnetic material decreases as the temperature  $T$  is increased. According to Langevin theory the volume magnetic susceptibility of the paramagnetic salt solution at high temperatures ( $kT \gg mB$ ) is given by[1,3],

$$\chi_{salt} = \frac{N \mu_0 m^2}{3kT} = \frac{N p^2 \mu_B^2 \mu_0}{3kT}$$

where  $k$  is Boltzmann's constant and  $N$  is the number of ions per unit volume and  $m = p\mu_B$ , where  $p$  is the magneton number. The  $1/T$  dependence of  $\chi$  is known as Curie's Law.

The total susceptibility of solution is due to water and dissolved salt or

$$\chi_{solution} = \chi_s + \chi_w$$

If we have water of mass  $M_w$  and dissolved mass of salt  $M_s$ , then mass susceptibility of the solution [1]

$$\chi_{Solution}^m = \frac{M_w}{M_s + M_w} \frac{\chi_w}{\rho_{water}} + \frac{M_s}{M_s + M_w} \frac{\chi_s}{\rho_{salt}}$$

## v Quincke's Method

Magnetic susceptibility of the diamagnetic or paramagnetic solution determine by the Quincke's method. A schematic diagram of Quincke's experimental set up is shown in Figure 1. A glass U-tube is filled with sample solution and is very narrow (about 2 - 3 mm in diameter) end is placed between the pole pieces of an electromagnet. The one end is narrow and other end of U-tube is wider because



the change of the level in narrow end does not affect the level of the wider end. Furthermore the length of tube is large so that wider end well outside the field of the magnet [1,2].

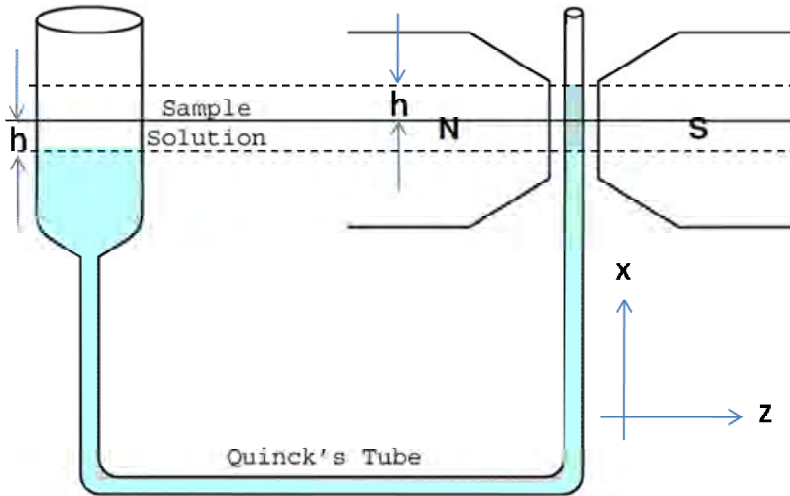
Due to the magnetic field, the sample gets magnetized or magnetic moment tries to align along magnetic field  $H$  is in the z-direction in the liquid.

The force acting per unit volume is equal to

$$f = \frac{d}{dx} \left[ \frac{1}{2} \mu_0 (\mu_r - \mu_a) H^2 \right] = \frac{1}{2} \mu_0 (\mu_r - \mu_a) \frac{d}{dx} H^2$$

The force acting on a volume element of area  $A$  and length  $dx$  of liquid is  $Afdx$ , so the total force  $F$  on the liquid is[1],

$$F = A \int f dx = \frac{A \mu_0}{2} (\chi - \chi_a) (H^2 - H_0^2)$$



**Figure 3:** Paramagnetic liquid in a uniform magnetic field[1].

The magnetic force on liquid in U-tube is balanced by pressure acting on surface area of  $A$  after a height difference ( $h$ ) between the level of the liquid surfaces in the two arms of the U-tube or[1]

$$\frac{A \mu_0}{2} (\chi - \chi_a) (H^2 - H_0^2) = Ah(\rho - \rho_a)g$$

$$\Rightarrow (\chi - \chi_a) = \frac{2h(\rho - \rho_a)g}{\mu_0 (H^2 - H_0^2)}$$

Where :

$\chi_a$  = a susceptibility of the air

$\rho_a$  = a density of the air

$\mu_0 = 4\pi \times 10^{-7}$  H/m is the permeability of free space

$g$  = Acceleration due to gravity

$\rho$  = density of the salt solution

$H_0$  = is the field at the other surface away from the magnet.

Because in practice,  $H_0$ , air density and air susceptibility are negligible hence [1]:

$$\chi = \frac{2gh\rho}{\mu_0 H^2}$$

The slope of the straight line graph between  $h$  and  $H^2$  allows us to calculate the susceptibility.

$$\chi = 2g\mu_0\rho \frac{h}{B^2}$$

### 31.6 Self Learning Exercise-I

- Q.1** What the magnitude of the angular momentum of the electron, in the ground state (According to Bohr theory,)?
- Q.2** Define mass magnetic susceptibility?
- Q.3** Define the Molar magnetic susceptibility?
- Q.4** What is magnetic moment of a current loop of area  $A$ ?
- Q.5** What is the volume magnetic susceptibility of the paramagnetic salt solution at high temperatures ( $kT \gg mB$ ) (According to Langevin theory)?

### 31.7 Procedure

#### (a) Sample Preparation:

1. First of all known mass of salt ( $MnSO_4 \cdot 4H_2O$ ,  $FeCl_3$ ) dissolve in a given volume of water.
2. Measurement of the density  $\rho$  of the solution by specific gravity bottle:

$$\rho = \rho_w (m_2 - m_1) / (m_3 - m_1)$$

Where:

$\rho_w$  = Density of water

Weight of specific gravity bottle ( $m_1$ ) =

Weight of specific gravity bottle + water ( $m_3$ ) =

Weight of specific gravity bottle + Salt solution ( $m_2$ ) =

**(b) Calibrate the magnetic field**

1. Adjust the 10 mm gap between the poles of the electromagnet.
2. Measure the magnetic field as a function of current flowing in electromagnet using the digital Hall probe and gauss meter.
3. Plot the calibration curve.

**(c) Measurement of the solution level (h) as a function of magnetic field (H)**

1. A schematic diagram of Quincke's experimental set up is shown in Figure 1. A glass U-tube is filled with sample solution and is very narrow (about 2 - 3 mm in diameter) end is placed between the pole pieces of an electromagnet.
2. U-tube adjusts such that the meniscus remains in the centre of the pole pieces. And Focus the microscope on the meniscus.
3. Now note the height of the lower meniscus of the solution as a function of applied field H.
4. After changing the magnetic field, wait for few minutes to take the observation.
5. Repeat above step for various concentrations of solution.
6. Plot the graph between h and  $H^2$  and calculate the slope.

**31.8 Observation**

**1. Preparation of solution:-**

Weight of salt to make paramagnetic solution = .....kg

Weight of distilled water = .....kg

Weight of specific gravity bottle ( $m_1$ ) = .....kg

Weight of specific gravity bottle + water ( $m_3$ ) = .....kg

Weight of specific gravity bottle + Salt solution ( $m_2$ ) = .....kg

2. Temperature of solution= 293 K

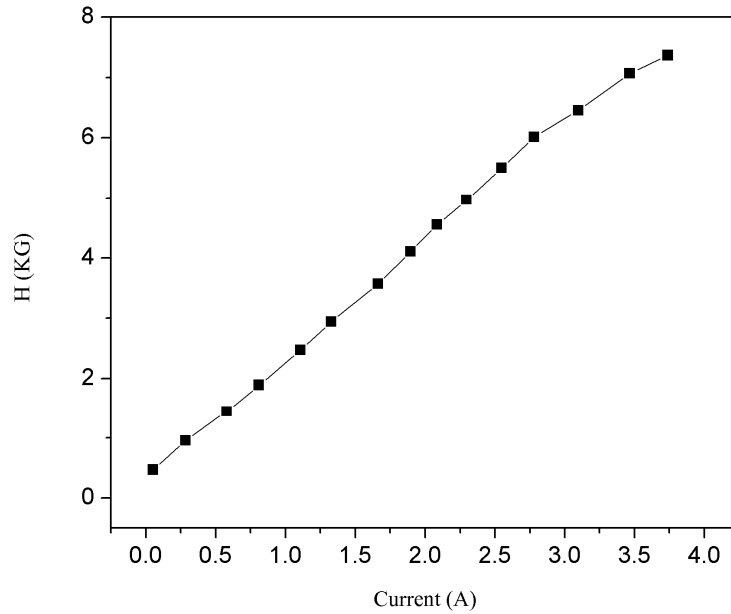
3. Calibration of magnetic field as a function magnetizing current[1]:-

S.No.	Current (A)	Magnetic Field H (kG)
1.	0	
2.		
3.		
4.		
5.		
6.		
7.		
8.		

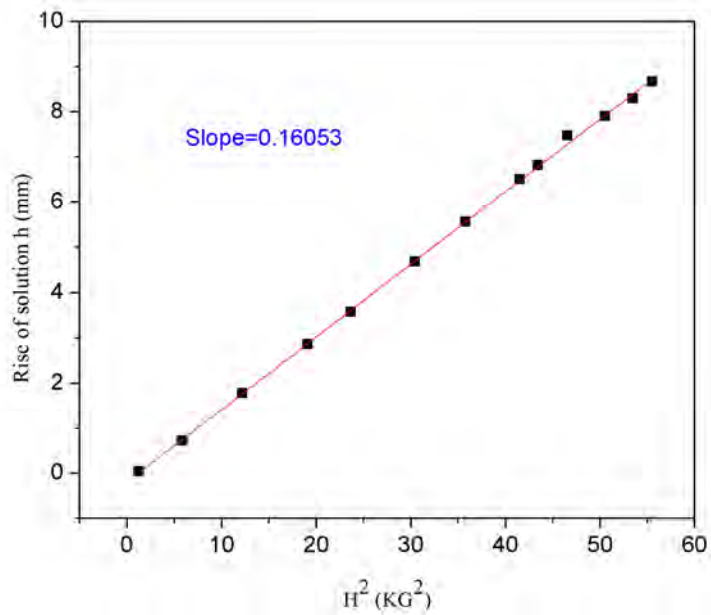
4. Rise of solution level (h) as a function of magnetic field (H)[1]:-

S.No.	Current (A)	H (kG)	$H^2$ (kG <sup>2</sup> )	Height of liquid level (mm)	Rise of solution h (mm)
1.	0				
2.					
3.					
4.					
5.					
6.					
7.					
8.					

## 31.9 Graph



**Figure 4:** Calibration of magnetic field as a function magnetizing current[1].



**Figure 5:** Rise of solution level (h) as a function of magnetic field (H)[1].

## 31.10 Calculations

**Calculation for  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$  as an example[1]:**

**(a) Density  $\rho$  of the solution :**

Weight of specific gravity bottle ( $m_1$ ) = 20g

Weight of specific gravity bottle + water ( $m_3$ ) = 45g

Weight of specific gravity bottle + Salt solution ( $m_2$ ) = 56 g

Density of water  $\rho_w = 1\text{g/cm}^3$

Density  $\rho$  of the solution is  $\rho = \rho_w (m_2 - m_1) / (m_3 - m_1)$

$$\rho = 36/25 = 1.44\text{g/cm}^3$$

**(b) Calculation for volume susceptibility of the solution :**

**Slope** (from figure 5) = 0.16053 mm/kG<sup>2</sup>

$$\text{Slope} = 0.016 \text{ m/ T}^2$$

$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$  is the permeability of free space.

If B( Tesla) is SI unit	If B( Gauss) is cgs unit
$\rho = 1440\text{kg/m}^3$	$\rho = 120/100 = 1.44\text{g/cm}^3$
$g = 9.80 \text{ m/s}^2$	$g = 980 \text{ m/s}^2$
Slope = .016m/ T <sup>2</sup>	Slope = 16.053x10 <sup>-9</sup> cm/G <sup>2</sup>
$\chi = 2g\mu_0\rho \frac{h}{B^2}$ or $\chi = 2g\mu_0\rho(\text{Slope})$	$\chi = 2g\rho \frac{h}{H^2}$ or $\chi = 2g\rho(\text{Slope})$
$2 \times 1440 \times 9.80 \times 0.016 \times 4\pi \times 10^{-7}$	$2 \times 1.44 \times 980 \times 16.053 \times 10^{-9}$
$\chi = 4\pi \times 45.15 \times 10^{-6}$	$\chi = 45.15 \times 10^{-6}$
<p><i>The S.I. unit of B is the Tesla, and c.g.s. units is Oersteds or Gauss. The conversion factors are: 1T = 10KOe = 10KOe = 104Gauss. <math>\chi(SI) = 4\pi \chi(cgs)</math></i></p>	

(c) **Mass susceptibility of the solution:**

$$\chi_{\text{Solution}}^m = 4\pi \times 45.15 \times 10^{-6} / 1440 =$$

$$\chi_{\text{Solution}}^m = 0.394 \times 10^{-6} \text{ m}^3/\text{kg}$$

(d) **Calculation for Mass susceptibility of salt  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$ :**

Mass susceptibility of salt can be written as:

$$\chi_{\text{masSalt}} = \frac{\chi_s}{\rho_{\text{salt}}} = \left( \chi_{\text{Solution}}^m - \frac{M_w}{M_s + M_w} \frac{\chi_w}{\rho_{\text{water}}} \right) \frac{M_s + M_w}{M_s}$$

Weight of  $\text{MnSO}_4 \cdot \text{H}_2\text{O} = 0.020 \text{ kg}$

Weight of distilled water = 0.033 kg

Density of water  $\rho_w = 1000 \text{ kg/m}^3$

Density  $\rho$  of the solution is  $\rho_s = 1440 \text{ kg/m}^3$

$$\chi_w = -4\pi \times 0.72 \times 10^{-6}$$

$$\chi_s = 4\pi \times 45.15 \times 10^{-6}$$

$$\chi_{\text{mass\_Salt}} = \left( \chi_{\text{Solution}}^m - \frac{0.033}{0.053} \left( \frac{-4\pi \times 0.72 \times 10^{-6}}{1000} \right) \right) \frac{0.053}{0.020}$$

$$\chi_{\text{mass\_Salt}} = (0.394 \times 10^{-6} - 0.006 \times 10^{-6}) \frac{0.053}{0.020}$$

$$\chi_{\text{mass\_Salt}} = 1.0282 \times 10^{-6} \text{ m}^3/\text{kg}$$

(e) **Calculation for Molar susceptibility of salt  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$**

Molecular weight of  $\text{MnSO}_4 \cdot \text{H}_2\text{O} (m_w) = 0.169 \text{ kg}$

$$\chi_{\text{Molar\_Salt}} = m_w \chi_{\text{mass\_Salt}}$$

$$\chi_{\text{Molar\_Salt}} = 1.7 \times 10^{-7} \text{ m}^3/\text{mol}$$

Standard Molar susceptibility of salt  $\text{MnSO}_4 \cdot \text{H}_2\text{O} = 1.4 \times 10^{-7} \text{ m}^3/\text{mol}[1]$

## 31.11 Result

1. Volume susceptibility of the solution = .....

2. Mass susceptibility of salt=.....
3. Molar susceptibility of salt=.....

### 31.12 Discussion

.....

.....

.....

.....

.....

### 31.13 Precautions and Sources of Error

1. The high current does not flow for more time into the magnet.
2. Gauss meter probe kept in between the poles such that it give positive value of field and remain at same position during your calibration
3. Hall probe should be perpendicular to field direction.
4. Take care the U-tube during cleaning and put properly in between magnetic poles.
5. After changing the magnetic field, wait for few minutes to take the observation.

### 31.14 Self Learning Exercise-II

- Q.1 The one end of the U-tube is narrow and other end is wider, why?
- Q.2 Hall probe should be perpendicular to field direction, why?
- Q.3 Does the magnetic susceptibility of a paramagnetic salt dependent on temperature?
- Q.4 Does the volume magnetic susceptibility of a paramagnetic salt dependent on concentration?
- Q.5 Does the Mass magnetic susceptibility of a paramagnetic salt dependent on concentration?

### 31.15 Glossary

**Meniscus:** The curved upper surface of a nonturbulent liquid in a vertical tube.



## 31.16 Answers to Self Learning Exercises

### *Answers to Self Learning Exercises -I*

- Ans.1:** Magnitude of the angular momentum  $L = mvr$  of the electron, in the ground state equal to  $\hbar$ .
- Ans.2:** Mass magnetic susceptibility is the ratio of the volume susceptibility to density of the material.
- Ans.3:** Molar magnetic susceptibility is the product of mass magnetic susceptibility  $\chi_m$  and molar mass  $M$  (kg/ mol).
- Ans.4:** Magnetic moment of a current loop is given by  $\mathbf{m} = \mathbf{IA}$
- Ans.5:** volume magnetic susceptibility of the paramagnetic salt solution at high temperatures ( $kT \gg mB$ ) is given by,

$$\chi_{salt} = \frac{N \mu_0 m^2}{3kT} = \frac{N p^2 \mu_B^2 \mu_0}{3kT}$$

### *Answers to Self Learning Exercise-II*

- Ans.1:** Because the change of the level in narrow end does not affect the level of the wider end.
- Ans.2:** Because measured magnetic field depend on angle between probe surface and magnetic field direction.
- Ans.3:** Yes,  $1/T$  dependence of  $\chi$  is known as Curie's Law.
- Ans.4:** Yes.
- Ans.5:** No.

## 31.17 Viva Questions

- Q.1** What do you mean by magnetization  $\mathbf{M}$  of a sample?
- Q.2** What is the magnetic field in a long solenoid ( $n$  turns per unit length and a current  $I$ )?
- Q.3** If the solenoid is filled with Iron then, the field inside the solenoid is less or greater?
- Q.4** Define the magnetic susceptibility.
- Q.5** What are the dimensions of magnetic susceptibility?

- Q.6** Broadly Classified the Magnetic materials on the basis of magnetic susceptibility.
- Q.7** Out of these materials, diamagnetic, paramagnetic and ferromagnetic, which one shows hysteresis?
- Q.8** What is the relation between magnetic susceptibility and relative permeability?
- Q.9** What is the value of  $\mu_r$  in vacuum and inside the material, generally?
- Q.10** What is the Bohr magneton?
- Q.11** What is the Curie's law ?
- Q.12** "Diamagnetism is a property of all materials" this statement is True or false?
- Q.13** What will be the potential energy of a dipole moment  $m$  is placed in a uniform magnetic field  $B$ ?
- Q.14** The ferromagnetism is due to spin or orbital motion?
- Q.15** What is the molar susceptibility of salt  $MnSO_4.H_2O$ ?
- Q.16** What is the origin of the diamagnetism?
- Q.17** The magnetization of paramagnetic material increase or decreases with increasing of temperature?
- Q.18** A paramagnetic material has diamagnetisms or not?
- Q.19** Hall probe should be perpendicular to field direction during observation, why?
- Q.20** Why does a paramagnetic material show greater magnetization (For the same magnetizing field) upon cooled?

### 31.18 Answers to Viva Questions

- Ans.1:** Magnetization of sample is equal to its net magnetic moment per unit volume.
- Ans.2:** The magnetic field in a solenoid  $B = \mu_0 nI$ .
- Ans.3:** Field will increase  $\mu_r$  times.
- Ans.4:** Magnetic susceptibility for linear material  $\chi = M/H$ .
- Ans.5:** Magnetic susceptibility is a dimensionless quantity.
- Ans.6:** Magnetic materials broadly classified as diamagnetic, paramagnetic and ferromagnetic. In terms of the susceptibility  $\chi$ , a material is diamagnetic

if  $\chi$  is negative, paramagnetic- if  $\chi$  is positive and small, and ferromagnetic if  $\chi$  is large and positive.

**Ans.7:** Ferromagnetic material shows the property of hysteresis, these materials are characterized by non-linear relation between **B** and **H**.

**Ans.8:** Relative permeability  $\mu_r = 1 + \chi$

**Ans.9:** Vacuum = 1, and inside the material generally  $\mu_r$  greater than 1.

**Ans.10:** Bohr Magnetron ( $\mu_b$ ) =  $eh/4\pi m = 9.27 \times 10^{-24}$  J/T

**Ans.11:**  $\chi = C/T$ , the constant C is called Curie's constant.

**Ans.12:** True,

**Ans.13:** Potential energy is  $U = -\mathbf{m} \cdot \mathbf{B}$ ,

**Ans.14:** Due to magnetic moment associated with spin.

**Ans.15:**  $1.4 \times 10^{-7}$  m<sup>3</sup>/mol.

**Ans.16:** Orbital motion of the electron.

**Ans.17:** Decrease.

**Ans.18:** Yes, but diamagnetism is weak as compared to paramagnetic contribution to the sample.

**Ans.19:** Because the magnitude depended on the angle between surface of hall probe and magnetic field direction.

**Ans.20:** At fixed applied magnetic field, when the temperature of the sample is decrease, the magnetization increase because thermal energy decreases consequently the magnetic moments easily aligned to field direction. Hence magnetization increases.

## References and Suggested Readings

1. <http://www.iiderkol.ac.in/~ph324ExptManuals/quincke`s%20manual.pdf>
2. <http://www.iiderkol.ac.in/~ph324/StudyMaterials/QuincksTube02.pdf>
3. Stephen Blundell, *Magnetism in Condensed Matter*, Oxford University Press Inc., New York, 2001.
4. B. D. Cullity, and C. D. Graham, *Introduction to Magnetic materials*, Second Edition, Wiley & Sons, Inc., Hoboken, New Jersey, 2008.

# UNIT-32

## Laser Experiment: Divergence of Laser Beam, Double Slit Interference Pattern, Diameter of Thin Wire

### Part -I: Measurement the Divergence of Laser Beam by the Size Spots & a Lens

- 32.1A Aim
- 32.2A Apparatus
- 32.3A Diagram
- 32.4A Formula
- 32.5A Theory and Description
- 32.6A Self Learning Exercise
- 32.7A Procedure
- 32.8A Observation
- 32.9A Calculations
- 32.10A Result
- 32.11A Discussion
- 32.12A Precautions and Sources of Error
- 32.13A Answers To Self Learning Exercise

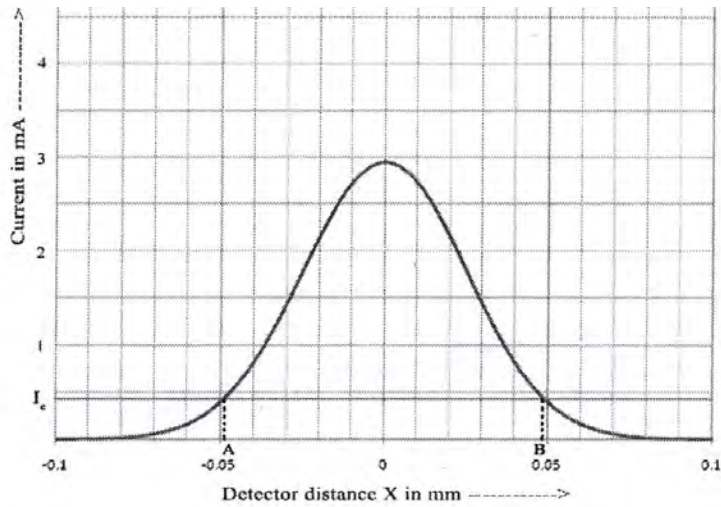
#### **32.1A Aim**

To measure the divergence of laser beam by the size-spots and a lens.

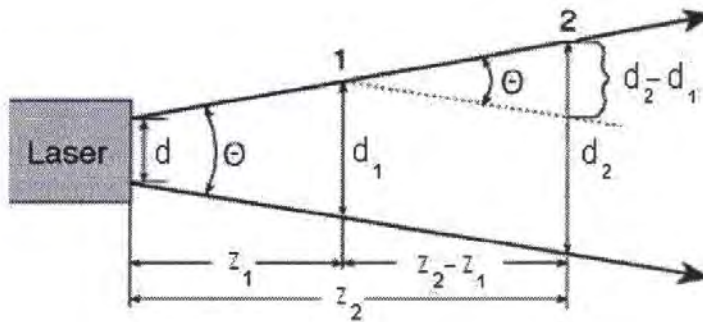
#### **32.2A Apparatus**

A laser beam source, optical bench arrangement, photodiode, micrometer, screw gauge, a lens, a laser stand, a screen and a meter rod etc.

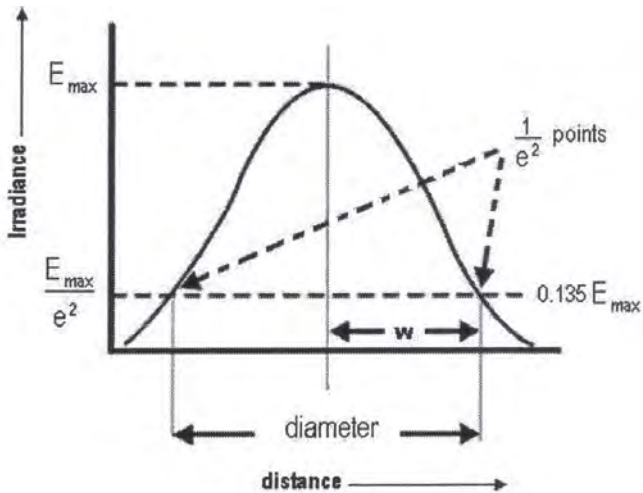
## 32.3A Diagram



**Figure 32.1: Beam profile of laser beam divergence.**



**Figure 2: Laser Beam Divergence.**



**Figure 3: Graph between diameter and irradiance.**

## 32.4A Formula

The divergence of a laser beam by the help of the size spots is measure by

$$\theta = \frac{(d_2 - d_1)}{(z_2 - z_1)}$$

Here  $\theta$  is the divergence angle and it is measure in the mradian.

Now the divergence of a laser beam by the help of a lens is measured by

$$\Theta = \frac{\left[ \frac{\{S_1 + S_2 + 2S_3\}}{2} \right]^{1/2}}{D}$$

Here  $\Theta$  is the divergence angle and  $s_1, s_2, s_3$  (Say) is the size of the spot by a lens at various distances.

## 32.5A Theory and Description

The term laser is the acronym for light amplification by stimulated of radiation. It is a mechanism for emitting electromagnetic radiation via the process of stimulated emission. The laser was the first device capable of amplifying light waves themselves. The emitted laser light is a spatially coherent, narrow low-divergence beam. When the waves of a beam of light have the same frequency, phase and direction, it is said to be coherent. There are lasers that emit a broad spectrum of light, or emit different wavelengths of light simultaneously. According to the some physicist of laser physics and technology, beam divergence of a laser beam is measure for how fast the beam expands far from the beam waist. A laser beam with narrow beam divergence is greatly used to make laser pointer devices. Generally the beam divergence of laser beam is measured using beam profiler.

Lasers usually emit beams with a Gaussian profile. A Gaussian beam is a beam of electromagnetic radiation whose transverse electric field and intensity distributions are described by Gaussian functions.

### **32.5.1 Beam Parameter:**

Beam parameters govern the behaviour and geometry of Gaussian beam. The important beam parameters are described below:

#### **32.5.1.1 Beam Divergence:**

The light emitted by a laser is confined to rather narrow cone. But, when the

beam propagates outward, it slowly diverges or fans out. For an electromagnetic beam, beam divergence is the angular measure of the increase in the radius or diameter with distance from the optical aperture as the beam emerges.

The divergence of a laser beam can be calculated if the beam diameters  $d_1$  and  $d_2$  at two separate distances are known. Let  $z_1$  and  $z_2$  are the distances along the laser axis, from the end of the laser to points.

Usually, divergence angle is taken as the full angle of opening of the beam. Then

$$\theta = \frac{(d_2 - d_1)}{(z_2 - z_1)}$$

Half of the divergence angle can be calculated as

$$\theta = \frac{(w_2 - w_1)}{(z_2 - z_1)}$$

Here  $w_2$  and  $w_1$  are the radii of the beam at  $z_1$  and  $z_2$ .

### **32.5.1.2 Spot Size:**

It is defined as the radius of the beam itself. The irradiance of the beam decreases gradually at the edges.

The distance across the centre of the beam for which the irradiance equals  $\frac{1}{e^2}$  of the maximum irradiance is defined as the beam diameter. The spot size ( $w$ ) of the beam is defined as the radial distance from the centre point of maximum irradiance to the  $\frac{1}{e^2}$  point.

Gaussian laser beam are said to be diffraction limited when their radial beam divergence is close to the minimum possible value, which is given by

$$\Theta = \frac{\theta}{2} = \frac{\lambda}{\pi w_0}$$

where  $\lambda$  is the wavelength of a given laser and  $w_0$  is the radius of the beam at the narrowest point, which is termed as the beam waist.

### **32.5.2 Theory For The Divergence of Beam by A Lens:**

A lens can be used to focus a laser beam to a small spot, or systems of lenses may be used to expand the beam and recollimate it. An idea thin lens in such an application will not change the transverse mode intensity pattern measured at the

lens but it will alter the radius of curvature of the phase fronts of the beam.

We have seen that a Gaussian laser beam of whatever order is characterized by a complex beam parameter  $q(z)$  which change as the beam propagates in an isotropic, homogeneous material according to  $q(z) = q_0 + z$ , where

$$q_0 = \frac{i\pi w_0^2}{\lambda} \text{ and } \frac{1}{q(z)} = \frac{1}{R(z)} - \frac{i\lambda}{\pi w^2}$$

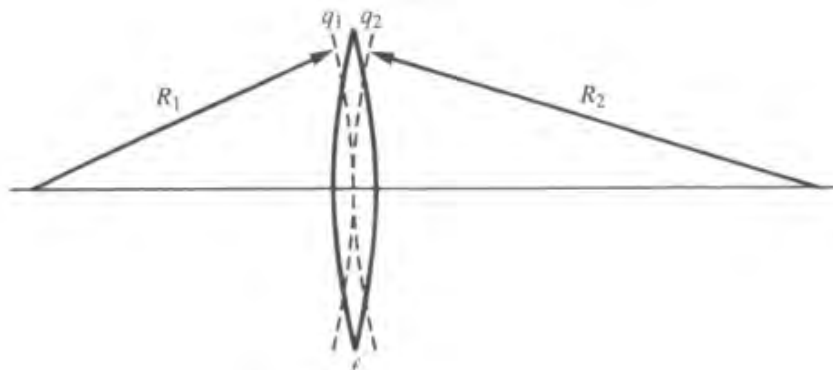
$R(z)$  is radius of curvature of (approximately) spherical phase front at  $z$  and is given by

$$R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

This Gaussian beam becomes a true spherical wave as  $w \rightarrow \infty$ . Now, a spherical wave changes its radius of curvature as it propagates according to  $R(z) = R_0 + z$  where  $R_0$  is its radius of curvature as  $z = 0$ . So the complex beam parameter of a Gaussian wave change in just the same way as it propagates as does the radius of curvature of a spherical wave.

When a Gaussian beam strikes a thin lens the spot size, which measures the transverse width of the beam intensity distribution, is unchanged at the lens. However, the radius of curvature of its wavefront is altered in just the same way as spherical wave. If  $R_1$  and  $R_2$  are the radii of curvature of the incoming and outgoing waves measured at the lens, as shown in Fig., then as in the case of a true spherical wave

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$



**Figure: Study of a Gaussian laser beam by a lens.**

So, for the change of the overall beam parameter, since  $w$  is unchanged at the lens, we have the following relationship between the beam parameters, *measured* at the



lens

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f}$$

### 32.6A Self Learning Exercise

- Q.1** How is a four-level laser better than a three-level laser? Give examples of each.
- Q.2** What is the wavelength of laser light from?  
(i) Ruby laser and (ii) He-Ne laser?  
How are the two lasers different?
- Q.3** How does Gaussian beam works in Lens Waveguides?

### 32.7A Procedure

*The procedure of the experiment for divergence of laser beam by size spots is done by following process:*

1. Arrange the laser and detector in an optical bench arrangement.
2. The laser is switched on and is made to incident on the photodiode.
3. Fix the distance,  $z$  between the detector and the laser source.
4. By adjusting the micrometer of the detector, move the spot in the horizontal direction, from left to right.
5. Note the output for each distance,  $x$  from the measuring device.
6. Then the beam profile is plotted with the micrometer distance.
7. The experiment is repeated for different detector distances.
8. Note the points in the graph where the intensity equals  $\frac{1}{e^2}$  of the maximum intensity, say it as  $I_e$ .
9. Find the micrometer distance across the beam corresponding to these points for a pair of detector distances  $z_1$  and  $z_2$ . Half of this distance is noted as  $w_1$  and  $w_2$ .
10. Then the divergence and spot size of the laser beam can be calculated from the equations.

**The procedure of the experiment for divergence of laser beam by a lens is done by following process:**

1. Arrange the laser source on a laser stand at one end of the optical bench.
2. Place a screen in front of laser source at a distance  $Z$ . Measure this distance with meter rod.
3. Paste m.m. graph on the screen. A bright spot of laser light will be formed on the graph paper by the help of lens which is pasted on the screen. Draw the boundary of spot using pencil.
4. Measure the size of the spot by counting the m.m. squares covered in it. Let it be  $s_1$ .
5. Now change the distance  $Z$  by  $Z+D$  and take reading of spot size made by the help of lens. Now next let the size is  $s_2$ .
6. Take three sets by changing  $D$ .
7. Put the values of  $s_1, s_2, s_3$  and  $D$  in the formula of divergence and calculate it.

### 32.8A Observation

**Table for the divergence of a beam:**

Least count of Screw gauge:

One pitch scale division (n) = .....mm

Number of divisions on head scale (m) =.....

Least Count=  $n/m$  =.....

$$z_1 = \dots \dots \dots \text{ cm}, z_2 = \dots \dots \dots \text{ cm}$$

For diameter of beam  $d_1$ :

Distance X in mm			Current in mA
P.S.R	H.S.R	TOTAL	

For diameter of beam  $d_2$ :

Distance X in mm			Current in mA
P.S.R	H.S.R	TOTAL	

### 32.9A Calculations

Divergence angle is calculated for size spot by the below given equation:

$$\theta = \frac{(d_2 - d_1)}{(z_2 - z_1)}$$

Divergence angle is calculated for a lens by the below given equation:

$$\Theta = \frac{\left[ \frac{\{S_1 + S_2 + 2S_3\}}{2} \right]^{1/2}}{D}$$

### 32.10A Result

Divergence Angle  $\theta$  = ..... mrad.

Divergence Angle  $\Theta$  = ..... mrad.

#### Percentage Error

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

### 32.11A Discussion

.....

.....

### 32.12A Precautions and Sources of Error

1. The laser source should be switched on only while taking the observation

and immediately switched off thereafter.

2. The laser tube axis should be horizontal.
3. Eyes should not be brought in front of laser source.

### 32.13A Answers to Self Learning Exercise

**Ans.1:** Three – level= Ruby laser

Four - level= He-Ne laser

**Ans.2:** (i) 6943 Å (ii) 6328 Å

Ruby laser is a three-level solid state laser whereas He-Ne is a four- level gas laser. Ruby laser has a pulsed output whereas He-Ne laser has a continuous output.

**Ans.3:** In a biperiodic lens sequence containing equally spaced lenses of focal lengths  $f_1$  and  $f_2$  the transfer matrix for  $n$  unit cells of the sequence is, from eq. (15.3)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^n = \frac{1}{\sin \phi} \begin{pmatrix} A \sin n\phi - \sin(n-1)\phi & B \sin n\phi \\ c \sin n\phi & D \sin n\phi - \sin(n-1)\phi \end{pmatrix}$$

So from eq. (16.88) and writing

$$q_{n+1} = (1/\sin \phi) \frac{[A \sin n\phi - \sin(n-1)\phi]q_1 + B \sin n\phi}{C \sin(n\phi)q_1 - D \sin n\phi - \sin(n-1)\phi}$$

The condition for stable confinement of the Gaussian beam by the lens sequence is the same as in the case of paraxial rays. This is condition that  $\phi$  remains real, i.e.  $|\cos \phi| \leq 1$ , where

$$\cos \phi = \frac{1}{2}(A + D) = 1 - \frac{d}{f_1} - \frac{d}{f_2} + \frac{d^2}{2f_1f_2}$$

which gives

$$0 \leq \left(1 - \frac{d}{2f_1}\right) \left(1 - \frac{d}{2f_2}\right) \leq 1$$

## Part -II: To Study Interference Due to a Double Slit

32.1B Aim

32.2B Apparatus

- 32.3B Diagram
- 32.4B Formula
- 32.5B Theory and Description
- 32.6B Self Learning Exercise
- 32.7B Procedure
- 32.8B Observation
- 32.9B Calculations
- 32.10B Result
- 32.11B Discussion
- 32.12B Precautions and Sources of Error
- 32.13B Answers to Self Learning Exercise

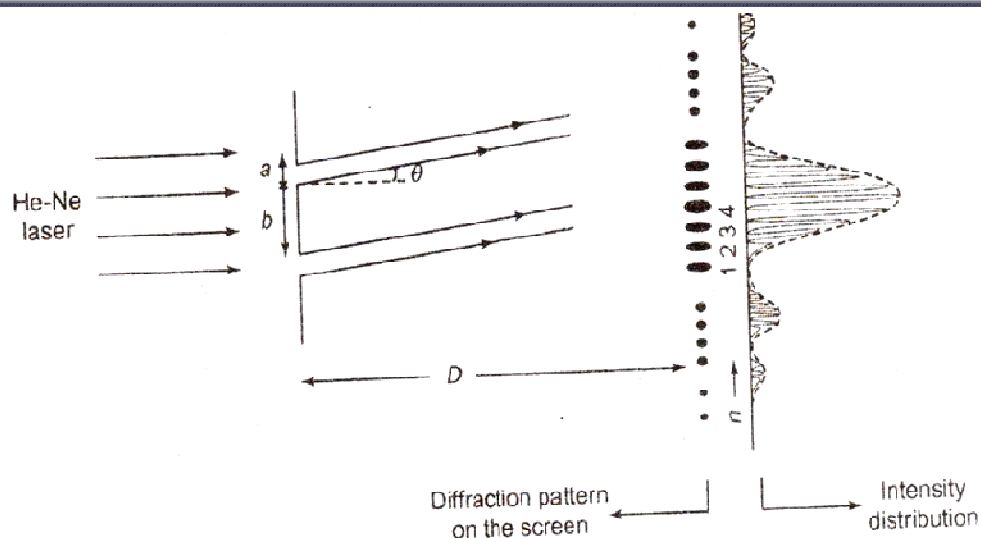
### 32.1B Aim

To determine the wavelength of the laser by using double slit experiment.

### 32.2B Apparatus

Laser, a double slit, an optical bench with uprights, a screen, a metre scale, stands and a travelling microscope.

### 32.3B Diagram



**Figure:** For the experiment arrangement of the double slit experiment.

### 32.4B Formula

For the calculation of wavelength we shall use this below given formula which is:

$$\lambda = \frac{(a + b) (x_2 - x_1)}{D (n_2 - n_1)}$$

where  $\lambda$  is the wavelength of the laser light,

and  $a$  is the width of the slit and  $b$  is the distance between the two slits,

and here  $x_2$  and  $x_1$  are the distance between the maxima of order  $n_2$  and  $n_1$  respectively.

### 32.5B Theory and Description

As a plane wave passes through a slit, it emerges as circular waves as if from a point source of light. If light from a laser (which is a plane wave) passes through two slits, then each slit acts as a point source of light. Because the incident wave is a plane wave and is monochromatic (i.e. one colour, one frequency), then the waves from each slit will be in phase (meaning a crest will emanate from each slit at the same time) and will have the same frequency and amplitude.

The waves from each source will interfere. At some points in space, total constructive interference occurs and a detector at this location will measure a wave with twice the amplitude,  $2A$ , as the wave from each source. At other points in space total destructive interference occurs and a detector at this location will measure a wave with zero amplitude (meaning no wave at all). At the majority of points in space, interference results in a wave that has an amplitude between 0 and  $2A$ —meaning that it is not total constructive or destructive interference.

For the calculation of wavelength we shall use this below given formula which is:

$$\lambda = \frac{(a + b) (x_2 - x_1)}{D (n_2 - n_1)}$$

If a graph is plotted between the order of maxima along x-axis and their respective distances along y-axis, a straight line is obtained. The slope of this line gives  $\frac{(x_2 - x_1)}{(n_2 - n_1)}$ . The distance between the corresponding points of the two slits  $(a+b)$  can be measured with the travelling microscope,  $D$  can be measured with a metre scale and  $\lambda$  can be easily determined.

## 32.6B Self Learning Exercise

- Q.1 Is light from sodium lamp polarized?
- Q.2 Give the conditions of interference.
- Q.3 Are the biprism fringes perfectly straight?
- Q.4 Is there any loss of energy in interference phenomenon?
- Q.5 What is difference between ordinary and extra-ordinary light?
- Q.6 What will happen if you replace monochromatic source by white light source?

## 32.7B Procedure

1. Level the bed of the optical bench with the help of the spirit level and the screws provided for the purposes. Adjust the position of the He-Ne source so that axis of the laser tube is horizontal.
2. Place the slit in between the laser source on an upright and make it vertical by means of the tangential screw.
3. Place the screen S at large distance from the slit and adjust its position as to get a sharp pattern.
4. With a pencil mark the position of as many minima as you can see and mark their orders as -1, -2, -3.....on the left side of the central maxima and as 1, 2, 3..... on the right side of the maxima with a travelling microscope.
5. Although the central maximum is the brightest spot. It is difficult to distinguish it from other maxima as the width of all the spots is same here. To avoid the confusion, mark the centre of all the equidistant bright spots with the soft pencil and label them as  $n=1, 2, 3, \dots$  starting from the left side. Choosing any point on left as zero, measures the distance of the interference maxima with the help of the metre scale and travelling microscope.
6. Measure the distance between the slits and the screen with the help of a metre scale and measuring-tape.
7. Measure the distance  $(a+b)$  with the travelling microscope or read it directly if a micrometre screw is provided with the double slit.
8. Plot a graph between  $n$  along x-axis and the distance of the corresponding maxima along y-axis.

### 32.8B Observation

Distance between the slit and the screen: .....cm

S. NO.	ORDER $n$	DISTANCE OF MAXIMA $x_n$ (in cm)
1		
2		
3		
4		
5		
6		

For measuring (a+b):

Reading	Microscope reading with the cross wire on			
	Left side of slit 1 A	Right side of slit 1 B	Left side of the slit 2 A	Right side of the slit 2 B
Main scale				
Vernier scale				
Total				

### 32.9B Calculations

.....

.....

### 32.10B Result

The wavelength of the laser is = .....

### 32.11B Discussion

.....

.....



### 32.12B Precautions and Sources of Error

1. Laser beam should be horizontal and not limited.
2. Laser beam should be looked upon directly with naked eye,
3. Once a sharp diffraction pattern is achieved, the set-up should not be disturbed.
4. The slit should be vertical.
5. The screen should be kept normal to the incident beam.
6. The distance of the minima should be measured accurately.

### 32.13B Answers To Self Learning Exercise

**Ans.1:** No, it is unpolarised.

**Ans.2:** The conditions of interference are given as:

- (i) The waves from the two sources must be of the same frequency.
- (ii) The two light waves must be coherent.
- (iii) The path difference between the overlapping waves must be less than the coherence length of the waves.
- (iv) If the two sets of waves are plane polarized, their planes of polarization must be the same.
- (v) The distance of the screen from the two sources must be large.
- (vi) The vector sum of the overlapping electric field vectors should be zero in the dark regions.

**Ans.3:** The biprism fringes are not perfectly straight but they are hyperbolic in nature. The eccentricity of the hyperbola is so large that they appear as straight in the field of view of eyepiece.

**Ans.4:** No, there is only redistribution of energy i.e. energy from dark places is shifted to bright places.

**Ans.5:** While passing through a double refracting crystal, the ray which obeys the laws of refraction is the ordinary ray while the other is the extra-ordinary ray.

**Ans.6:** In case of white light, the interference pattern consists of a central white fringe surrounded by a few coloured fringes.

## Part -III: Measurement the Diameter of A Thin Wire.

32.1C Aim

32.2C Apparatus

32.3C Diagram

32.4C Formula

32.5C Theory and Description

32.6C Self Learning Exercise

32.7C Procedure

32.8C Observation

32.9C Calculations

32.10C Result

32.11C Discussion

32.12C Precautions and Sources of Error

32.13C Answers To Self Learning Exercise

### 32.1C Aim

To measure the diameter of a thin wire.

### 32.2C Apparatus

Optical bench with uprights, a monochromatic source of light (sodium lamp), micrometre eyepiece, slit (may be already fitted with bench) and thin wire.

### 32.3C Diagram

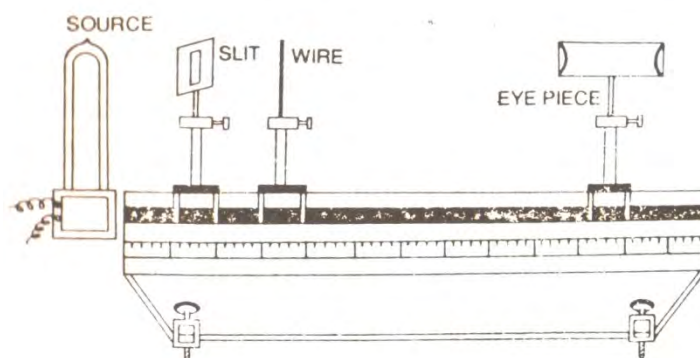
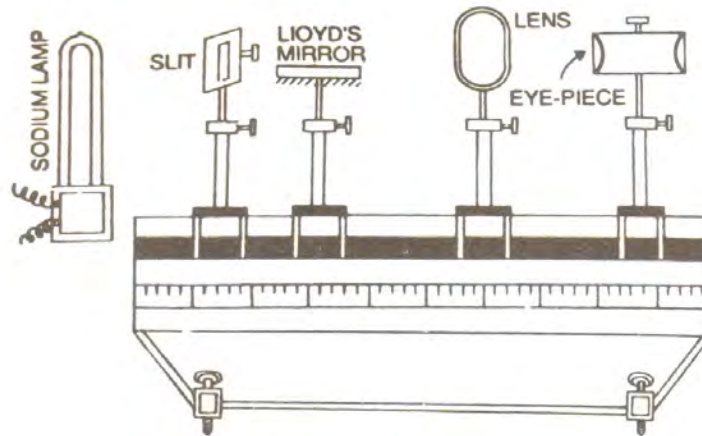


Figure 32.1: Figure for apparatus arrangement.



**Figure 32.2: Action of thin wire**



**Figure 32.3: Arrangement of bench.**

### 32.4C Formula

The thickness  $t$  of a thin wire can be found by using the following formula

$$d = \lambda \left[ \frac{D_2 - D_1}{\beta_2 - \beta_1} \right]$$

where

$\lambda$  = wavelength of light used

$D_1$  = first distance of eyepiece from the wire

$D_2$  = second distance of eyepiece from the wire

$\beta_1$  = average fringe width at a distance  $D_1$

$\beta_2$  = average fringe width at a distance  $D_2$

### 32.5C Theory and Description

The optical bench used in the experiment consists of a heavy cast iron base supported on four leveling screws. There is a graduated scale along its one arm.

The bench is provided with four uprights which can be clamped anywhere and the position can be read by means of vernier attached to it. Each of the uprights is subjected to the following motions:

- (i) Motion along bench,
- (ii) Motion right angle to bench
- (iii) Rotation about the axis of the upright
- (iv) The slit and the thin wire can be rotated in their own vertical planes.

The arrangement of bench is given in figure 32.3.

The action of thin wire is shown in figure 32.2. When a thin wire is placed in the path of monochromatic light coming from the slit S, the light is reflected. Now the interference of light takes place. The two edges of the wire acts as the two sources. The interference fringes are observed on the screen AB or they can be seen in eyepiece.

### 32.6C Self Learning Exercise

- Q.1** Write the expression for the difference in the wavelength of sodium light using Fabry-Perot interferometer.
- Q.2** Give the application of rare-earth ion laser.

### 32.7C Procedure

1. Level the bed of the optical bench with the help of spirit level and leveling screws.
2. Place slit, thin wire and eyepiece at the proper uprights and adjust them at the same height. (See figure 32.1)
3. The wire should be adjusted vertical and parallel to the slit at its centre.
4. The slit and cross wire of eyepiece are made vertical.
5. The micrometre eyepiece is focused on cross-wires.
6. Monochromatic light is allowed to incident on the slit and the bench is so adjusted that light comes straight along its length.
7. The micrometre eyepiece is moved away from the wire. By moving the eyepiece at right angle to the optical bench, interference fringed are observed.

#### 32.7.1 Measurement of fringe-width $\beta$ :

- (i) Find out the least count the micrometre screw.

- (ii) The micrometre eyepiece is placed in front of thin wire. This is moved to such a distance where fringes are distinct, bright and widely spaced.
  - (iii) The vertical cross wire of the eyepiece is fixed at the centre of one of bright fringes and its reading is noted on the main scale as well as on micrometre screw.
  - (iv) The cross wire is moved to different fringes and the corresponding readings are noted. The width of five fringes found on an averaged and then fringe width  $\beta$ , is calculated.
  - (v) The distance  $D_1$  of the eyepiece from thin wire is noted.
  - (vi) The micrometre eye-piece is moved to another distance  $D_2$  where distinct and bright fringes are observed.
- Now repeat procedure (4) to obtain the fringe-width  $\beta_2$  at this distance.

### 32.8C Observation

Wave length of light used  $\lambda =$

First distance of the eyepiece from the wire  $D_1 =$

Second distance of the eyepiece from the wire  $D_2 =$

Pitch of the screw =

No. of divisions on micrometre screw =

Least count of micrometre screw =

**Table 1:** For fringe width  $\beta_1$  when eyepiece is at a distance  $D_1$  from the wire:

No. of Fringe	Micrometer reading			Width of 5 Fringes	Mean of 5 fringes	Fringe width $\beta_1$
	Main Scale Reading	Vernier Scale Reading	Total Reading			
1						
2						
3						
4						
5						

6						
7						

**Table 2:** For fringe width  $\beta_2$  when eyepiece is at a distance  $D_2$  from the wire:

No. of Fringe	Micrometer reading			Width of 5 Fringes	Mean of 5 fringes	Fringe width $\beta_2$
	Main Scale Reading	Vernier Scale Reading	Total Reading			
1						
2						
3						
4						
5						
6						
7						

### 32.9C Calculations

The diameter  $d$  of a thin wire can be found by using the following formula

$$d = \lambda \left[ \frac{D_2 - D_1}{\beta_2 - \beta_1} \right]$$

### 32.10C Result

The diameter of the wire = .....

### 32.11C Discussion

.....  
 .....

### 32.12C Precautions and Sources of Error

1. The diffraction fringes formed on either side of the interference fringes should be clearly differentiated.

2. The wire should be vertical and parallel to the slit at its centre.
3. The distance D should be taken as large as possible.
4. The setting of uprights should be at the same level.
5. Cross wire should be fixed at the centre of the fringe while taking observations for fringe width.
6. The fringes should be measured at fairly large distance.
7. Motion of eyepiece should be perpendicular to the lengths of the bench.

### 32.13C Answers To Self Learning Exercise

**Ans.1:** 
$$2(\lambda_1 - \lambda_2) = \frac{\lambda_1 \lambda_2}{(t_2 - t_1)} = \frac{\lambda_{mean}^2}{(t_2 - t_1)}$$

**Ans.2:** The major application is in the different forms of material processing such as spot welding, drilling and making other lasers. Since they can be focused to tiny spots, these lasers are used for resister trimming, memory repair and in circuit mask. They are also used in cutting out specialized circuits.

It is also used in the field of military exploitation. They are used for target designation, range finding etc. Laser gyros are used in many aircraft and in weapon systems. Attempts are being made to use them in detection of submerged inertial confinement fusion. *Nd* lasers also find wide application in general laboratory work and in scientific fields. Since frequency multiplication into ultraviolet and green can be done with these lasers, they are good sources for pumping tunable dye lasers and other varieties of diagnostic and laser probes.

### Glossary

**Spot size:** It is defined as the radius of the beam itself. The irradiance of the beam decreases gradually at the edges

**Photo Detector:** For processing the signal at the receiver end of the fiber link, we require a device to convert the light signals to electrical wave forms. This task is done by the *photo – detectors*.

**Beam divergence:** Beam divergence is the angular measure of the increase in the radius or diameter with distance from the optical aperture as the beam emerges.

**Interference filters:** An interference filter is an optical system that will transmit a

very narrow range of wavelengths and thus provides a monochromatic beam of light.

### Viva Questions

- Q.1** Give the application of He-Ne laser.
- Q.2** What is the population inversion?
- Q.3** What is the pumping?
- Q.4** How many types are producing population inversion?
- Q.5** What is optical pumping?
- Q.6** What is direct conversion?
- Q.7** Give two applications of laser.
- Q.8** What is the use of laser in medical field?
- Q.9** Give the importance of the Optical Fiber.
- Q.10** What are concentric resonator?
- Q.11** Give the brief theory on the Circular Fringes.
- Q.12** How many types of interference phenomenon? Give its conditions.
- Q.13** What are Interference Fringes?
- Q.14** What is diffraction of light?
- Q.15** What is the pumping?
- Q.16** What is coherence?
- Q.17** What is LASER?
- Q.18** Give the application of Excimer Laser.

### Answers to Viva Questions

**Ans.1:** There are some fields, where helium-neon laser is applied, which are given below:

- (1) Interferometry.
- (2) Laser printing.
- (3) Bar-code reading.
- (4) As pointing and directional reference beams.

**Ans.2:** Under ordinary conditions of thermal equilibrium, the number of atoms in the higher energy state is considerably smaller than the number in the number in



the lower energy state. This is known as population inversion.

**Ans.3:** The method of producing population inversion is called pumping. The population inversion can be achieved by exciting the medium with suitable form of energy.

**Ans.4:** There are five types of producing population inversion or pumping, which is given below:

(i) Optical pumping, (ii) Electric discharge, (iii) Inelastic atom-atom collision, (iv) Direct conversion, (v) Chemical reactions.

**Ans.5:** If we supply luminous energy to the medium for causing population inversion, then the pumping is called the optical pumping.

**Ans.6:** It is a method of producing population inversion. A direct conversion of electrical energy into radiant energy occurs in light emitting diodes (LEDs). This method of pumping occurs in Semiconductor Lasers.

**Ans.7:** (i) Laser is used for three-dimensional photography or holography.

(ii) Lasers can be extremely useful tool in controlled fusion research (Laser-fusion).

**Ans.8:** The laser beam is used in delicate surgery as cornea grafting. With laser beam, the surgery is completed in much shorter time. The laser beam is also used in the treatment of kidney, stone, cancer and tumor and in depositing and cutting the blood cells in brain operations.

**Ans.9:** The following are the main importance of optical fibers:

(i) *Attenuation* in a fibre is markedly lower than that of coaxial cable or twisted pair and is constant over a very wide range. So transmission within wide range of distance is possible without repeaters etc.

(ii) *Smaller size and lighter weight.* Optical fibres are considerably thinner than coaxial cable or bundles twisted-pair cable. So they occupy much less space.

**Ans.10:** In this type of resonator, two spherical mirrors with exactly the same radius  $r$  are in operation. The reflectors are separated by a distance  $l$ . The centres of curvature of the reflectors  $C_1$  and  $C_2$  are coincident. Hence, we can say that  $l = 2r$ .

**Ans.13:** They are alternately bright and dark patches of light obtained in the region of superposition of two wave trains of light.

**Ans.14:** For diffraction phenomenon we assume that light travels in the form of waves and bends round the corners of an obstacle. This phenomenon of deviation of light from rectilinear propagation and bending round the corners of an obstacle is known as Diffraction of light.

**Ans.15:** The method of producing population inversion is called pumping. The population inversion can be achieved by exciting the medium with suitable form of energy.

**Ans.16:** A wave which appears to be a pure sine wave for an infinitely large period of time or in an infinitely extended space is said to be a perfectly coherent wave.

**Ans.17:** It is a device by which an intense, monochromatic, collimated and highly-coherent light beam can be obtained.

**Ans.18:** Excimer lasers are used in medical field, material processing, pumping of dye lasers and in photolithography. Excimer lasers can provide pulse intensities of the order of  $10^{13} W/m^2$ . They produce sharp edges in cutting process. This property can be gainfully employed in medical applications, such as surgery and corneal sculpting which can provide optical correction to the eye without using glasses. Excimer lasers are also used for pumping organic dye lasers, because all such dyes have extended absorption, into the ultraviolet region.

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## Unit-33

### Laser Experiment:

#### (i) The Optical Rotation of Quartz

#### (ii) Numerical Aperture of a Fiber

#### Part (i) To Measure the Optical Rotation of Quartz using Laser light Structure of the unit

33.1A Aim

33.2A Apparatus

33.3A Diagram

33.4A Formula

33.5A Theory and Description

33.6A Procedure

33.7A Observation

33.8A Calculations

33.9A Result

33.10A Discussion

33.11A Precautions and Sources of Error

33.12A Self Learning Exercise

33.13A Glossary

33.14A Answers to Self Learning Exercises

33.15A Viva Questions

33.16 A Answers to Viva Questions

References and Suggested Readings

#### **33.1A Aim**

To measure the optical rotation of Quartz.

## 33.2A Apparatus

A specified quartz crystal, sodium and mercury sources of light, a filament lamp, spectrometer, grating, polarizing and analyzing Nicols or Polaroid, collimating and condensing lenses, aperture etc.

## 33.3A Diagram

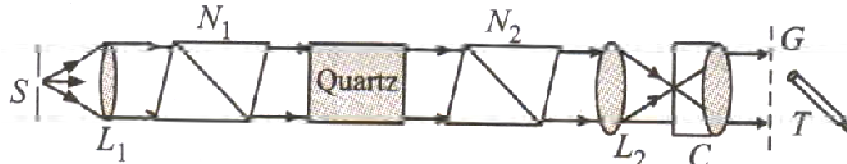


Figure 33.1: Experiment set up to study optical rotation of quartz.

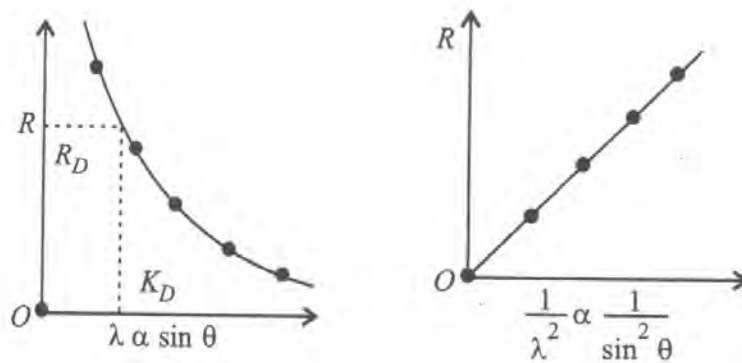


Figure 33.2: Graph of experiment with grating.

## 33.4A Formula

For initial crossed setting of Nicols or Polaroids the optical rotation is given by:

$$R = \frac{(2n+1)\pi}{2} \quad (33.1)$$

where  $n=0, 1, 2, 3, \dots$

For initial crossed setting of Nicols or Polaroids (polariser-analyser) the optical rotation

$$R' = \frac{2n\pi}{2} \quad (33.2)$$

where  $n=0, 1, 2, 3, \dots$

The spectrometer is calibrated by using grating relation is given below:

$$\lambda = \frac{\sin \theta}{nN} \quad (33.3)$$

Where  $n =$  the order of the spectrum;

$N =$  number of rulings per centimeter on grating;

$\theta =$  the angle of diffraction for wavelength  $\lambda$

### 33.5A Theory and Description

When a polarizer and analyzer are crossed, no light emerges out of the analyzer. When a quartz plate cut with its faces parallel to the optic axis is introduced between  $N_1$  and  $N_2$  such that light falls normally upon the quartz plate, the light emerges out of  $N_2$ .

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration gradually rotated as shown in figure 33.1 and 33.2. The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light.

Certain substances rotate the plane of polarizing of a wavelength through a definite angle. Such substances are optically active and quartz is one of them. Quartz crystal cut its faces at right angles to the optic axis is used to study its optical rotation. The light is incident normally on its face. Quartz of one mm thickness of this type rotates the plane of polarization of sodium light through  $21.7^\circ$ . Hence a thick quartz crystal shows sufficient rotation. Biot expresses that the rotation is inversely proportional to the square of wavelength of light incident on the crystal, i.e.

$$R = \frac{K}{\lambda^2} \quad (33.4)$$

Where  $K$  is constant whose value is given by the slope of  $\left[ R, \left( \frac{1}{\lambda^2} \right) \right]$  plot.

If light of continuous wavelength is examined through parallel Nicols or Polaroids by a spectrometer, then on interposing quartz between polarizer and analyzer dark bands appear for a rotation equal to an odd multiple of  $\left( \frac{\pi}{2} \right)$ . This rotation is given by relation (33.1). If Polaroids are crossed then rotation is given by relation (33.2). The value of integer ( $n$ ) is determined by using sodium light for which rotation is known.

For experiment set-up we have the figure 33.1. Here  $S$  is a strong source of

light of continuous spectrum, a filament bulb of high wattage serves the purpose well. The lens  $L_1$  renders the light parallel which is incident on a Polarizer  $N_1$  (Polaroid or Nicol). The light then passes through the experimental quartz crystal of nearly  $8\text{ cm}$  length. The emergent light falls on analyser  $N_2$  (Polaroid and Nicol) and is condensed by a condensing lens  $L_2$  on the slit of the collimator C of a spectrometer. G is the grating and T is the telescope of the spectrometer. The source S can be replaced by a sodium or mercury lamp as and when required.

Since spectrometer is required to measure wavelengths corresponding to the dark bands appearing due to optical rotation, the calibration of spectrometer in terms of wavelengths is the very important part of experiment. The spectrometer is calibrated in terms of standard wavelengths is given in observation table by using grating or prism. Set a spectrometer for making prismatic or grating measurements. The spectrometer is calibrated to give wavelengths.

### 33.6A Procedure

Now switch on the incandescent lamp and bring S in the focal plane of  $L_1$ . For correct setting the aperture of  $L_1$  appears filled with light when viewed through the collimator, the items  $N_1, Q$  and  $N_2$  are kept away. Next adjust  $L_2$  so that light is condensed on the slit aperture. Finally polariser ( $N_1$ ) and analyzer ( $N_2$ ) are adjusted in parallel setting. For this,  $N_2$  is rotated to show maximum intensity when viewed through the collimator.

Now for observation we have three methods for study, which are given below:

#### 33.7A.1 Experiment with Grating:

A continuous spectrum is formed by using grating for the incandescent lamp. The quartz crystal ( $Q$ ) is now interposed between polariser ( $N_1$ ) and analyzer ( $N_2$ ), dark bands appear. The angle of diffraction ( $\theta$ ) corresponding to each dark band is noted. Find the value of  $(\sin \theta)$  and tabulate the order of band and corresponding  $(\sin \theta)$  from red end of the spectrum. In grating spectrum red end lies farther than violet, while reverse is the case with prismatic spectrum. Read the wavelength ( $\lambda'_s$ ) corresponding to  $(\sin \theta)$  for each dark band from the calibration curve of the spectrometer.

For finding the value of rotation for each dark band substitute order  $n = 0, 1, 2, 3, \dots$  in relation (33.1) and calculate  $R$  for each value. Tabulate  $(R, \lambda)$  and plot the graph between  $\left[R, \left(\frac{1}{\lambda^2}\right)\right]$ . In grating spectrum ( $\lambda \propto \sin \theta$ ) for the same order of spectrum therefore variations  $(R, \sin \theta)$  and  $\left[R, \left(\frac{1}{\sin^2 \theta}\right)\right]$  may be plotted in place of  $(R, \lambda)$  and  $\left[R, \left(\frac{1}{\lambda^2}\right)\right]$ , respectively. Determine the slope of  $\left[R, \left(\frac{1}{\lambda^2}\right)\right]$  and calculate constant ( $k$ ) of relation (33.4). Determine rotations for the bands in red, yellow and violet regions as  $R_r, R_y$  and  $R_v$ , respectively and calculate the rotary dispersive power of quartz by the following relation.

$$\omega = \frac{(R_v - R_r)}{R_y} \quad (33.5)$$

Exact rotation is calculated with reference to rotation of yellow line of sodium. Knowing the thickness of the crystal and the fact that 1 mm thick quartz rotates the plane of polarization through  $21.7^\circ$  or  $\pi \times \frac{21.7}{180}$  radian, the value of  $n$  is calculated by relation (33.1). Now the other values of rotation are calculated by assigning  $(n + 1), (n + 2), (n + 3), \dots$  to subsequent dark bands. It may be seen that the order of  $n$  for 1 cm thick quartz crystal is nearly 0.7. The analyzer  $N_2$  is rotated through  $90^\circ$  from its parallel setting. The position of the dark bands is change.

### 33.7A.2 Experiment with Prism:

A continuous spectrum is formed with the help of a prism is set in the position of minimum deviation. Interpose the quartz between the polarizer and analyzer and get the dark bands. Note the positions of the dark bands on eye piece micrometer scale and spectrometer dial and calculate the corresponding wavelength from the calibration. Determine rotations for each dark band as described above and plot the variations  $(R, \lambda)$  and  $\left(R, \frac{1}{\lambda^2}\right)$ . Repeat the experiment for parallel and crossed setting of polariser and analyzer.

#### 33.7A Observation

Least Count of Spectrometer: .....

**Table 1: For Experiment with grating:**

S. No.	Colour	Readings for RHS			Readings for LHS			Angle $2\theta = b - a$	Angle $\theta$	$\sin \theta$
		Main scale Reading	Venire scale Reading	Total reading	Main scale Reading	Venire scale Reading	Total reading			
1										
2										
3										
4										
5										
6										

**Table 2: For Experiment with prism:**

S.No	Direct Slit Reading (a)			Reading for minimum deviations(b)			Angle of minimum deviation for wavelength $\lambda$ (b ~ a)
	Main scale Reading	Venire scale Reading	Total reading	Main scale Reading	Venire scale Reading	Total reading	
1							
2							
3							
4							

### 33.8A Calculations

-----  
 -----



### 33.9A Result

The Optical rotation of Quartz is.....

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

### 33.10A Discussion

.....

.....

.....

### 33.11A Precautions and Sources of Error

1. The width of slit should be adjusted at its optimum value.
2. The collimator should be adjusted for parallel rays.
3. Leveling of spectroscopy is very important factor for the source of error.
4. The mercury lamp should be placed on the axis of collimator to obtain the best intensity.
5. The slit should be free from dust particles otherwise black horizontal lines will appear across the spectrum.

### 33.12A Self Learning Exercise

- Q.1** How can you produce elliptically polarized light using quarter wave plate?
- Q.2** Give the classification of the wave plates.
- Q.3** Give the analysis of elliptically polarized light using Babinet Compensator.
- Q.4** What is elliptically polarized light?
- Q.5** Give the brief description about polarizing angle.

### 33.13A Glossary

**Polarization:** The phenomenon where light wave exhibits lack of symmetry about its direction of propagation is known as polarization of light.

**Optical Rotation:** Optical rotation is the phenomenon of rotating the plane polarization of light about the direction of propagation of light, when passed through certain crystals.

**Specific Rotation:** Specific rotation for the given wavelength of light at a given temperature is defined as the rotation produced by one decimeter length of the solution of unit concentration.

**Dextrorotatory:** These are the substance which rotates the plane of polarization towards right.

**Laevorotatory:** These are the substance which rotates the plane of polarization towards left.

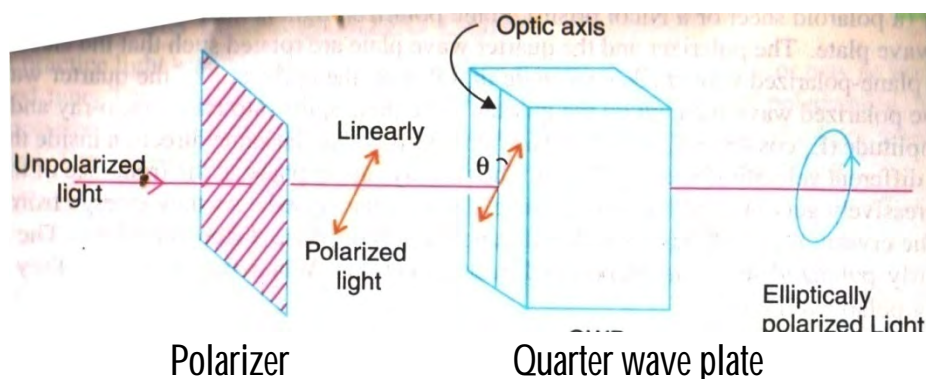
### 33.14A Answers to Self Learning Exercise

#### Ans.1: Production of Elliptically Polarized Light:

Here for the production of elliptically polarized light from unpolarized light, we use two optical devices which are

(a) Quarter wave plate and (b) Polarizer

First of all Unpolarized light is converted to plane polarized light by allowing it to pass through a polarizer (a Polaroid sheet or a Nicol prism). The plane polarized light is then made incident on a quarter wave plate (see figure 1). The quarter wave plate or the polarizer is rotated such that the electric vector  $E$  of plane polarized light wave makes an angle  $\theta$  ( $\neq 45^\circ$ ) with the optic axis of the quarter wave plate. The incident ray divides into o-ray and e-ray of amplitudes  $E \sin \theta$  and  $E \cos \theta$ . The rays travel along the same direction in the crystal with different velocities. The two rays are polarized in orthogonal planes. They are in phase at the front face but progressively get out of phase as they travel through the crystal. When they emerge out of the crystal they will have a path difference of  $\frac{\lambda}{4}$  or a phase difference of  $90^\circ$ . When they combine, they produce elliptically polarized light.



**Figure: For production of elliptically polarized light.**

**Ans.2:** Here we will study two types of retardation plates or wave plates which are given below:

**(1) Quarter Wave Plate:**

A quarter wave plate is thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness adjusted such that it introduce a quarter wave ( $\frac{\lambda}{4}$ ) wave path difference (or a phase difference of  $\{\frac{\pi}{2}\}$ ) between the extra ordinary ray and ordinary ray propagating through it.

When a plane polarized light wave is incident on a birefringent crystal having the optic axis parallel to its refracting face, the wave splits into e-wave and o-wave. The two waves travel along the same direction but with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them. Thus, for a quartz wave plate

$$(\mu_e - \mu_o)d = \frac{\lambda}{4} \quad \therefore d = \frac{\lambda}{4(\mu_e - \mu_o)}$$

A quarter wave plates introduces between e-ray and o-ray a phase difference  $\delta$  given by

$$\delta = \frac{2\pi}{\lambda} \Delta = \frac{\pi}{2} = 90^\circ$$

**A quarter-wave plate is used to produce elliptically and circularly polarized light.** It converts plane polarized light into elliptically or circularly polarized light depending upon the angle that the incident light vector makes with optic axis of the quarter wave plate.

**(2) Half Wave Plate:**

A half wave plate is a thin plate, having the optic axis parallel to its refracting faces and its thickness chosen such that a half-wave( $\frac{\lambda}{2}$ ) path difference (or a phase difference of  $(180^\circ)$  between extra ordinary ray and ordinary ray.

When a plane polarized light wave is incident on quartz crystal having the optic axis parallel to its refracting faces, it splits into two waves: ordinary wave and extra ordinary wave. The two waves travel along the same direction inside the crystal but with different velocities. As a result, when these waves emerge from the rear face of the crystal, an optical path difference would be developed between them

$$(\mu_e - \mu_o)d = \frac{\lambda}{2} \quad \therefore d = \frac{\lambda}{2(\mu_e - \mu_o)}$$

A half wave introduces a phase difference between ordinary ray and extra ordinary ray, which is given below

$$\delta = \frac{2\pi}{\lambda} \Delta = 180^\circ$$

**Ans.4:** When two plane polarized waves are superimposed, the resultant light vector rotates in a plane perpendicular to the direction of propagation. If the resultant light vector has constant magnitude, then the tip of the light vector traces a circle and the light is said to be circularly polarized.

However, if the magnitude of the resultant light vector varies periodically during its rotation, the tip of the light vector traces an ellipse and the light is said to be elliptically polarized.

**Ans.5:** Malus found that an ordinary beam of light on reflection from a transparent medium becomes partially polarized. The degree of polarization increase as the angle of incidence is increased. At a particular value of angle of incidence, the reflected beam becomes completely polarized. The angle of incidence is called *polarising angle* for that medium.

Therefore, the angle of incidence, at which when ordinary light is incident on transparent refracting medium, the reflected light becomes maximum rich in plane polarized light is called polarizing angle.

The value of polarizing angle varies with the wavelength of light and hence complete polarization can be obtained only monochromatic light.

### 33.15A Viva Questions

- Q.1 What is Polarization?
- Q.2 What is Specific rotation?
- Q.3 What is Optical Rotation?
- Q.4 What is Dextrorotatory?
- Q.5 What is Laevorotatory?
- Q.6 Define the Plane of polarization.
- Q.7 Define Malus Law.

- Q.8** Is light from sodium lamp polarized?
- Q.9** What is difference between ordinary and extra-ordinary light?
- Q.10** How can you detect elliptically polarized light?

### 33.16A Answers to Viva Questions

**Ans.1:** The phenomenon where light wave exhibits lack of symmetry about its direction of propagation is known as polarization of light.

**Ans.2:** Specific rotation for the given wavelength of light at a given temperature is defined as the rotation produced by one decimeter length of the solution of unit concentration.

**Ans.3:** Optical rotation is the phenomenon of rotating the plane polarization of light about the direction of propagation of light, when passed through certain crystals or solutions.

**Ans.4:** These are defined as the substance which rotates the plane of polarization towards right. Example: cane sugar, tartaric acid, camphor.

**Ans.5:** These are defined as the substance which rotates the plane of polarization towards left. Example: fructose, turpentine etc.

**Ans.6:** The plane in which the vibrations of polarized ray are confined is known as the plane of vibration. A plane perpendicular to this plane of vibration is known as the plane of polarization.

**Ans.7:** Malus law states that when a beam of completely plane polarized light is incident on an analyzer, the resultant intensity of light ( $I$ ) transmitted from the analyser varies directly as the square of cosine of the angle ( $\phi$ ) between the plane of transmission of analyser and polariser

$$I \propto \cos^2 \phi$$

**Ans.8:** No, it is unpolarized.

**Ans.9:** While passing through a double refracting crystal, the ray which obeys the laws of refraction is the ordinary ray while the other is the extra-ordinary ray.

**Ans.10:** When elliptically polarized light passes through a Nicol, acting as an analyzer, the transmitted intensity varies from maximum to minimum twice for the full rotation of the analyzer.

## References and Suggested Readings

1. Optics by A.K. Ghatak.
2. B. Sc. Practical Physics by C. L. Arora
3. Fundamental of optics by Jenkins and White.
4. Physics Practical Vol. 2 by R.K. Shukla and Anchal Srivastav.
5. A Text Book of Physics Practical by R. Glazebrook.

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## Part (ii) To Measure The Numerical Aperture of a Fiber using Laser

### Structure of the Unit

- 33.1B Aim
  - 33.2B Apparatus
  - 33.3B Diagram
  - 33.4B Formula
  - 33.5B Theory and Description
  - 33.6B Procedure
  - 33.7B Observation
  - 33.8B Calculations
  - 33.9B Result
  - 33.10B Discussion
  - 33.11B Precautions and Sources of Error
  - 33.12B Self Learning Exercise
  - 33.13B Glossary
  - 33.14B Answers to Self Learning Exercises
  - 33.15B Viva Questions
  - 33.16B Answers to Viva Questions
- References and Suggested Readings

### 33.1B Aim

To determine the numerical aperture of a given optical fiber.

### 33.2B Apparatus

LED, 20 X microscope objective, microscope objective holder, photo detector, X-stage, an XYZ stack with a V groove mounted on it, a plastic fiber of approximate length one meter, a travelling microscope specially modified to serve both as holder for output fiber end as well as to provide a platform for the X-stage, 5V DC supply, function generator with D.C. offset and A.C. amplitude, optical power meter, resistances and capacitances.

### 33.3B Diagram

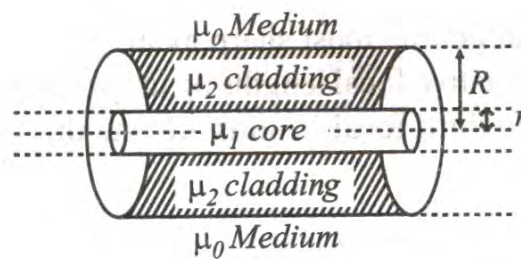


Figure 33.1: Basic form of optical fiber.

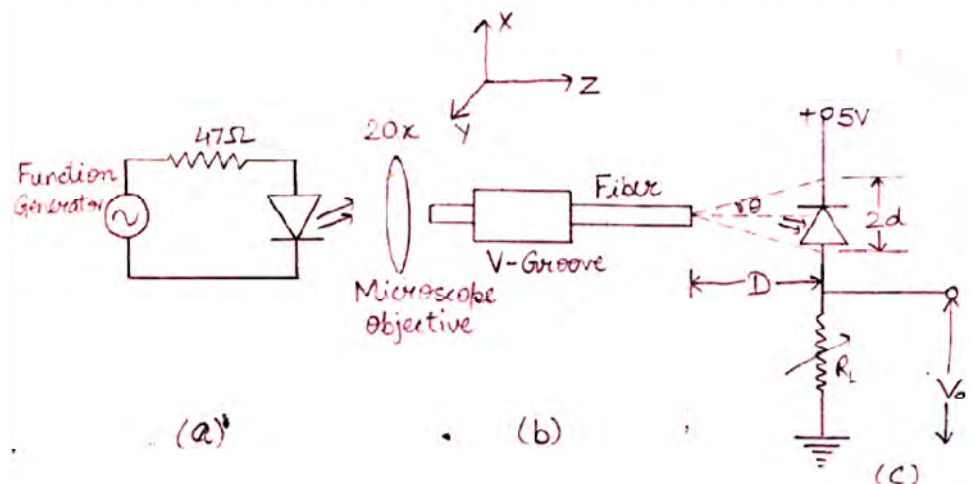
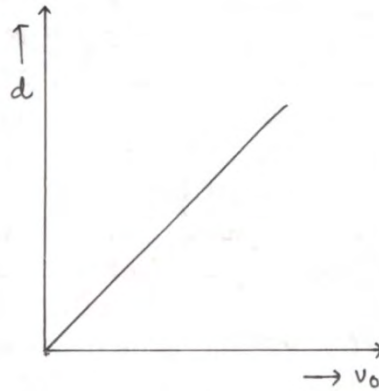


Figure 33.2: Experimental Set-up.



**Figure 33.3:** Graph between  $v_0$  and  $d$ .

### 33.3B Diagram

We have for the numerical aperture

$$\tan \theta = \frac{d_{NA}}{D}$$

Here  $\theta$  is small so we consider

$$\tan \theta \cong \sin \theta$$

$$\text{Hence Numerical Aperture} = \sin \theta = \frac{d_{NA}}{D}$$

where  $d_{NA}$  = position of the photo detector

$D$  = distance between the tip of the fibre and the photodetector surface

### 33.5B Theory and Description

The optical fiber communication has become important due to its following merits:

1. **Large Band Width:** Optical signal frequencies vary from about  $10^{13}$  to  $10^{16}$  Hz and therefore the attenuation is very low as compared to the highest frequency waves of about  $10^7$  Hz used in radio communication.
2. The thickness of fiber is of  $\mu m$  order.
3. The optical fibers are electrically isolated.
4. The signal passing through the optical fiber cannot intercept.
5. The power loss is low.
6. The optical fiber system has low cost, rugged, reliable and low cost of maintenance.



We can see the basic form of the optical fiber in figure 33.1.

The light emitting diode is placed just behind the  $20\times$  microscope objective mounted on a microscope objective holder that is screwed onto a breadboard. The fibre is placed firmly in the  $V$  – groove mounted on a  $XYZ$  stack such that only a small length (*not more than 5mm*) of the free end of the fibre projects outside the groove. The free end of the fibre projects the microscope objective so that the light from the LED can be coupled into the fiber. The  $XYZ$  stack is a stack of micropositioners which can provide translational movement along the three perpendicular directions  $X, Y$  and  $Z$  and is screwed on the breadboard.

The other end of the fiber faces the photo-detector which receives the light coming out of the fiber and converts it into electrical signal. The corresponding voltage across the load resistor can be displayed on the CRO screen. The photo detector is mounted on an  $X$ -stage which is fitted to the base of a travelling microscope. A calibrated screw fixed to the travelling microscope can provide the translational motion to the stage along  $X$ -direction. The movement of the stage along the  $Y$ -direction can be brought about by the calibrated screw fixed to the stage.

### **33.6B Procedure**

For the numerical aperture we have following given points:

1. First of all we make the connections as shown in figure 33.2 (a).
2. Then set the D.C. offset and amplitude to zero volts.
3. Now slowly adjust the D.C. offset to get 30mA D.C. current in the transmitter circuit.
4. Next press the sine function knob and the frequency at 1 kHz in the function generator, and slowly increases the amplitude of sinusoidal signal so as to get 1.4 V (p-p).
5. Now couple the light from the LED into the fiber in the following manner:  
Manipulate the  $X$  and  $Y$  (transverse) stages of  $XYZ$  stack so that maximum power output is obtained from the output end of the fiber. This power output can be read on an optical power meter, placed at the output fiber end. Now further maximum the output by manipulating  $Z$  stage. This procedure should

be repeated till the maximum possible power output is obtained. At this point the input fiber end is positioned at the focus of the microscope objective and maximum light is coupled into the fiber.

6. Next referring to figure 33.2(c) fixes the potentiometer  $R_L$  at its maximum value of  $100\text{ k}\Omega$ . Now make the connections as shown in given figure 33.2 (c).
7. Align the output end of the fiber vertically above the photo detector. Manipulate the X-stage so that the fiber tip is exactly at the centre of the photo detector surface.
8. Now move the photo detector in a direction exactly perpendicular to the fiber and for each position  $d$  of the photo detector, which can be read on the horizontal scale of travelling microscope, note the voltage  $V_0$  across the load resistance  $R_L$  in one channel of the CRO.
9. Plot a graph between  $d$  and  $V_0$  by the observation table.
10. Measure the distance ( $D$ ) between the tip of the fibre and the photo detector surface on the vertical scale of the travelling microscope. The numerical aperture can be found out thus

$$\tan \theta = \frac{d_{NA}}{D}$$

Because  $\theta$  is small so

$$\tan \theta \cong \sin \theta$$

Hence

$$NA = \sin \theta = \frac{d_{NA}}{D}$$

where

$d_{NA}$  = position of the photo detector

$D$  = distance between the tip of the fibre and the photodetector surface

### 33.7B Observation

Least count = .....

S.No.	D (in cm)	Voltage $V_0$ (in volts)
1		
2		
3		
4		

5		
6		
7		
8		
9		
10		

### 33.8B Calculations

Calculation is based on upper given reading.

### 33.9B Result

The numerical aperture of given plastic fiber is.....

$$\%error = \frac{\text{Experimental value} - \text{Standard value}}{\text{standard value}} \times 100$$

### 33.10B Discussion

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### 33.11B Precautions and Sources of Error

1. Keep 100K potentiometer at its maximum value before making the connections.
2. The fiber ends should be properly cleared and cleaned.
3. Care should be taken to read the positions of equipments.
4. A major source of error is to presence of dust and particles on the fiber end.

### 33.12B Self Learning Exercise

- Q.1 What is Numerical Aperture?
- Q.2 What are the applications of fibers?
- Q.3 How can you produce elliptically polarized light using quarter wave plate?

**Q.4** Give the classification of the wave plates.

**Q.5** Calculate refractive indices of the core and cladding material of a fiber from the following data:

$$NA = 0.22, \Delta = 0.012$$

**Q.6** At a distance of 1 km, spot size=9800 cm; calculate the cut-off parameter and also the number of modes for a fibre of core diameter 50  $\mu\text{m}$ .  $\mu_1 = 1.47, \mu_2 = 1.45$  (cladding), operating wavelength = 0.85  $\mu\text{m}$ .

### 33.13B Glossary

**LED:** It is a  $p - n$  junction semiconductor diode. It emits light, when it is forward biased.

**Fiber:** Fibers are the light equivalent of microwave waveguides with the additional advantages of a very wide bandwidth.

**NA:** It can be defined as

$$NA = \sin \theta_0 (\text{max.}) = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}$$

**Photo Detector:** For processing the signal at the receiver end of the fiber link, we require a device to convert the light signals to electrical wave forms. This task is done by the *photo - detectors*.

### 33.14B Answers to Self Learning Exercise

**Ans.1:** Numerical aperture has been observed that for a given fiber diameter, the numerical aperture of GRIN fiber is generally smaller than the one on a SI fiber. For a GRIN fiber, with an attenuation between 5 and 10  $\text{dB}/\text{km}$ , the  $NA$  will tend to run between 0.16 to 0.2, whereas a SI fibre of about the same physical size, with a loss of the order of 12  $\text{dB}/\text{km}$ , will tend to have  $NA$  of the order of 0.2 to 0.35.

**Ans.2:** There are some applications of optical fiber, which are given below:

(1) When the bandwidth of transmission medium is very large then we use a single mode fiber. Example: Undersea cable system.

(2) When the system bandwidth requirements are between 200 MHz and 2GHz km, a graded index multimode fiber would be the best choice. For example: in intra-city trunks between telephone central offices.

(3) When the system bandwidth requirements are lower, a step-index multimode

fiber would be better.

**Ans.5:** We know in optical fiber

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = 0.012$$

$$NA = \mu_1 \sqrt{2\Delta}$$

$$\text{Or } \mu_1 = \frac{NA}{\sqrt{2\Delta}} = \frac{0.22}{\sqrt{2 \times 0.012}} = 1.42$$

$$\Delta = \frac{1.42 - \mu_2}{1.42} = 0.012$$

Or

$$\mu_2 = 1.40$$

### 33.15B Viva Questions

- Q.1** What is optical fiber?
- Q.2** Give the classification of Optical Fiber.
- Q.3** Define LED.
- Q.4** Give the importance of the Optical Fiber.
- Q.5** What is acceptance angle?
- Q.6** Give the definition of alternating Current.
- Q.7** Give the definition of Direct current.
- Q.8** Give the definition of Resistance.
- Q.9** Give the definition of Capacitance.
- Q.10** The D. C. Offset and voltage amplitude set at zero. Why?

### 33.16B Answers to Viva Questions

**Ans.1:** Optical fibers are the light equivalent of microwave waveguides with the additional advantages of a very wide bandwidth. Physically an optical fiber is a very thin and flexible medium, having a cylindrical shape consisting of three sections: (i) the core, (ii) the cladding and (iii) the jacket.

**Ans.2:** The classification of fibers is given below:

- (i). Stepped index multimode fibre (SI)
- (ii). Graded index multimode fiber (GRIN)

(iii). Single mode fibre.

**Ans.3:** The full name of LED is Light Emitting Diode. It is a  $p - n$  junction semiconductor diode. It emits light, when it is forward biased.

**Ans.4:** The following are the main importance of optical fibers:

(1) *Attenuation* in a fibre is markedly lower than that of coaxial cable or twisted pair and is constant over a very wide range. So transmission within wide range of distance is possible without repeaters etc.

(2) *Smaller size and lighter weight.* Optical fibers are considerably thinner than coaxial cable or bundles twisted-pair cable. So they occupy much less space.

**Ans.5:** When any light wave impinging on the core within this maximum external incident angle  $\theta_0$  is coupled into the fiber and will propagate. This angle is called the acceptance angle. It is unique for a particular fiber but different for different fibers.

**Ans.6:** It is the form in which electric power is gave to residences. The usual wave form of the alternating current is Sine Wave. In this type of current, electric charge is periodically reverses direction.

**Ans.7:** It is the unidirectional flow of electric charge. Direct current is produced by sources such as batteries, thermocouples, solar cells.

**Ans.8:** The ability of a system to store an electric charge is known as capacitances.

**Ans.9:** A measure of the degree to which conductor opposes an electric current through that conductor.

**Ans.10:** Because of definite and accurate values of readings.

## References and Suggested Readings

1. A Text Book of Physics Practical by R. Glazebrook,
2. B.Sc. Practical Physics by C. L. Arora,
3. Practical Physics by G. L. Squires.
4. Physics Practical Vol. 2 by R. K. Shukla & Anchal Srivastav.

# UNIT-34

## Lissajous Figure Using C.R.O.

### Structure of the Unit

- 34.1 Aim
  - 34.2 Apparatus
  - 34.3 Diagram
  - 34.4 Formula
  - 34.5 Model Graph
  - 34.6 Theory and Description
  - 34.7 Self Learning Exercise-I
  - 34.8 Procedure
  - 34.9 Observation
  - 34.10 Graph
  - 34.11 Calculations
  - 34.12 Result
  - 34.13 Discussion
  - 34.14 Precautions and Sources of Error
  - 34.15 Self Learning Exercise -II
  - 34.16 Glossary
  - 34.17 Answers to Self Learning Exercises
  - 34.18 Viva Questions
  - 34.19 Answers to Viva Questions
- References and Suggested Readings

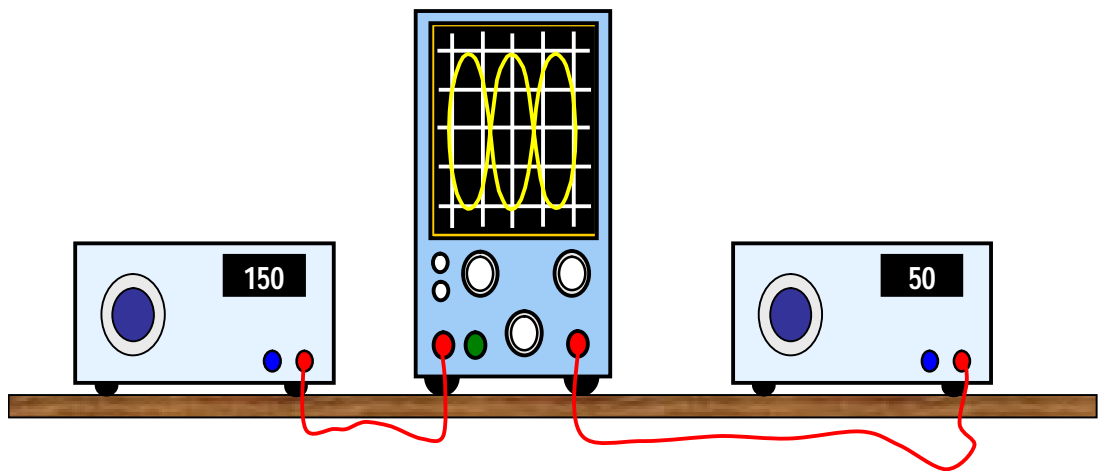
### 34.1 Aim

Measurement of phase differences and frequency using CRO (Lissajous figure).

## 34.2 Apparatus

- CRO,
- Signal generator,
- Patch cords etc.

## 34.3 Diagram



## 34.4 Formula

Voltage

$$V = (\text{no. of boxes in cm.}) \times (\text{selected Volts/cm scale})$$

Time period

$$T = (\text{no. of squares in cm}) \times (\text{selected Time/cm scale})$$

Then frequency is given by

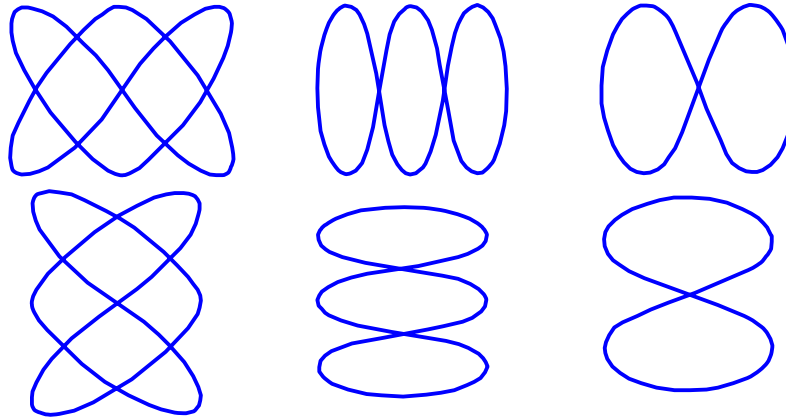
$$f (\text{Hz}) = 1/T(\text{sec})$$

The phase shift

$$\theta = \frac{\text{Phase shift in cm.}}{\text{One period in cm.}} \times 360^\circ$$



## 34.5 Model Graph



## 34.6 Theory and Description

### Introduction to CRO

The cathode-ray oscilloscope (CRO) is a common laboratory instrument that provides accurate time and amplitude measurements of voltage signals over a wide range of frequencies. Its reliability, stability, and ease of operation make it suitable as a general purpose laboratory instrument. The heart of the CRO is a cathode-ray tube shown schematically in Figure 1.

The cathode ray is a beam of electrons which are emitted by the heated cathode (negative electrode) and accelerated toward the fluorescent screen. The assembly of the cathode, intensity grid, focus grid, and accelerating anode (positive electrode) is called an *electron gun*. Its purpose is to generate the electron beam and control its intensity and focus. Between the electron gun and the fluorescent screen there are two pairs of metal plates - one oriented to provide horizontal deflection of the beam and one pair oriented to give vertical deflection to the beam. These plates are thus referred to as the *horizontal* and *vertical deflection plates*. The combination of these two deflections allows the beam to reach any portion of the fluorescent screen. Wherever the electron beam hits the screen, the phosphor is excited and light is emitted from that point. This conversion of electron energy into light allows us to write with points or lines of light on an otherwise darkened screen.

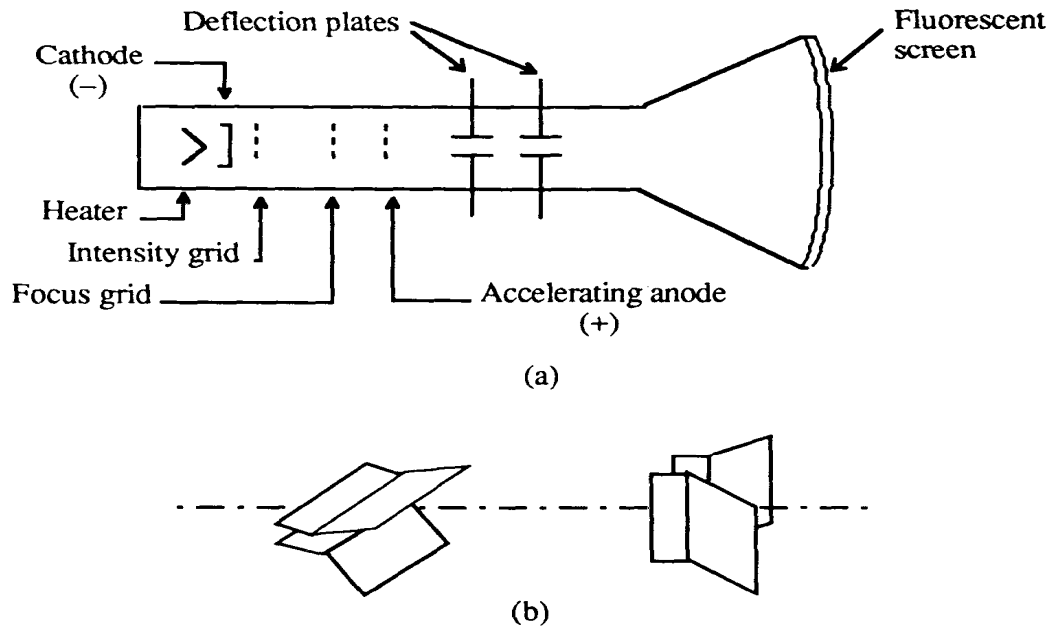


Figure 1. Cathode-ray tube: (a) schematic, (b) detail of the deflection plates.

In the most common use of the oscilloscope the signal to be studied is first amplified and then applied to the vertical (deflection) plates to deflect the beam vertically and at the same time a voltage that increases linearly with time is applied to the horizontal (deflection) plates thus causing the beam to be deflected horizontally at a uniform (constant) rate. The signal applied to the vertical plates is thus displayed on the screen as a function of time. The horizontal axis serves as a uniform time scale.

The linear deflection or sweep of the beam horizontally is accomplished by use of a *sweep generator* that is incorporated in the oscilloscope circuitry. The voltage output of such a generator is that of a sawtooth wave as shown in Figure. Application of one cycle of this voltage difference, which increases linearly with time, to the horizontal plates causes the beam to be deflected linearly with time across the tube face. When the voltage suddenly falls to zero, as at points (a) (b) (c), etc...., the end of each sweep - the beam flies back to its initial position. The horizontal deflection of the beam is repeated periodically, the frequency of this periodicity is adjustable by external controls.

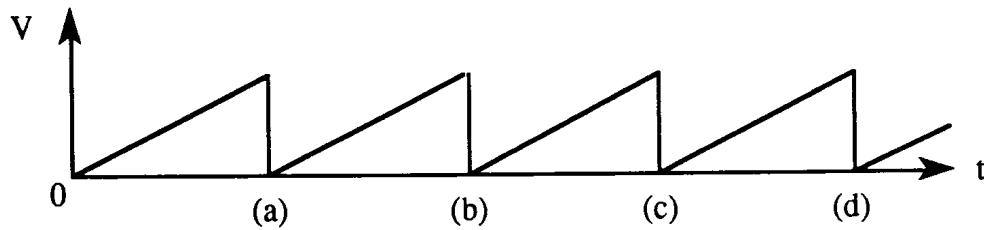


Figure. 2. Voltage difference  $V$  between horizontal plates as a function of time  $t$ .

To obtain steady traces on the tube face, an internal number of cycles of the unknown signal that is applied to the vertical plates must be associated with each cycle of the sweep generator. Thus, with such a matching of synchronization of the two deflections, the pattern on the tube face repeats itself and hence appears to remain stationary. The persistence of vision in the human eye and of the glow of the fluorescent screen aids in producing a stationary pattern. In addition, the electron beam is cut off (blanked) during fly back so that the retrace sweep is not observed.

### CRO Operation

A simplified block diagram of a typical oscilloscope is shown in Figure. In general, the instrument is operated in the following manner. The signal to be displayed is amplified by the vertical amplifier and applied to the vertical deflection plates of the CRT. A portion of the signal in the vertical amplifier is applied to the **sweep trigger** as a triggering signal. The sweep trigger then generates a pulse coincident with a selected point in the cycle of the triggering signal. This pulse turns on the sweep generator, initiating the saw tooth wave form. The saw tooth wave is amplified by the horizontal amplifier and applied to the horizontal deflection plates. Usually, additional provisions signal are made for applying an external triggering signal or utilizing the 60 Hz line for triggering. Also the sweep generator may be bypassed and an external signal applied directly to the horizontal amplifier.

### CRO Controls

The controls available on most oscilloscopes provide a wide range of operating conditions and thus make the instrument especially versatile. Since many of these controls are common to most oscilloscopes a brief description of them follows.

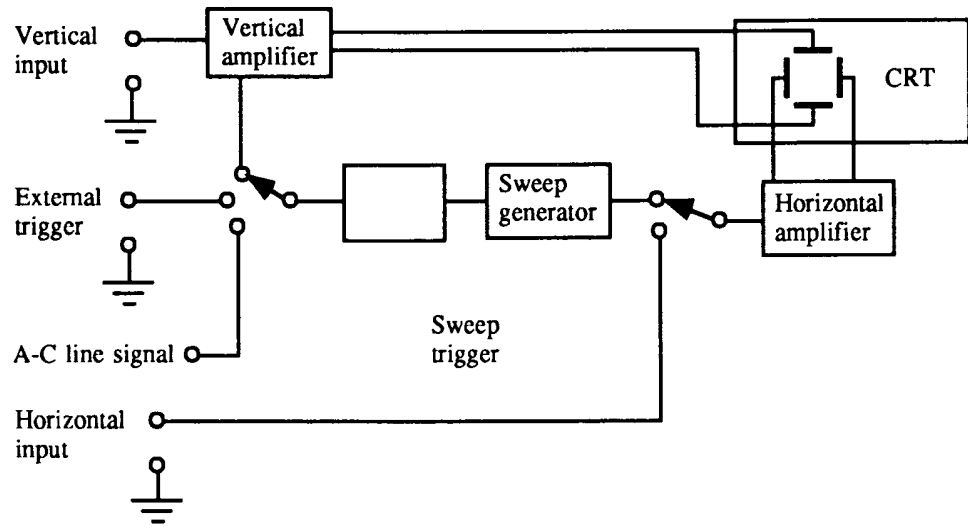


Figure 3. Block diagram of a typical oscilloscope.

- ***CATHODE-RAY TUBE***

**Power and Scale Illumination:**

Turns instrument on and controls illumination of the graticule.

**Focus:**

Focus the spot or trace on the screen.

**Intensity:**

Regulates the brightness of the spot or trace.

- ***VERTICAL AMPLIFIER SECTION***

**Position:**

Controls vertical positioning of oscilloscope display.

**Sensitivity:**

Selects the sensitivity of the vertical amplifier in calibrated steps.

**Variable Sensitivity:**

Provides a continuous range of sensitivities between the calibrated steps. Normally the sensitivity is calibrated only when the variable knob is in the fully clockwise position.

**AC-DC-GND:**

Selects desired coupling (ac or dc) for incoming signal applied to vertical amplifier, or grounds the amplifier input. Selecting dc couples the input directly to the amplifier; selecting ac send the signal through a capacitor before going to the amplifier thus blocking any constant component.

- ***HORIZONTAL-SWEEP SECTION***

**Sweep time/cm:**

Selects desired sweep rate from calibrated steps or admits external signal to horizontal amplifier.

**Sweep time/cm Variable:**

Provides continuously variable sweep rates. Calibrated position is fully clockwise.

**Position:**

Controls horizontal position of trace on screen.

**Horizontal Variable:**

Controls the attenuation (reduction) of signal applied to horizontal aplifier through Ext. Horiz. connector.

- ***TRIGGER***

The trigger selects the timing of the beginning of the horizontal sweep.

**Slope:**

Selects whether triggering occurs on an increasing (+) or decreasing (-) portion of trigger signal.

**Coupling:**

Selects whether triggering occurs at a specific dc or ac level.

**Source:**

Selects the source of the triggering signal.

**INT** - (internal) - from signal on vertical amplifier

**EXT** - (external) - from an external signal inserted at the **EXT. TRIG. INPUT.**

**LINE** - 60 cycle triger

**Level:**

Selects the voltage point on the triggering signal at which sweep is triggered. It also allows automatic (auto) triggering of allows sweep to run free (free run).

## ***CONNECTIONS FOR THE OSCILLOSCOPE***

### **Vertical Input:**

A pair of jacks for connecting the signal under study to the Y (or vertical) amplifier. The lower jack is grounded to the case.

### **Horizontal Input:**

A pair of jacks for connecting an external signal to the horizontal amplifier. The lower terminal is grounded to the case of the oscilloscope.

### **External Trigger Input:**

Input connector for external trigger signal.

**Cal. Out:** Provides amplitude calibrated square waves of 25 and 500 millivolts for use in calibrating the gain of the amplifiers.

Accuracy of the vertical deflection is + 3%. Sensitivity is variable.

Horizontal sweep should be accurate to within 3%. Range of sweep is variable.

### **Operating Instructions:**

Before plugging the oscilloscope into a wall receptacle, set the controls as follows:

1. Power switch at off
2. Intensity fully counter clockwise
3. Vertical centering in the center of range
4. Horizontal centering in the center of range
5. Vertical at 0.2
6. Sweep times 1

Plug line cord into a standard ac wall receptacle (nominally 118 V). Turn power on. Do not advance the Intensity Control.

Allow the scope to warm up for approximately two minutes, then turn the Intensity Control until the beam is visible on the screen.

**WARNING:** Never advance the Intensity Control so far that an excessively bright spot appears. Bright spots imply burning of the screen. A sharp focused spot of high intensity (great brightness) should never be allowed to remain fixed in one position on the screen for any length of time as damage to the screen may occur.

Adjust Horizontal and Vertical Centering Controls. Adjust the focus to give a sharp trace. Set trigger to internal, level to auto.

**Lissajous figure:**

Lissajous patterns are formed when you combined periodic waves moving back and forth with periodic waves moving up and down. This exhibit does this electronically allowing the visitor to control the frequency of the X and Y motions independently. The resulting pattern can be observed on an oscilloscope and viewed on moving speakers. If the frequencies are high enough, the speakers produce sound. The combination of tones can be correlated with the scope pictures. The oscilloscope can measure voltage, frequency, and phase angle. Two-channel measurements are very useful and are presented. We also can get lissajous figures in oscilloscope other than to get the waveform figures. In electronic applications we can generate Lissajous patterns by applying different signals to the horizontal and vertical inputs of an oscilloscope. In facts, this technique was often used to measure frequencies in the days before frequency meters. A signal of a known frequency was applied to the vertical input. The resulting pattern was a function of the ratio of the two frequencies.

A lissajous figure is produced by taking two sine waves and displaying them at right angles to each other. This easily done on an oscilloscope in XY mode. If the oscilloscope has the x-versus-y capability, one can apply one signal to the vertical deflection plates while applying a second signal to the horizontal deflection plates. The horizontal sweep section is automatically disengaged at this time. The resulting waveform is called Lissajous figure. This mode can be used to measure phase or frequency relationships between two signals. When the two sine waves are of equal frequency and in-phase, diagonal line to the right will produced. When the two sine waves are of equal frequency and 180 degrees out-of-phase a diagonal line to the left will produced. When the two sine waves are of equal frequency and 90 degrees out-of-phase a circle will produced. When the horizontal and vertical

sine wave frequencies differ by a fixed amount, this is equivalent to constantly rotating the phase between them

➤ **Measurement of Voltage Using CRO**

A voltage can be measured by noting the Y deflection produced by the voltage; using this deflection in conjunction with the Y-gain setting, the voltage can be calculated as follows:

$$V = (\text{no. of boxes in cm.}) \times (\text{selected Volts/cm scale})$$

➤ **Measurement of Current and Resistance Using a CRO**

Using the general method, a correctly calibrated CRO can be used in conjunction with a known value of resistance R to determine the current I flowing through the resistor.

➤ **Measurement of Frequency Using a CRO**

A simple method of determining the frequency of a signal is to estimate its periodic time from the trace on the screen of a CRT. However this method has limited accuracy, and should only be used where other methods are not available. To calculate the frequency of the observed signal, one has to measure the period, i.e. the time taken for 1 complete cycle, using the calibrated sweep scale. The period could be calculated by

$$T = (\text{no. of squares in cm}) \times (\text{selected Time/cm scale})$$

Once the period T is known, the frequency is given by

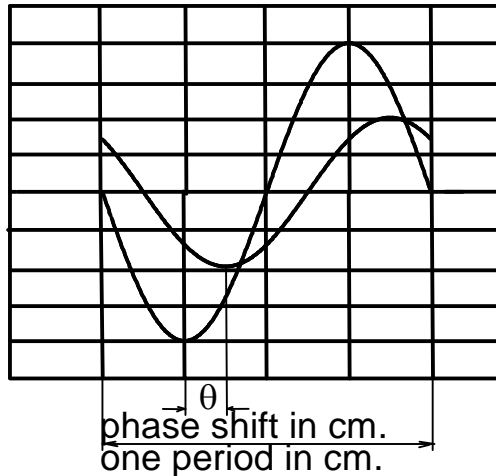
$$f (\text{Hz}) = 1/T(\text{sec})$$

➤ **Measurement of Phase**

The calibrated time scales can be used to calculate the phase shift between two sinusoidal signals of the same frequency. If a dual trace or beam CRO is available to display the two signals simultaneously (one of the signals is used for synchronization), both of the signals will appear in proper time perspective and the amount of time difference between the waveforms can be measured.

This, in turn can be utilized to calculate the phase angle  $\theta$ , between the two signals.





Phase shift between two signals

Referring to figure, the phase shift can be calculated by the formula;

$$\theta = \frac{\text{Phase shift in cm.}}{\text{One period in cm.}} \times 360^\circ$$

Note that the calculation does not involve the actual calibrated time base setting. In fact, the observed waveforms can be varied using the horizontal amplifier venire adjustment to obtain as many boxes for one full scale as desired. Another method for fast calculation is to multiply the scale factor by the phase difference (in cm) where the scale factor is degrees per box or degrees per cm.

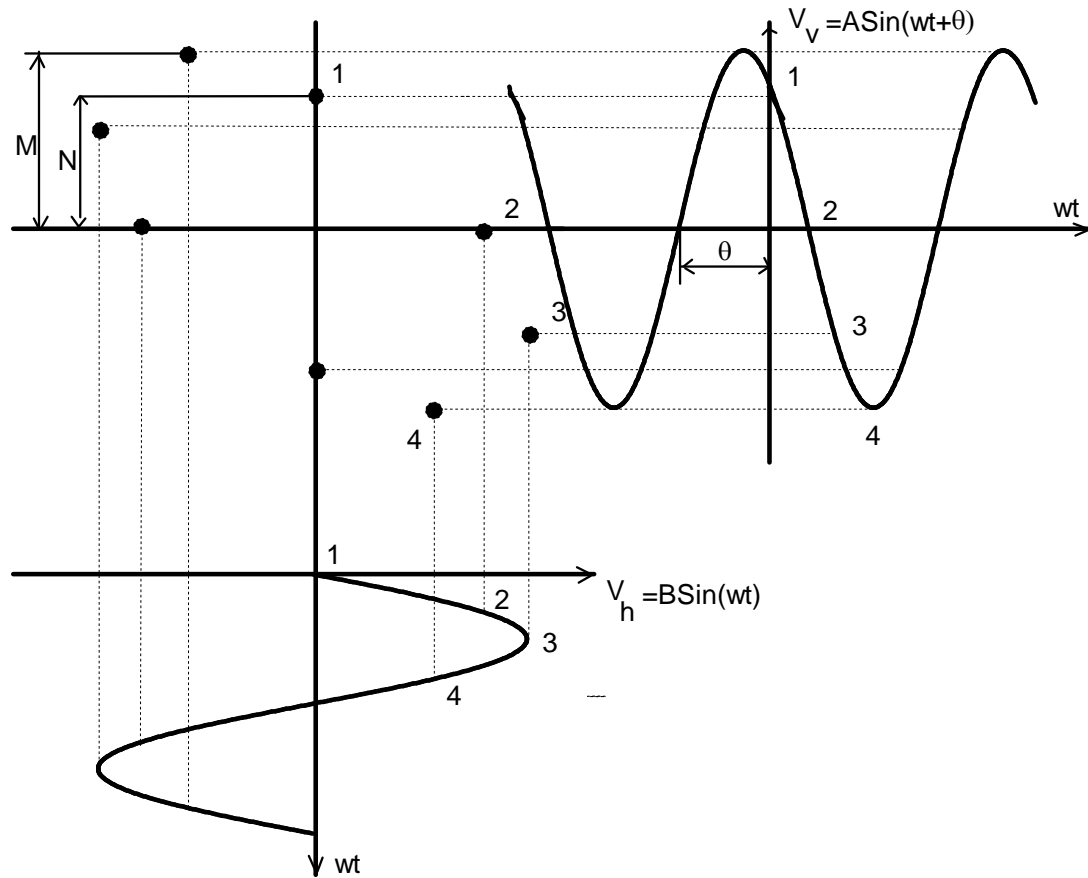
### Use of Lissajous Patterns to Calculate Phase Shift

Lissajous patterns are obtained on the scope simultaneously by applying the two sinusoidal inputs to be compared at the vertical and horizontal channels. The phase shift is then determined using measured values taken from resulting Lissajous pattern. This pattern on the CRT screen may be either a straight line or a circle or an ellipse depending on the amount of phase shift.

Figure shows the resulting closed curve if the phase shift is between  $90^\circ$  and  $180^\circ$ . This pattern is an ellipse (inclined at  $135^\circ$  if the two amplitudes are the same). The angle of inclination at which the ellipse is generated is of no importance in the phase angle calculation. Noting that the vertical signal amplitude at instant 1 is

$N = A \sin(\theta)$ ,  $\theta$  can be computed by

$$\theta = 180^\circ - \sin^{-1}(N/M)$$

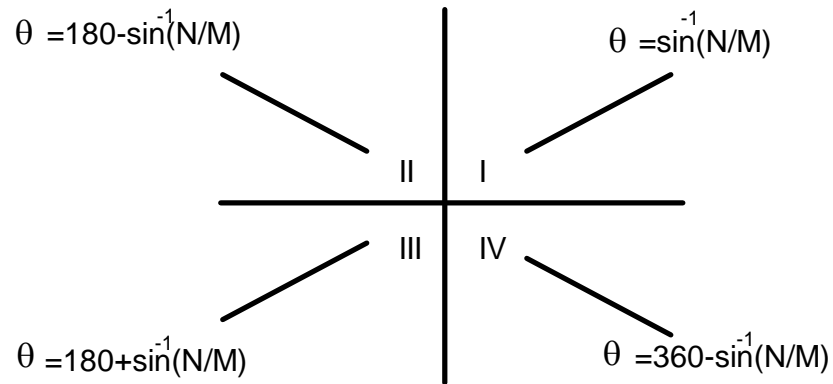


### Lissajous Pattern

Phase angles between  $0^\circ - 90^\circ$ , the ellipse has a positive slope and angle calculated by the following formula:

$$\theta = \sin^{-1}(N / M)$$

The actual scale settings do not change the ratio (N/M). Hence try to get an ellipse of maximum possible size on the CRO for increased accuracy. For phase angles of  $90^\circ - 180^\circ$ , the ellipse has a negative slope and the angle calculated by the above method must be subtracted from  $180^\circ$  to obtain the phase shift. Phase angles between  $180^\circ - 270^\circ$  result in Lissajous patterns similar to those for  $\theta : 90^\circ - 180^\circ$ , and can't be directly distinguished. One technique for determining, if the phase shift is less or more than  $180^\circ$ , is to add an extra slight phase shift to the signal  $V_v$ . If the phase angle measured increases, then the angle is less than  $180^\circ$ . If it decreases, the angle is greater than  $180^\circ$ . Figure shows how to compute the required phase angle.



**Phase angle calculation in 4 quadrants**

### Use of Lissajous Patterns for Frequency Measurements:

If a well calibrated CRO time base is not available, a signal generator can be used to measure the frequency of an unknown sinusoidal signal. It is connected to the vertical channel (or horizontal) and the calibrated signal source is fed to the horizontal channel (or vertical). The frequency of the signal generator is adjusted so that a steady Lissajous pattern is obtained. The Lissajous pattern can be very involved to analyze. However, for the frequency measurement, all that is needed is the number of tangencies (points at the edge of arcs) along the vertical and horizontal lines.

The frequency relationship between the horizontal and vertical inputs is given by;

$$\frac{f_h}{f_v} = \frac{\text{No. of tangencies (vertical)}}{\text{No. of tangencies (horizontal)}}$$

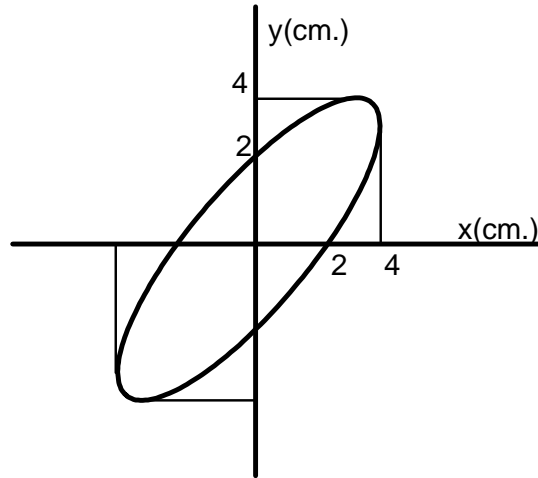
from which  $f_v$ , the unknown frequency can be calculated.

### 34.7 Self Learning Exercise-I

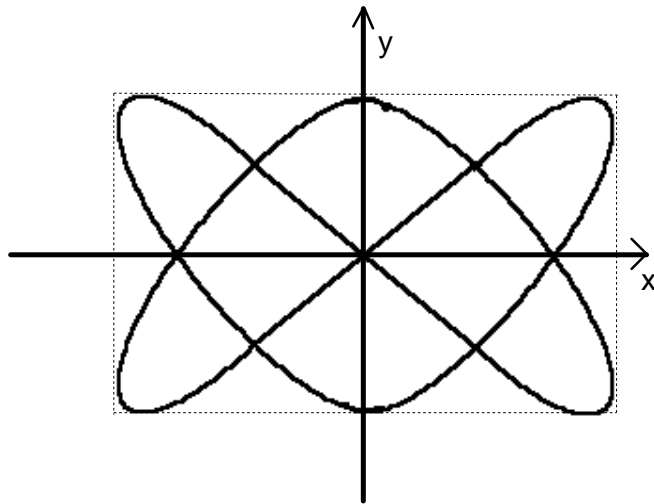
- Q.1 Write down the full form of CRO?
- Q.2 What is time period?
- Q.3 Write down the procedure of measurement of Phase using CRO.
- Q.4 Draw the lissajous pattern in CRO.

### 34.8 Procedure

1. The Lissajous pattern shown in figure is observed on the CRT screen. Find the phase shift between the signals applied to the X and Y inputs of the scope.



2. Figure shows a Lissajous pattern observed on the CRT screen. Determine the frequency relationship between the signals applied to the X and Y inputs of the scope.



3. Two sinusoidal inputs having the same amplitudes but different period, are applied to the X and Y inputs of the CRO. Draw the Lissajous pattern which will be observed on the CRT, for

$$T_y = 4T_x$$

4. The signals given  $V_1$  and  $V_2$  are applied to the X and Y inputs of the scope. Sketch the Lissajous pattern and calculate the phase difference between the two signals.

$$V_1 = 10\cos(\omega t)$$

$$V_2 = 15\sin(\omega t - 180)$$

### **34.9 Observation**

The output of CRO is traced with the help of tracing paper.

### **34.10 Graph**

Plot the required graph.

.....  
.....

### **34.11 Calculations**

.....  
.....  
.....

### **34.12 Result**

Hence measurement of phase and frequency has been done and output is traces using CRO.

### **34.13 Discussion**

.....  
.....  
.....  
.....

### **34.14 Precautions and Sources of Error**

1. To clear the measurements from the screen, press the blue shift button and then press the clear button.
2. Disconnect the digital telescope from the function generator and turn it off.
3. Make sure all probes and connectors are disconnected from the equipment and neatly placed on the shelf above the work area.
4. Be sure to turn off oscilloscope.

### **34.15 Self Learning Exercise -II**

**Q.1** What is lissajous pattern in CRO?

**Q.2** Write down the procedure to find the lissajous pattern in CRO.

**Q.3** What is the frequency relationship between the horizontal and vertical inputs.

### 34.16 Glossary

**Alternating current** : The electric current that changes its direction periodically.

**Frequency** : The number of oscillations completed in 1 second by an oscillating body.

**Direct current** : An electrical current which always flows in one direction.

### 34.17 Answers to Self Learning Exercises

#### *Answers to Self Learning Exercise-I*

**Ans.1:** Cathode-ray oscilloscope (CRO)

**Ans.2:** Voltage  $V = (\text{no. of boxes in cm.}) \times (\text{selected Volts/cm scale})$

Time period  $T = (\text{no. of squares in cm}) \times (\text{selected Time/cm scale})$

**Ans.3:** See Section 6

**Ans.4:** See Section 6

#### *Answers to Self Learning Exercise-II*

**Ans.1:** When sine-wave signals of different frequencies are input to the horizontal and vertical amplifiers a stationary pattern is formed on the CRT when the ratio of the two frequencies is an intergral fraction such as  $1/2, 2/3, 4/3, 1/5$ , etc. These stationary patterns are known as Lissajous figures and can be used for comparison measurement of frequencies.

**Ans.2:** See Section 8

**Ans.3:** 
$$\frac{f_h}{f_v} = \frac{\text{No. of tangencies (vertical)}}{\text{No. of tangencies (horizontal)}}$$

### 34.18 Viva Questions

**Q.1** What is Cathode ray oscilloscope (CRO)?

**Q.2** What are the basic component of a CRO?

**Q.3** What is the function of probe in CRO?

**Q.4** How many types of probe used in CRO?

- Q.5** What are the functions of different probes used in CRO?
- Q.6** What is the function of Attenuator in CRO?
- Q.7** Which device is used for the source of emission of electrons in a CRT?
- Q.8** What is the function of Aquadag in a CRO?
- Q.9** What is the function of electron gun assembly used in CRT?
- Q.10** What is the function of electron gun used in CRT?
- Q.11** What is lissajous pattern in CRO?

### **34.19 Answers to Viva Questions**

**Ans.1:** Cathode ray oscilloscope is a instrument used for display, measurement and analysis of waveforms and other phenomenon in electrical and electronic circuit.

**Ans.2:** CRO Circuit consists of following components:

1. Vertical deflection system
2. Horizontal deflection system
3. Synchronization circuit
4. Blanking circuit
- 5 Intensity modulation
- 6 position control
7. Focus control
8. Cathode ray tube
9. Calibration circuit

**Ans.3:**The probe performs the very important function of connecting the test circuit to oscilloscope without altering, loading or otherwise disturbing the test circuit.

**Ans.4:** There are three types of probe used in CRO:

1. Direct probe
2. Isolationprobe
3. Detector probe

**Ans.5:** Direct Probe: direct probe avoids stay-pick up which may create problems when low level signals are being measured. It is used for low frequency or low

impedance circuit. Isolation probe: Isolation probe is used in order to avoid the undesirable circuit loading effects of the shielded probe. Detector probe: when analyzing the response to modulated signals used in Communications equipment like AM, FM and TV receivers, the detector probe functions to separate the low frequency modulation component from the high frequency carrier.

**Ans.6:** The voltage in input terminal of the vertical amplifier causes the beam to deflect off the CRT screen, is quite low in amplitude. So that high amplitude signals may be displayed, an attenuator network is placed between the vertical input terminals of the vertical amplifier. The main function of the attenuator is to reduce the amplitude of the vertical input signal before applying it to vertical amplifier.

**Ans.7:** A barium and strontium oxide coated cathode is used for the source of emission of electrons in a CRT.

**Ans.8:** An Aquadage is used in a CRO to collect secondary emission electrons.

**Ans.9:** The electron gun assembly produces a sharply focused beam of electrons which are accelerated to high velocity .this focused beam of electrons strikes the fluorescent screen with sufficient energy to cause a luminous spot on the screen.

**Ans.10:** The source of focused and accelerated electrons beam is the electron gun. The electron gun emits electrons and forms them into a beam consist of a heater, a cathode, a grid, a pre-accelerating anode, a focusing anode and an accelerating anode.

**Ans.11:** When sine-wave signals of different frequencies are input to the horizontal and vertical amplifiers a stationary pattern is formed on the CRT when the ratio of the two frequencies is an intergral fraction such as  $1/2$ ,  $2/3$ ,  $4/3$ ,  $1/5$ , etc. These stationary patterns are known as Lissajous figures and can be used for comparison measurement of frequencies.

## References and Suggested Readings

1. All about Lissajous figures by T.B. Greenslade, 1993.
2. Experimental science by G.M. Hopkins, 1911.



# **Unit-35**

## **To Study The Brewster Angle and Refractive Index of a Given Materials**

### **Structure of the Unit**

- 35.1 Aim
  - 35.2 Apparatus
  - 35.3 Diagram
  - 35.4 Formula
  - 35.5 Theory and Description
  - 35.6 Self Learning Exercise-I
  - 35.7 Procedure
  - 35.8 Observation
  - 35.9 Graph
  - 35.10 Calculations
  - 35.11 Result
  - 35.12 Discussion
  - 35.13 Precautions and Sources of Error
  - 35.14 Self Learning Exercise-II
  - 35.15 Glossary
  - 35.16 Answers to Self Learning Exercises
  - 35.17 Viva Questions
  - 35.18 Answers to Viva Questions
- References and Suggested Readings

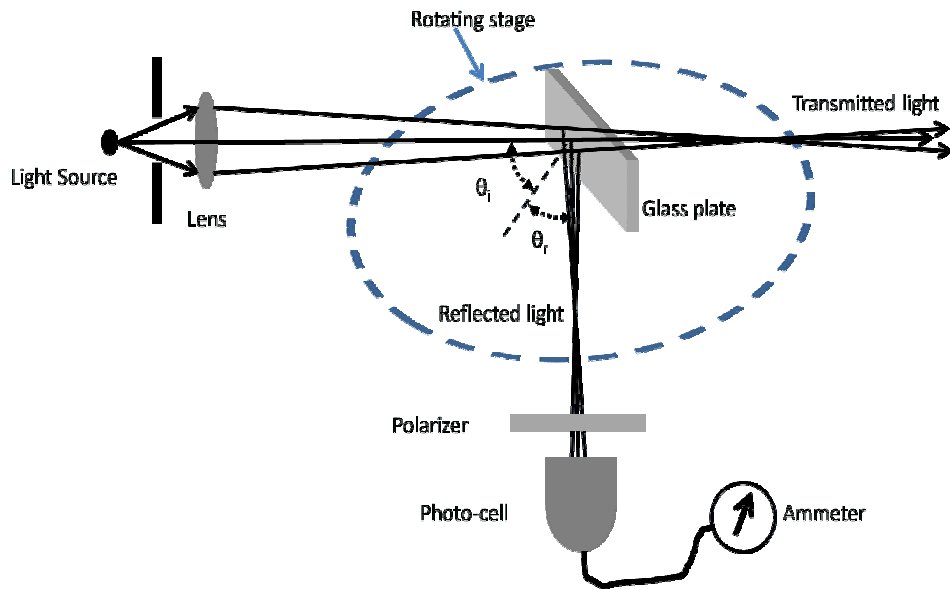
### **35.1 Aim**

To study the Brewster angle and refractive index of a given materials.

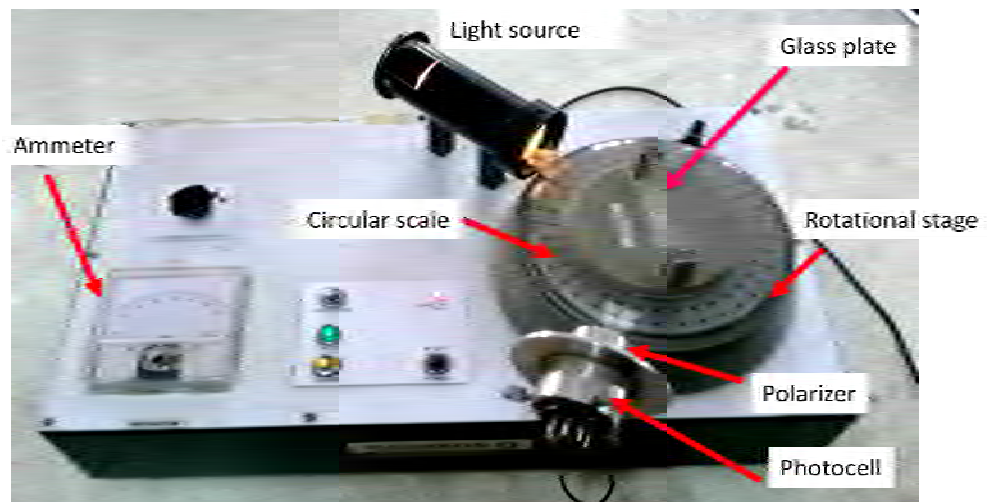
## 35.2 Apparatus

Light source, glass plate, Convex lens, Rotating stage, Polarizer, Photocell, and Ammeter.

## 35.3 Diagram



*Figure 1: Schematic diagram to study the Brewster angle and refractive index of a glass plate.*



**Figure2:** Experimental setup for Verification of Fresnel's law of reflection and reflection

## 35.4 Formula

### Brewster's law and Refractive index:

The angle of incidence for maximum polarization depends only on the refractive index.

$$\tan(\theta_p) = \frac{n_t}{n_i} = n_{ii}$$

### Degree of polarization (P) :

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Where:  $n_i$  = Refractive index of incident medium (air,  $n=1$ )

$n_t$  = Refractive index of refractive medium (glass,  $n=1.5$ )

$\theta_i$  = Angle of incidence

$\theta_t$  = Angle of refraction

$\theta_p$  = Brewster's angle or polarization angle

$I_{\max}$  = Maximum intensity

$I_{\min}$  = Minimum intensity

### Reflectance :

When electric fields  $\mathbf{E}$  is parallel to the plane-of-incidence, than Fresnel's Equations for reflection and transmission coefficients:

$$r_{\parallel} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)},$$

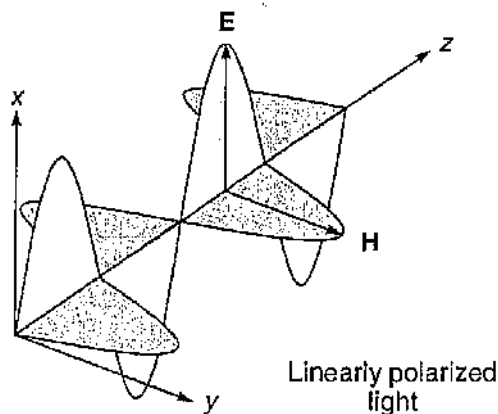
$$t_{\parallel} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)},$$

$$R_{\parallel} = r_{\parallel}^2, R_{\perp} = r_{\perp}^2$$

## 35.5 Theory and Description

Light is an electromagnetic wave having electric and magnetic fields vibrating sinusoidally at right angle to each other and to the direction of propagation? In

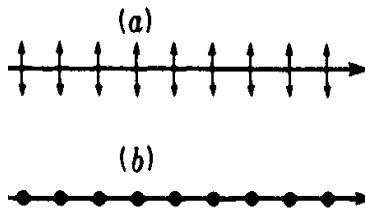
figure 3, the electric field  $E_x$  and magnetic field  $B_y$  are varies sinusoidally with  $z$ , at a given time.



**Figure 3:** A linearly x-polarized electromagnetic wave, propagating in the z-direction [1].

**i. Polarized and unpolarized light**

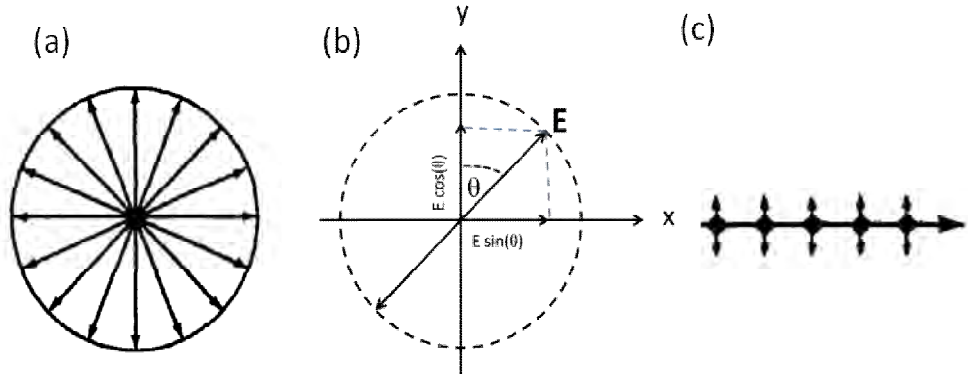
The direction of polarization of an electromagnetic wave is represented by oscillations of its electric field. Figure 4 shows a common way of representing of these vibrations. Figure 4, representing the two plane-polarized components. Dots represent the linear vibrations normal to plane of the paper, and double-pointed arrows represent vibrations lies into the plane of the paper.



**Figure 4:** Pictorial representations of plane-polarized light beams (side and end views) [2].

If the incoming light is unpolarized (sun, light bulb), then there is no constant phase relation because sudden, random changes in  $\mathbf{E}$  occurring in time intervals of  $10^{-8}$ s. Every orientation of  $\mathbf{E}$  is to be regarded as equally probable, in plane normal to the direction of propagation (see *Figure. 5(a)*). Because electric field is a vector quantity, hence it can represent by two orthogonal components ( $E_x = E \cos(\theta)$ ,  $E_y = E \sin(\theta)$ ) along two mutually perpendicular directions (  $x$ , and  $y$  -axis) (*Figure. 5(b)*). So finally the unpolarized light can be represented as two vibrations at right angles

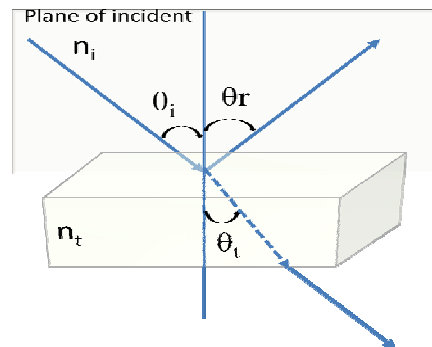
with equal amplitudes but no coherence of phase (*Figure 5(c)*).



**Figure 5:** (a) Pictorial representations of unpolarized light beams, (b) each vibration can be assumed as two components in the x and y directions. (c) Pictorial representations of the unpolarized light beams [2].

## ii. The Law of Reflection and Refraction

Suppose a light ray entering from lower-index medium  $n_i$  (air) and to a higher-index medium  $n_t$  (glass) as shown in figure 6. (Or reverse of it also true).



**Figure 6** The incident, reflected, and refracted rays all lie in the same plane.

So according to law of reflection "The incident ray, the perpendicular to the surface, and the reflected ray all lie in a plane called the plane-of-incidence; the angle-of-incidence equals the angle-of-reflection or  $\theta_i = \theta_r$  [1].

Here we can infer from figure 6, that the ray after entering into a higher-index medium, slightly bends toward the normal, called refracted ray. The relation between these angles and material properties define by Snell's Law.

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_t}{n_i} = n_{ti}$$

Here  $\theta_i$  and  $\theta_r$  are the angle of incidence and reflection respectively.

**Table 1:** Approximate value of refractive index (at wavelength of 589 nm) for various Substances\*

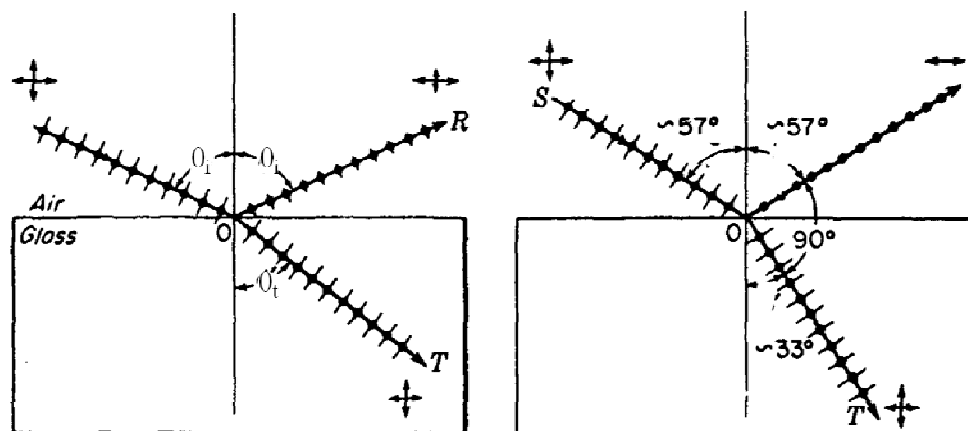
S. No.	Material	Refractive Index
1	Vacuum	1
2	Air	1.0003
3.	<b>Glass</b>	<b>~1.5</b>
4.	Water	1.33
5	Diamond	2.42

\*Values vary with physical conditions--purity, pressure, temperature etc [3].

Here we did not consider the polarization state of incident light and we can't get any information about the incident, reflected, and transmitted intensities by simply using Snell's law. To answer these questions we need detailed study on reflection and refraction and how these are related with vector nature of light (Polarization).

### iii. Polarization by reflection

When an unpolarized light is incident on a medium (like glass, water) at an angle, then at interface light will be reflected ray and a refracted ray according to Snell's law. We can infer from figure 7(a) that the reflected light is partially plane-polarized. But at incident angle, about  $57^\circ$  (Figure .7 (b)) it is plane-polarized. This angle called polarizing angle and angle between reflected and refracted rays is  $90^\circ$ .



**Figure : 7** (a) Polarization by reflection and refraction, (b) Brewster's law for the

polarizing angle [2].

According to Brewster's law, the angle of incidence ( $\theta_i = \theta_p$ ) where maximum polarization occur depends only on the refractive index

$$\tan(\theta_p) = \frac{n_t}{n_i} = n_{ti}$$

We know from dispersion relation, that the refractive index generally depends on the wavelength, and Brewster's angle depend on refractive index, hence Brewster's angle depends on wavelength of the incident light. But for ordinary glass the refractive index does not change so much in whole visible spectrum, therefore the polarizing angle does not change much for glass in visible spectrum.

#### iv. The Fresnel Equations

The Fresnel equations describe the effects of the polarization direction of an incoming electromagnetic plane wave on the reflection and transmission intensities (or amplitudes) at interface between two different dielectric media. The reflected and transmitted field amplitudes are related to the incident amplitude via the angles of  $\theta_t$  transmission and reflection  $\theta_r$ . The amplitude reflection (or transition (t)) coefficient (r) defined as the ratio of the reflected (or transmitted) to incident electric-field amplitudes [3].

$$r = [E_{0r} / E_{0i}] \text{ or } t = [E_{0t} / E_{0i}]$$

In this section we will assume that our dielectric mediums are linear, isotropic, homogeneous, with  $\mu \sim \mu_0$  (like air, glass). So that we can use the simple form of Fresnel's equations. Let  $\mathbf{E}$  is perpendicular to the plane-of-incidence than Fresnel's Equations for reflection and transmission amplitude coefficients are [3]:

$$r_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

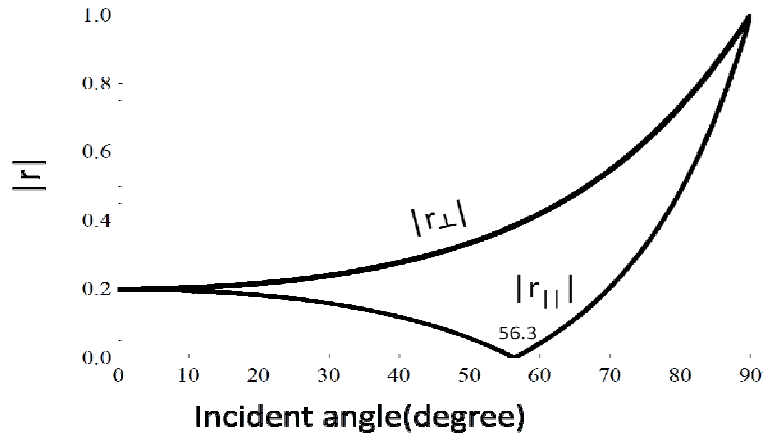
$$t_{\perp} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

And when  $\mathbf{E}$  is parallel to the plane-of-incidence than Fresnel's equations for reflection and transmission amplitude coefficients are [3]:

$$r_{\parallel} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\parallel} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

Figure 8 shows the amplitude coefficients of reflection, when E is parallel or perpendicular to the plane-of-incidence as a function of incident angle for air-glass interface ( $n_i = 1, n_t = 1.5$ ).



**Figure.8:** The amplitude coefficients of reflection as a function of incident angle for air- glass interface ( $n_i = 1, n_t = 1.5$ )[3].

But the experimentally measurable quantity is the light intensities not amplitude. Hence in particular, we are interested to determining the intensities of light that are reflected and refracted. The reflectance R is the ratio of the reflected intensity to the incident intensity which is equal to square of the amplitude coefficients of reflection.

$$R = r^2$$

So reflectance gives information about how much light of incident will be reflect for a given incident angle and material. When E is parallel to the plane-of-incidence than reflectance is

$$R_{\parallel} = r_{\parallel}^2 = \left[ \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} \right]^2 \quad (1)$$

When E is perpendicular to the plane-of-incidence than reflectance is

$$R_{\perp} = r_{\perp}^2 = \left[ \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)} \right]^2 \quad (2)$$

When there is no absorption than the total incident intensity is the sum of reflected

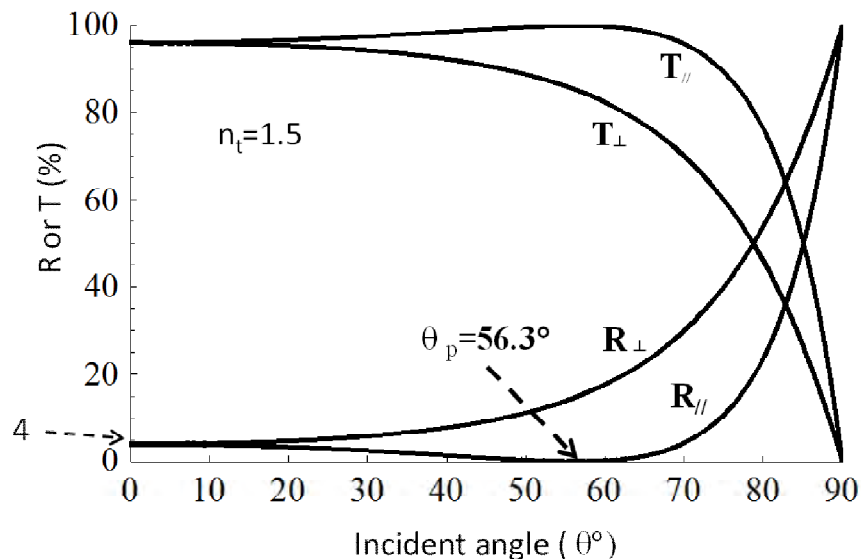


and transmitted intensities or

$$R+T=1 \implies T=1-R \quad (3)$$

So if we know the reflectance then we can calculate the transmittance. In figure 10, we can see the behaviour the reflectance and transmittance as a function of incident angle for air- glass interface ( $n_i = 1, n_t = 1.5$ ). The essential features of reflectance curve are briefly described as follows: At normal incidence ( $\theta_i=0$ ) about 4 % of the intensity of a beam of for both the polarizations is reflected. At a particular angle of incidence ( $\theta_p = 56.3^\circ$ ) called Brewster's angle, the reflected light is completely plane-polarized (in electric vector perpendicular to the plane incidence ( $R_\perp = 15\%, R_\parallel = 0\%$ ))

In our experiment we will use a glass plate kept in front of unpolarized light source (light bulb, see fig. 1 and 2). We know that if, the incoming light is unpolarized, and in that case we can represent it by two orthogonal, equal-amplitude and incoherent in polarized sates (see fig. 5). One polarization state is parallel to plane of incident and other one is perpendicular to plane of incident. Here equal in amplitude means that the incident intensity ( $I_i = \text{Square of amplitude}$ ) is equally share by these two polarization states (parallel and perpendicular to plane of incident) is  $I_{i\parallel} = I_{i\perp} = I_i$



**Figure.9:** Reflectance and transmittance. as a function of incident angle for air-glass interface ( $n_i = 1, n_t = 1.5$ )[3].

After incident on glass plate these two polarization states are reflected according to equations (1) and (2). The amount of these reflected intensity can be written as:

$$I_{r\parallel} = R_{\parallel} I_i = R_{\parallel} I_i / 2, \text{ and}$$

$$I_{r\perp} = R_{\perp} I_i = R_{\perp} I_i / 2$$

Further the reflected light passes through a polarizer and is detected with a photocell. The photocell converts the light intensity into electrical current and it is measured by an ammeter. The measured current is proportional to the intensity falling on the photocell. We can see in figure 9, that the reflectance  $R_{\perp}$  is generally greater than or equal to  $R_{\parallel}$ . Hence

$$I_{r\perp} = R_{\perp} I_i / 2 \geq I_{r\parallel} = R_{\parallel} I_i / 2$$

To verify Fresnel's law we will rotate the glass plate by means of changing the incident angle (10, 20, ..., 80 degrees) and corresponding to each incident angle we will record the maximum ( $I_{\max}$ ) and minimum ( $I_{\min}$ ) value of current by rotating the polarizer. The current will be maximum when reflected intensity  $I_{r\perp}$  is parallel to the polarizer axis. Similarly, current will be minimum when reflected intensity  $I_{r\parallel}$  is parallel to the polarizer axis.

$$I_{\max} \propto I_{r\perp} \text{ and } I_{\min} \propto I_{r\parallel}$$

The degree of polarization  $P$  of the reflected light can be calculated by using the  $I_{\max}$  and  $I_{\min}$  (intensities of the parallel and perpendicular components).

$$P(\theta_i) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

The photocell current  $I_{\max}$  and  $I_{\min}$  depends on the angle of incidence, hence  $P$  also depends on the incident angle of the light. At a particular angle of incidence ( $\theta_p = 56.3^\circ$ ) called Brewster's angle, the reflected light is completely plane-polarized, having an electric vector perpendicular to the plane of incidence ( $R_{\perp} = 15\%$ ,  $R_{\parallel} = 0\%$ ).

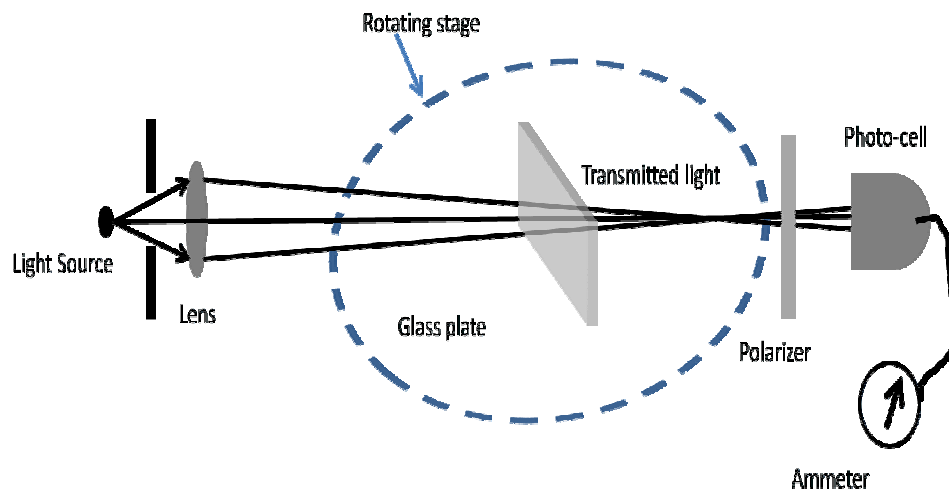
Since  $R_{\parallel} = 0$ , then  $I_{\min} = 0$  and  $P = 1$ , this is the maximum value of  $P$ . At all other incident angles, except Brewster's angle,  $P$  is always less than one. If we plot  $P$  as a function of the incident angle for glass, then the maximum of  $P$  versus  $\theta_i$  curve gives the Brewster angle of the glass (see figure 3).

## 35.6 Self Learning Exercise-I

- Q.1** What is the dimension of the refractive index?
- Q.2** What is the refractive index of glass?
- Q.3** What is the speed of light in a medium having a refractive index of 'n'?
- Q.4** Does the refractive index depend on wavelength (frequency) of incident light?
- Q.5** Does the refractive index of glass depend on the incident angle of light?

## 35.7 Procedure

1. Arrange light source, lens, glass plate, polariser and the photocell, according to figure 12.
2. Align the photocell such that the light after polariser falls on its full area (Generally the photocell and polarizer are mounted on same arm).
3. After alignment, rotate the glass plate such that the angle between the incident light rays and the normal of the surface of the glass plate is 10 degree.
4. Now rotate the arm containing photocell and polarizer, so that the light reflected light from glass plate fall on photo cell through polarizer (see Figure1 and 10).



**Figure10:** *The arrangement of Fresnel's experiment.*

5. Now rotate the polarizer about its own axis, and take the reading of maximum current ( $I_{\max}$ ) and minimum current ( $I_{\min}$ ) of ammeter .
6. Now take the reading of maximum current ( $I_{\max}$ ) and minimum current ( $I_{\min}$ ) of ammeter for different incident angles (i.e. 20,30,40,50,60,70, and 80) repeating the steps 3 to 5.
7. Plot the degree of polarization (P) versus different values of  $\theta_i$  .

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

8. Find out the value of incident angle corresponding to the maximum value of P, this angle called as Brewster angle  $\theta_p$ .
9. Find out the  $\tan(\theta_p)$  and calculate the refractive index of the glass plate.

### 35.8 Observation

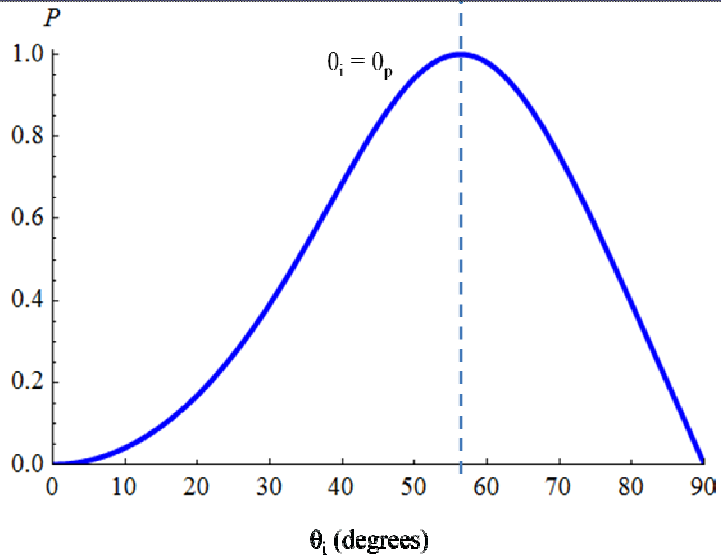
Least count of rotating stage = ..... (Degree)

Refractive index of glass plate=..... (Given).

*Table 2: For Fresnel's relations and degree of polarization*

S. No.	Incident angle (degrees)	Photo cell Current ( $\mu\text{A}$ )		$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$
		$I_{\max}$	$I_{\min}$	
1.	10			
2.	20			
3.	30			
4.	40			
5.	50			
6.	60			
7.	70			
8.	80			

## 35.9 Graph



**Figure11:** Degree of polarization as a function of incident angle.

## 35.10 Calculations

### Model Calculations:

Suppose the maxima of  $P$  versus  $\theta_i$  curve occur at  $\theta_i = 56^\circ$

Hence Brewster angle is  $(\theta_p) = 56^\circ$

So  $\tan(\theta_p) = \text{refractive index of glass plate} = \tan(56) = 1.48$

## 35.11 Result

- (a) The degree of polarization for reflected light from glass plate is shown in figure (.....) gives the Brewster angle  $\theta_p = \dots$  degree.
- (b) The refractive index for glass plate is  $= \tan(\theta_p) = \dots$

## 35.12 Discussion

.....

.....

.....

.....

.....

### 35.13 Precautions and Sources of Error

1. Glass plate should be cleaned.
2. Do not touch the glass plate and polarizer otherwise the finger print affect the reflection efficiency.
3. The intensity of light source should be constant.
4. Take more reading near polarization angle.
5. The reflected light from glass plate should be fall on photocell properly.

### 35.14 Self Learning Exercise-II

**Q.1** Does the refractive index of glass depend on the polarization of incident light?

**Q.2** A medium having refractive index 'n' always changes the frequency (in vacuum) ' $f_0$ ' of the light when it travel into the medium.

**Q.3** A medium having refractive index 'n' always changes the wavelength (in vacuum) ' $\lambda_0$ ' of the light when it passing through the medium.

**Q.4** Does the refractive index depend on temperature and pressure?

**Q.5** How much percentage of light is reflected by glass upon normal incident?

### 35.15 Glossary

**Photocell:** A transducer used to detect and measure light and other radiations.

**Dispersion:** Separate (light) into spectral rays.

**Quasi:** Partly similar

### 35.16 Answers to Self Learning Exercises

#### *Answers to Self Learning Exercise I*

**Ans.1:** The refractive index is a dimensionless quantity.

**Ans.2:**  $n_g \sim 1.5$

**Ans.3:**  $v=c/n$

**Ans.4:** Generally refractive index depends on wavelength called dispersion of light.

**Ans.5:** No

## ***Answers to Self Learning Exercise II***

**Ans.1:** No

**Ans.2:** No, frequency remain same

**Ans.3:** Wavelength and velocity changes.

**Ans.4:** Yes.

**Ans.5:** 4%

### **35.17 Viva Questions**

**Q.1** What is the nature of the light wave?

**Q.2** Which event of light support that light is a transverse wave?

**Q.3** What is the Snell's Law?

**Q.4** What do you mean by unpolarized light?

**Q.5** What do you mean by plane-of-incidence?

**Q.6** Do electromagnetic waves carry energy like other waves?

**Q.7** What do you mean by plane of polarization of EM wave?

**Q.8** What are coherence light sources?

**Q.9** What do you mean amplitude reflection coefficient?

**Q.10** What is the Brewster`s law?

**Q.11** Does the Brewster`s angle depends on the wavelength of incident light?

**Q.12** What percentage of incident light is reflected from glass plate, when

$$\theta_i = \theta_B ?$$

**Q.13** What is the value of polarization angle for glass?

**Q.14** What are the parallel and perpendicular components of reflectance for normal incidence of light?

**Q.15** What is the relation between reflected, incident intensity and reflectance?

**Q.16** What is the percentage of transmitted light from air-glass interface upon normal incident?

**Q.17** What is the angle between reflected and refracted rays when  $\theta_i = \theta_B$ ?

**Q.18** Is it true that the incident unpolarized light is reflected at  $90^\circ$  is partially

polarized?

**Q.19** What do you mean by degree of polarization?

**Q.20** What is the value of the degree of polarization (**P**) at  $\theta_i = \theta_B$ .

### 35.18 Answers to Viva Questions

**Ans.1:** Light is a transverse electromagnetic (EM) wave

**Ans.2:** Polarization of light.

**Ans.3:**  $\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_t}{n_i} = n_{ti}$

**Ans.4:** For unpolarized light, every orientation of **E** is to be regarded as equally probable, in plane normal to the direction of propagation.

**Ans.5:** Law of Reflection maintains that the incident ray, the perpendicular to the surface, and the reflected ray all lie in a plane called the plane-of-incidence.

**Ans.6:** Yes

**Ans.7:** The direction of polarization of an electromagnetic wave is represented by oscillations of its electric field.

**Ans.8:** The light sources having same frequency (monochromatic) and their relatively phase difference should be zero or remain constant with time.

**Ans.9:** The amplitude reflection coefficient (**r**) defined as the ratio of the reflected to incident electric-field amplitudes.  $r = [E_{0r} / E_{0i}]$

**Ans.10:** Brewster's law correlate the polarization angle with the refractive index of the medium or  $\tan(\theta_p) = \frac{n_t}{n_i} = n_{ti}$

**Ans.11:** Yes, As the refractive index is a function of the wavelength, and Brewster's angle depend on refractive index, hence Brewster's angle depends on wavelength of the incident light.

**Ans.12:** 15 %

**Ans.13:**  $56.3^\circ$

**Ans.14:**  $R_{\parallel} = R_{\perp} = \left( \frac{n_t - n_i}{n_t + n_i} \right)^2$



**Ans.15:**  $I_{\text{ref}} = R I_{\text{incident}}$

**Ans.16:** 96%

**Ans.17:**  $90^\circ$

**Ans.18:** No, because at  $\theta = 0$  and  $90^\circ$ , the reflectance  $R_{\perp} = R_{\parallel}$ , hence reflected light at  $0^\circ$  and  $90^\circ$  is always unpolarized.

**Ans.19:**  $P = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})}$

**Ans.20:**  $P=1$

### References and Suggested Reading

1. Ajoy Ghatak, *Optics*, Tata McGraw-Hill, New Delhi, 2009.
2. Francis A. Jenkins and Harvey E. White, *Fundamentals of optics*, The McGraw-Hill, 2001.
3. Eugene Hecht, *Optics*, Addison Wesley, 2002.

# **UNIT-36**

## **Driven Mechanical Oscillator**

### **Structure of the Unit**

- 36.1 Aim
- 36.2 Apparatus
- 36.3 Diagram
- 36.4 Formula
- 36.5 Model Graph
- 36.6 Theory And Description
- 36.7 Self Learning Exercise1
- 36.8 Procedure
- 36.9 Observation
- 36.10 Graph
- 36.11 Calculations
- 36.12 Result
- 36.13 Discussion
- 36.14 Precautions and Sources Of Error
- 36.15 Self Learning Exercise 2
- 36.16 Glossary
- 36.17 Answers To Self Learning Exercises
- 36.18 Viva Questions
- 36.19 Answers To Viva Questions

## References and Suggested Readings

### 36.1 Aim

To investigate amplitude and phase as a function of driving frequency for a driven damped oscillator.

### 36.2 Apparatus

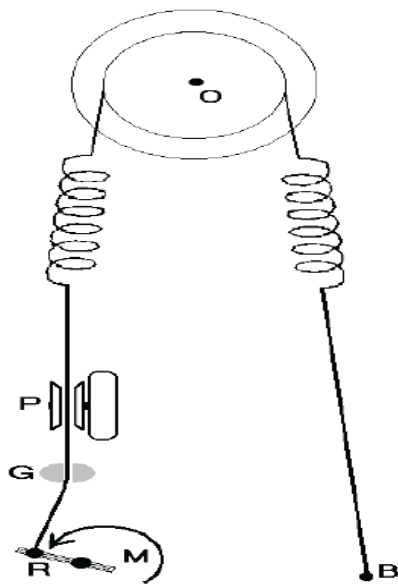
- Pulley and spring system
- Two Rotary Motion Sensors
- Retort Stand
- Voltmeter
- Power Supply
- Electric Motor
- Computer with Data Studio software
- Calibrated Metal Bars (for setting magnet distance)

### 36.3 Diagram

In this experiment the oscillating apparatus (aluminum disk and pulley) are the same as from the under damped oscillator lab. The oscillator is attached to a rotary motion sensor which is used to record the angular displacement and the driving torque is applied by an electric motor  $M$  (shown in figure). The electric motor rotates a metal rod,  $R$  about a horizontal axis. The rod,  $R$ , is attached to one end of the string and the other is attached to one of the springs. As the rod rotates, the sinusoidal driving force (and hence torque) is applied to the oscillator.

The string that attaches the rod and spring, passes through a guide,  $G$  (a fixed rod with a hole in it), and around the pulley,  $P$ , attached to a secondary rotary motion sensor. This sensor is used to determine the driving frequency  $\omega$ . The sensor attached to the aluminum disc will be used to determine the resulting amplitude  $A$  of oscillation, that is, the maximum angular displacement from equilibrium.

Comparison of the output graphs from the two sensors permits determination of the phase angle  $\theta$ .



## 36.4 Formula

The driven harmonic oscillator can be written as

$$-kx - b \, dx/dt + F_m \cos(\omega_1 t) = m \, d^2x/dt^2$$

$$\text{Or } m \, d^2x/dt^2 + b \, dx/dt + kx = F_m \cos(\omega_1 t)$$

The solution of above equation is

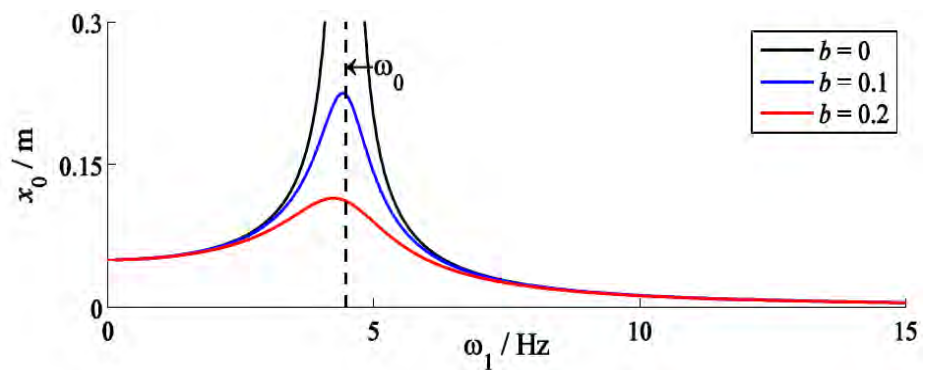
$$x_0(t) = (F_m/G) \cos(\omega_1 t + \phi)$$

$$\text{where } G = \sqrt{m^2(\omega_1^2 - \omega_0^2)^2 + b^2 \omega_1^2}$$

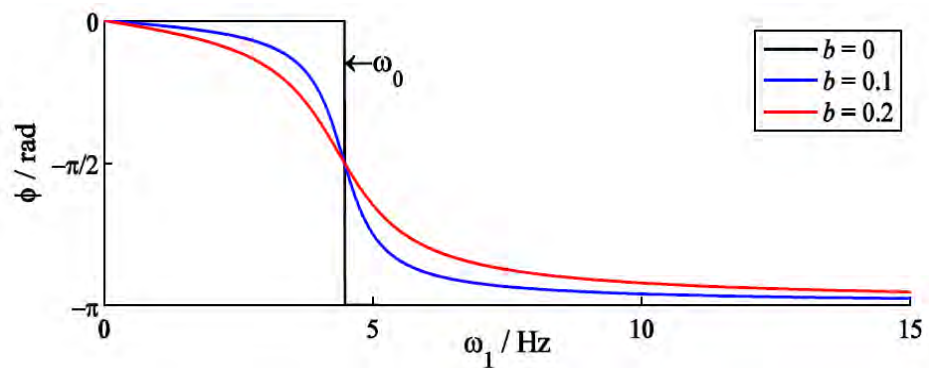
$$\text{and } \phi = \cos^{-1}(b \omega_1 / G)$$

## 36.5 Model Graph

**Amplitude response curve**



**Phase curve**



## 36.6 Theory and Description

Hooke's Law for a mass attached to a spring states that

$$F = -kx,$$

where  $x$  is the displacement of the mass from equilibrium,  $F$  is the restoring force exerted by the spring on the mass, and  $k$  is the (positive) spring constant. If this force causes the mass  $m$  to accelerate, then the equation of motion for the mass is

$$-kx = ma$$

Substituting for the acceleration

$$a = d^2x/dt^2$$

we can rewrite as

$$-kx = m d^2x/dt^2$$

$$d^2x/dt^2 + \omega_0^2 x = 0$$

where  $\omega_0 = \sqrt{k/m}$  is called the resonance angular frequency of oscillation. Equation is the differential equation for a simple harmonic oscillator with no friction. Its solution includes the sine and cosine functions, since the second derivatives of these functions are proportional to the negatives of the functions. Thus, the solution

$$x(t) = A \sin(\omega_0 t) + B \cos(\omega_0 t)$$

satisfies differential equation. The parameters  $A$  and  $B$  are two constants which can be determined by the initial conditions of the motion. The natural frequency  $f_0$  of such an oscillator is

$$f_0 = \omega_0 / 2\pi = (1/2\pi) \sqrt{k/m}$$

In the simple case described above, the oscillations continue indefinitely. We know, however, that the oscillations of a real mass on a spring eventually decay because of friction. Such behavior is called damped harmonic motion. To describe

it mathematically, we assume that the frictional force is proportional to the velocity of the mass (which is approximately true with air friction, for example) and add a damping term,  $-b dx/dt$ . Our equation for the damped harmonic oscillator becomes

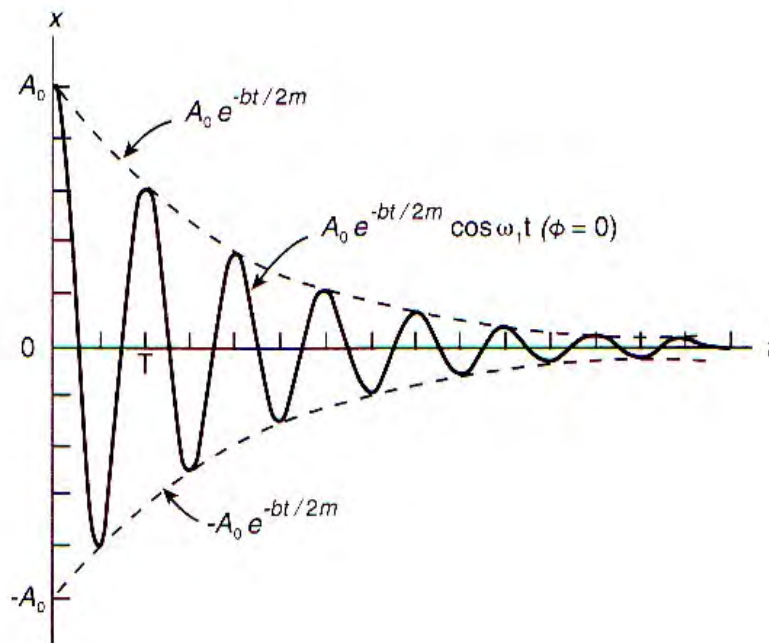
$$-kx - b \, dx/dt = m \, d^2x/dt^2$$

Or  $m \, d^2x/dt^2 + b \, dx/dt + kx = 0$

The solution of above equation is

$$x(t) = A_0 e^{-bt/2m} \cos(\omega_d t + \phi)$$

The parameter  $A_0$  is the initial amplitude of the oscillations, and  $\phi$  is the phase angle; these two constants are determined by the initial conditions of the motion. The oscillations decay exponentially in time, as shown in the below graph.



In addition, the angular frequency of oscillation is shifted slightly to

$$\begin{aligned} \omega_d &= \sqrt{[k/m - (b/2m)^2]} \\ &= \sqrt{[\omega_0^2 - (b/2m)^2]} \end{aligned}$$

Now imagine that an external force which varies cosinusoidally (or sinusoidally) in time is applied to the mass at an arbitrary angular frequency  $\omega_1$ . The resultant behavior of the mass is known as driven harmonic motion. The mass vibrates with relatively small amplitude, unless the driving angular frequency  $\omega_1$  is near the resonance angular frequency  $\omega_0$ . In this case, the amplitude becomes very large. If the external force has the form  $F_m \cos(\omega_1 t)$ , then our equation for the driven harmonic oscillator can be written as

$$-kx - b \, dx/dt + F_m \cos(\omega_1 t) = m \, d^2x/dt^2$$

Or  $m \, d^2x/dt^2 + b \, dx/dt + kx = F_m \cos(\omega_1 t)$

The solution of above equation is

$$x(t) = (F_m/G) \cos(\omega_1 t + \phi)$$

where  $G = \sqrt{m^2(\omega_1^2 - \omega_0^2)^2 + b^2\omega_1^2}$

and  $\phi = \cos^{-1}(b\omega_1/G)$

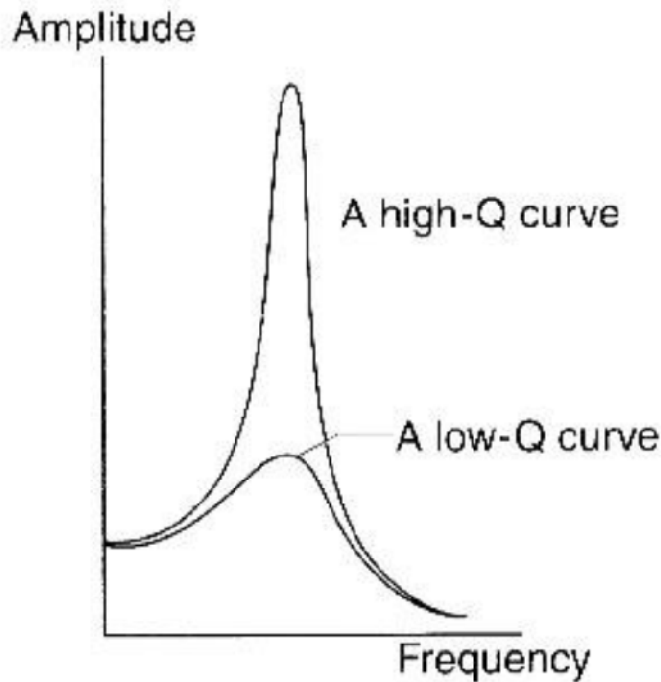
The factor  $G$  determines the shape of the resonance curve, which we wish to measure in this experiment. When the driving angular frequency  $\omega_1$  is close to the resonance angular frequency  $\omega_0$ ,  $G$  is small, and the amplitude of oscillation becomes large. When the driving angular frequency  $\omega_1$  is far from the resonance angular frequency  $\omega_0$ ,  $G$  is large, and the amplitude of oscillation is small (shown in model graph).

### **The quality factor**

The sharpness of the resonance curve is determined by the quality factor (or  $Q$  value),  $Q$ . If the frictional force, measured by the parameter  $b$ , is small, then  $Q$  is large, and the resonance curve is sharply peaked.

If the frictional force is large then,  $Q$  is small, and the resonance curve is broad (Figure).





The general definition of  $Q$  is

$$Q = 2\pi \times (\text{energy stored}) / (\text{energy dissipated in one cycle})$$

(If the energy dissipated per cycle is small, then  $Q$  is large, and the resonance curve is sharply peaked.) Physics texts derive the relationship between  $Q$  and the motion parameters,

$$Q = m \omega_1 / b,$$

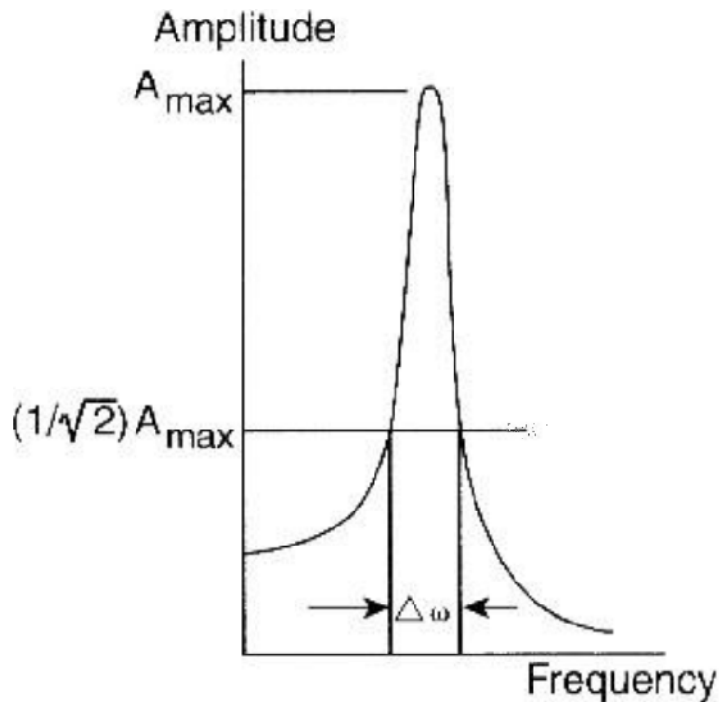
and relate  $Q$  to the sharpness of the resonance peak,

$$Q = \omega_1 / \Delta\omega,$$

where  $\Delta\omega = \omega_{\text{high}} - \omega_{\text{low}}$  is the difference in angular frequencies at which the amplitude has dropped to  $1/\sqrt{2}$  of its maximum value (see Figure).

Note that  $Q$  also controls the exponential damping factor,  $e^{-bt/2m}$ . We can also write

$$e^{-bt/2m} = e^{-\omega_1 t/2Q}$$



The reciprocal of the factor which multiplies  $t$  in the exponent,  $2m/b=2Q/\omega_1$ , is the time required for the amplitude of oscillation to decay to  $1/e$  of its initial value (the so-called "e-folding time").

### 36.7 Self Learning Exercise-1

**Q.1** Define simple pendulum.

**Q.2** Why the word 'SIMPLE' is used before the pendulum?

**Q.3** Define the following terms:

- (a) amplitude
- (b) oscillation or vibration
- (c) frequency
- (d) time period

### 37.8 Procedure

1. Open Data Studio and select 'Create Experiment'.

2. Under 'Setup' check the box angular position and select angular position and select radians for the units of measurement. Set sample rate at 200 Hz and from the drop down menu next to 'linear position' select 'Large Pulley'. Choose the same settings for the other sensor (each sensor must be set separately: to move to the other sensor window, click the arrowhead, i.e., small triangle, at the right-hand side of the sensor setup window).

Close the 'Setup' window.

1. Using the calibrated bar, adjust the distance between the magnet and the pulley to 4 mm. Record this distance.
2. Set the DVM to the DC volts setting.
3. On the power supply (with the power off); turn the voltage dial to a minimum and the current to a maximum. Then switch on the power supply.
4. To observe the phase of oscillation:

Set the voltage to approximately 1 V. This provides 1 V DC to the electric motor, which rotates slowly, providing a small driving frequency. Observe how the two pulleys rotate; you should see that they are rotating in phase. If you are not sure what to look for feel free to ask your TA for some guidance. Set the voltage to 6 V, this will provide a much larger driving frequency. Observe the rotation of the pulley; you should now see that they are rotating out of phase.

#### 5. **Collecting Data:**

Set the power supply to 1 V, and collect data points (angular displacement vs. time) from both rotary motion sensors for a minimum of 4 periods of oscillations. Save the data to your own personal USB drive or email it to yourself by going to File and then Export Data and saving the desired run(s). You should clearly label the files as driving or output pulley appropriately.

Do a data run for every 0.5 V interval (approximately) from 1 V to 6 V, but **DO NOT** exceed 6 V. Record the voltages and associated uncertainty (see suggested table on the following page). Do some extra runs for voltages near

the maximum amplitude (resonance) where the amplitude changes most rapidly as a function of driving frequency.

6. Repeat the above "Collecting Data" step for a magnet separation of 1 mm.
7. From the exported files create:
  - Sample graph(s) of angular displacement vs. time for one or two runs.
  - A graph of amplitude as a function of driving frequency, including error bars, for both magnet distances (two curves on one graph).
  - A graph of the phase angle vs. driving frequency, including error bars, for both magnet distances.

## 36.9 Observation

### Obsevation Table

Voltage (V)	Driving period (s)	Driving frequency (Hz)	Amplitude (rad.)	Phase difference (s)	Phase shift (fraction of a cycle)	Phase angle (rad)

### 36.10 Graph

Plot the required graph on graph paper .

### 36.11 Calculations

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### 36.12 Result

1. Sample graphs of angular displacement vs. time for one or two runs (including the displacements of both angular motion sensors).
2. Amplitude vs. driving frequency graphs for both magnet distances.
3. Phase angle vs. driving frequency graphs for both magnet distances.
4. A table showing your voltages, driving periods/frequencies, amplitudes and phase shifts.
5. Discussion of how calculations were done, how errors were estimated, etc.

### 36.13 Discussion

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### 36.14 Precautions and Sources of Error

1. Connections should be tight.

2. Note down the parameters without parallax error.
3. Verify the circuit before starting experiment.

## 36.15 Self Learning Exercise 2

**Q.1** What is the relation between frequency and time period

**Q.2** What are the units of frequency?

**Q.3** Define Frequency-response diagram (curve)

## 36.16 Glossary

**Anharmonic oscillator:** An oscillator that has a nonlinear relationship between force and displacement.

**Autonomous dynamical system:** A continuous dynamical system described by a set of ordinary differential equations in which time does not appear explicitly. If time appears explicitly, the dynamical system is said to be nonautonomous.

**Forced oscillations; forced vibration:** Motion that occurs if a system is continuously driven by an external agency. Forced harmonic oscillations occur when the external agency is time-harmonic.

**Free oscillations; free vibration:** This is the natural response of a system to some initial displacement and/or impulse (initial velocity). The response is completely defined by the properties of the system

**Frequency-response diagram (curve); amplitude–frequency-response (curve); response curve:** A plot showing how the steady-state amplitude of displacement changes with frequency.

**Harmonic oscillator:** An oscillator that has a linear relationship between force and displacement.

**Periodic motion:** This is a type of motion  $x(t)$  such that  $x(t) = x(t + T)$ , where  $T$  is the fundamental period of the motion.

## 36.17 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-1*

**Ans.1:** An ideal simple pendulum is defined as 'single isolated particle suspended by a weightless, flexible and inextensible string with a friction-less support'.

**Ans.2:** Because the pendulums used in the wall clocks are 'COMPOUND PENDULUMS', in which a metallic rod is used in place of the thread.

**Ans.3:(a)** Amplitude: the maximum displacement from the mean (equilibrium) position.

(b) Oscillation: the motion from one extreme position to the other and then back to the original one.

(c) Frequency: number of vibrations per second.

(d) Time period: time taken for one vibration.

### *Answers To Self Learning Exercise-2*

**Ans.1:**  $T=1/F$

**Ans.2 :** Vibrations / sec, cycles / sec (c.p.s.) or Hertz.

**Ans.3:** A plot showing how the steady-state amplitude of displacement changes with frequency.

## 36.18 Viva Questions

**Q.1** Define 'g'?

**Q.2** What is the difference between 'g' and 'G'?

**Q.3** What is the value of 'g' at the C.G. of the earth?

**Q.4** How the value of 'g' changes as we move from the surface towards the C.G. of the earth?

**Q.5** Where the 'g' is greater, at equator or poles?

- Q.6** Define vibratory system?
- Q.7** Can you replace the thread by a metallic wire?
- Q.8** What is restoring force?
- Q.9** What is restoring force (net force) acting on the bob?
- Q.10** What is net force on the bob, at equilibrium (mean) position?
- Q.11** Define equilibrium.
- Q.12** Can you replace the thread by a rubber band?
- Q.13** Why the pendulum stops after some time?
- Q.14** How P.E. and K.E. of the pendulum interchange into each other during vibrations?
- Q.15** From where the length of the pendulum is measured?

### **36.19 Answer to Viva Questions**

**Ans.1:** Acceleration due to gravity.

**Ans.2:** The value of  $G$  (gravitational constant) remains constant throughout the universe, whereas the value of ' $g$ ' decreases with the increase in the height.

**Ans.3:** Zero.

**Ans.4:** As a rule it should decrease gradually but due to variable density of the earth, it increases up to a small depth and then decreases.

**Ans.5:** At the poles (where the earth is slightly compressed).

**Ans.6:** Back and forth or to and from motion between two fixed positions.

**Ans.7:** No, because the wire is not flexible.

**EXPLANATION:** By definition of simple pendulum, the string must be perfectly flexible. The thread is flexible but a metallic wire is not. That is why the wire can be used in place of thread.



**Ans.8:** The force which tends to bring a vibrating body towards the mean position.

**Ans.9:** If the air friction is neglected, restoring force on the bob =  $mg\sin\theta$ .

**Ans.10:** Zero, since at mean position the weight of the bob is perfectly balanced by the tension (T) in the string.

**Ans.11:** A body is said to be in equilibrium when its linear and angular accelerations are zero or when  $F = 0$  &

EXPLANATION: when a body is

(a) at rest or

(b) moving with uniform linear velocity its linear acceleration is zero.

(c) not rotating at all or

(d) rotating at a constant rate it is in equilibrium.

**Ans.12:** No, because it is not inextensible. By definition the string must be inextensible.

**Ans.13:** Its energy is lost as heat.

**Ans.14:** (a) In the form of P.E. at extreme positions.

(b) In the form of K.E. at mean position and

(c) In the form of P.E. and K.E. between mean and extreme positions.

**Ans.15:** The length of the pendulum is measured from the centre of gravity of the bob to the point of suspension.

## References And Suggested Readings

1. Driven Harmonic Oscillator (Advanced Physics Lab 1) by Don Heiman, 2009.
2. A Brief Introduction to Resonance in . by David Van Baak, 2009.

# **UNIT-37**

## **Thermal Conductivity of a Bad Conductor**

### **By Lee's Disc Method**

#### **Structure of the Unit**

- 37.1 Aim
- 37.2 Apparatus
- 37.3 Diagram
- 37.4 Formula
- 37.5 Theory And Description
- 37.6 Self Learning Exercise1
- 37.7 Procedure
- 37.8 Observation
- 37.9 Graph
- 37.10 Calculations
- 37.11 Result
- 37.12 Discussion
- 37.13 Precautions And Sources Of Error
- 37.14 Self Learning Exercise 2
- 37.15 Glossary
- 37.16 Answers To Self Learning Exercises\
- 37.17 Viva Questions
- 37.18 Answers To Viva Questions

#### References and Suggested Readings

#### **37.1 Aim**

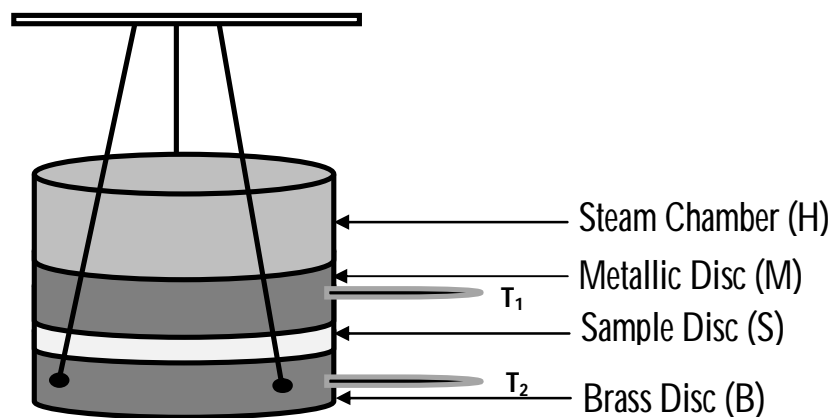
To determine the coefficient of thermal conductivity of a bad conductor using Lee's disc apparatus.

## 37.2 Apparatus

Lee's disc apparatus, a brass disc and a metallic disc (can be brass) of large diameter compared to its thickness, sample of bad conductor in the form of the disc with same diameter, two mercury thermometers, stop watch, Vernier, screw gauge and a balance.

## 37.3 Diagram

The sample disc S is sandwiched between two metallic discs (commonly brass) B and M. The brass disc B is hung from the stand with the help of three strings. On the metallic disc M, a heating chamber, H with facility for steam-in and steam-out is hired.



*Fig.19: Lee's disc apparatus [1].*

## 37.4 Formula

Thermal conductivity is defined as the property of a material to conduct heat. For general scientific use, thermal conductance is the quantity of heat that passes in unit time through a plate of particular area and thickness when its opposite faces differ in temperature by 1 °C.

Thermal conductivity of a bad conductor in Lee's experiment is given by

$$K = \frac{mc}{(\theta_{1s} - \theta_{2s})} \left( \frac{d\theta}{dt} \right)_{\theta_{2s}} \left( \frac{r + 2d}{2r + 2d} \right) \frac{d_o}{\pi r^2}$$

- Let  $\theta_{1s}$  and  $\theta_{2s}$  be the temperatures in steady state of metallic disc M and brass disc B measured by thermometer  $T_1$  and  $T_2$  respectively. So, the temperature difference between the two ends of bad conductor disc S is  $(\theta_{1s} - \theta_{2s})$ .
- Thickness of the bad conductor S is ' $d_o$ '

- Radius of bad conductor disc S is ' $r_0$ '
- Mass of the brass disc B is ' $m$ '
- The heat capacity of brass disc B is ' $c$ '
- Radius of brass disc B is ' $r$ '
- Thickness of brass disc B is ' $d$ '
- Rate of cooling of brass disc B at  $\theta_{2s}$  is  $\left(\frac{d\theta}{dt}\right)_{\theta_{2s}}$

Here  $\left(\frac{r+2d}{2r+2d}\right)$  is correction factor

The correction factor is introduced because the cooling measurement is not exactly the same environment of steady state measurement.

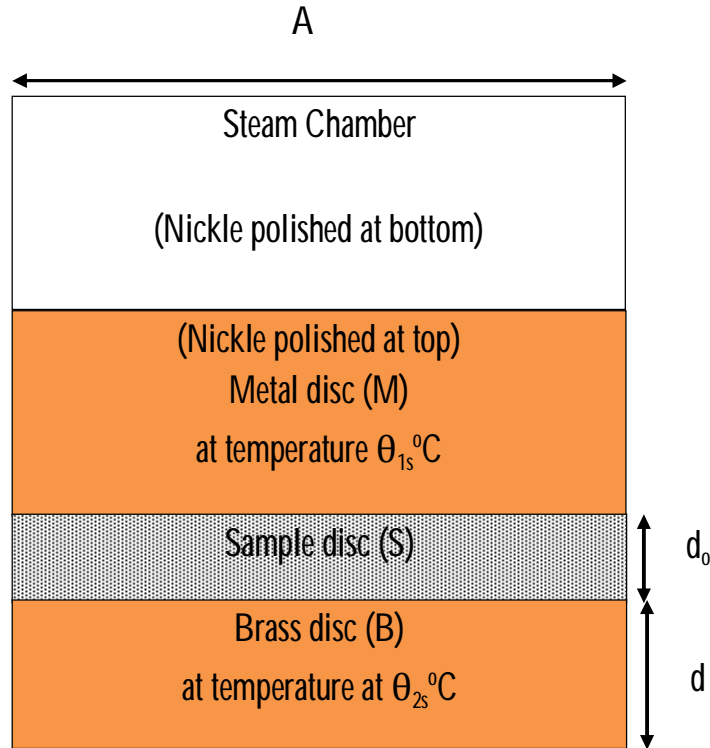
### 37.5 Theory and Description

**(i) Thermal conductivity:** Thermal conductivity describe the transport of energy in the form of heat. It is ability of material to conduct the heat as a result of temperature gradient under steady state boundary condition. It appears primarily in Fourier's Law for heat conduction [2]. Thermal conductivity is actually a tensor, which means it is possible to have different values in different directions. Thermal conductivity an indication of the rate at which energy is transferred by the diffusion process. It depends on the physical structure of matter, atomic and molecular, which is related to the state of the matter [3].

**(ii) Significances of thermal conductivity:** The analysis of thermal conductivity is very important for the determination of combustion and pyrolysis of wood exposed to fire [4-5], Increase in the conversion rate and provision of more electricity for on-board controls, and in the design of insulator for refrigerators, coolers and food flask [6]. Thus it very crucial to know the thermal conductivity for modelling heat transfer processes in materials. Thermal conductivity, which is the ability of any material to conduct heat is of great significance in heat transfer modelling [7]. The data of thermal and electrical properties of technically important materials is of great relevance to the Design and development engineers. Very often immediate measurements are needed to perform as the literature is not adequate. Also there are very few laboratories which perform the experiments with needed expertise and well equipped experimental set up [8]. Thus there is a need for the development of a suitable

reproducible method for the estimation of thermal conductivity of materials in a developing economy [9].

**(iii) The Lee's Disc experiment:** This experiment determines an approximate value for the thermal conductivity ( $K$ ) of a poor conductor like glass, cardboard, etc.



**Figure 2:** Lee's Disc apparatus assembly.

In this method a disc made of the poor conductor of radius  $r_0$  and thickness  $d_0$  is sandwiched between two good conductivity metal discs (of the same metal). One of these two metallic discs is heated by heat chamber. The assembly allows to come to steady state, so that the heat lost by the lower disc to convection is the same as the heat flow through the poorly conducting disc.

The change in temperature  $\theta_1$  of upper disc (which is attached with heat chamber) is noted with time. Meanwhile at the same time the temperature of the lower disc  $\theta_2$  (which is attached in air) is also recorded. The observations are till the steady state is achieved ( $\theta_{1s}$  and  $\theta_{2s}$ ). Then the poor conductor is removed and the lower metal disc is allowed to heat up to the 10 rate of cooling of lower disc own self is measured in air (without heat chamber and bad conductor). Then the slope  $=d\theta/dt$  of the cooling curve is measured graphically where the curve passes

through temperature  $\theta_{2s}$ .

At the steady state, rate of heat transfer (H) by conduction is given by;

$$H = KA \left[ \frac{\theta_2 - \theta_1}{d_0} \right]$$

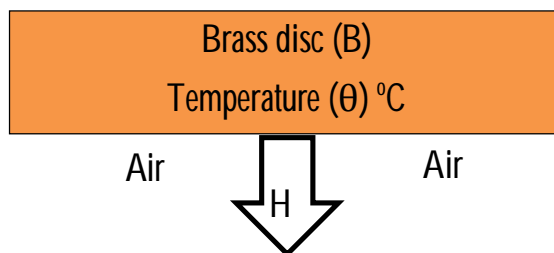
Where,

- K is thermal conductivity of the sample
- A is cross sectional area,
- $(\theta_{1s} - \theta_{2s})$  is temperature difference across the sample disc in steady state.
- $d_0$  is thickness of the bad conductor

The sample is an insulator. The sample material is taken in the shape of disc. Disc is taken with large cross sectional area ( $A = \pi r^2$ ) compared to the area exposed to the air at the edge ( $a = 2\pi r d$ ) in order to reduce the energy loss. Rate of energy transfer across the sample can be increased by keeping 'd<sub>0</sub>' small and 'A' large. If the thickness of sample disc will be less than the assembly will reach to the steady state quickly.

The temperature difference across the sample disc can be measured correctly if the thin sample of disc is sandwiched between two metallic plates mostly taken brass discs. The temperatures of the brass discs are measured by thermometer  $T_1$  and  $T_2$  which are found  $\theta_{1s}$  and  $\theta_{2s}$  respectively in steady state. This temperature difference across the sample is due to heat conduction through the sample disc.

In the steady state the rate of heat conducted through the brass disc must be equal to the rate of heat loss due to cooling from the bottom of the brass disc by air convection. By measuring the rate of cooling (at the steady state temperature  $\theta_{2s}$ ) of the brass disc due to air convection, the rate of heat loss can be determined (as it shown in figure 3).



**Figure 3:** To determine the rate of cooling of brass disc in the air.



**Figure 4:** Lees's Disc apparatus[10]

If the brass disc cools down at a rate,  $\frac{d\theta}{dt}$  then the rate of heat loss is given by:

$$H = mc \frac{d\theta}{dt}$$

Where, m is mass of the brass disc.

c is specific heat capacity of brass.

At steady state, heat conducted through the bad conductor per second will be equal to heat radiated per second from the exposed portion of the metallic disc.

$$KA \left[ \frac{\theta_2 - \theta_1}{d_o} \right] = mc \frac{d\theta}{dt}$$

So, the coefficient of heat conduction of bad conductor can be given by

$$K = \frac{mc}{(\theta_{1s} - \theta_{2s})} \left( \frac{d\theta}{dt} \right)_{\theta_{2s}} \frac{d_o}{\pi r_o^2}$$

$\theta_{1s}$  and  $\theta_{2s}$  are the steady state temperatures of disc in touch with heat chamber and disc in touch with air respectively.

Thickness of the bad conductor S is ' $d_0$ '

Radius of bad conductor disc S is ' $r_0$ '

Mass of the brass disc B is ' $m$ '

The heat capacity of brass disc B is ' $c$ '

Radius of brass disc B is ' $r$ '

Thickness of brass disc B is ' $d$ '

Rate of cooling of brass disc B at  $\theta_{2s}$  is  $\left(\frac{d\theta}{dt}\right)_{\theta_{2s}}$

### 37.6 Self Learning Exercise-1

- Q.1** What is use of The Lee's disc apparatus?
- Q.2** Which property of material is indicated by thermal conductivity?
- Q.3** The quantity of heat required to raise the temperature of unit mass of the substance through 1K is called?
- Q.4** 1 Calorie = ----- joule?
- Q.5** What is the specific heat capacity of water in KJ/Kg-K?

### 37.7 Procedure

#### Part 1: To find steady state temperature of brass disc with the sample disc

1. Measure the mass of the brass disc (B) take it as  $m$  Kg.
2. Measure the mass of sample disc (S) using a balance and take it as  $m_0$  gm.
3. Measure the diameter and hence the radius ( $r_0$ ) of the sample disc (S) using Vernier callipers.
4. Measure the thickness of the sample disc (S) using a screw gauge and take it as  $d_0$ .
5. Arrange the brass disc, sample disc and metallic disc as shown in the Figure 1.
6. Insert the thermometers  $T_1$  and  $T_2$  into the grooves of metallic disc (M) and brass disc (B), which measure the temperatures  $\theta_1$  and  $\theta_2$  respectively.
7. Place the heating chamber on the above assembly and suspend it as shown in Figure 1.
8. Start the heater, by sending steam through the heating chamber.



9. Monitor the temperatures  $\theta_1$  and  $\theta_2$  at a regular interval till they reach the steady state. Note down these observations in table 1.
10. Find out the steady state temperatures  $\theta_{1s}$  and  $\theta_{2s}$  from this table.

**Part 2: To determine the cooling rate of brass disc (B):**

11. Remove the metallic plate (M) and sample disc (S) from the assembly.
12. Place the heating chamber directly on the brass disc (B).
13. Allow the brass disc (B) to heat at least about 10 °C above the steady state temperature ( $\theta_{2s}$ ) measured in the first part of the experiment. Then remove the heating chamber.
14. Switch on the stop watch and measure the temperature of brass disc at an interval of 30 seconds as it cools down and take it as  $\theta$  in table 2.
15. Allow the brass disc to cool down to at least 5 °C below the steady state temperature,  $\theta_{2s}$ .
16. Plot a graph of temperature  $\theta$  of brass disc as a function of time. A typical plot is shown in Fig. 2.
17. Draw tangential line to the curve, corresponding to the temperature,  $\theta_{2s}$  and determine its slope.

$$\text{This slope} = \left( \frac{d\theta}{dt} \right)_{\theta_{2s}}$$

18. Calculate thermal conductivity, 'K' using given formula.

### 37.8 Observations

**Table 1: Measurement of Radius of brass disc**

S. No.	Main Scale reading (cm)	Vernier scale reading (cm)	Total (cm)	Average r (cm)
<b>1.</b>				
<b>2.</b>				
<b>3.</b>				
<b>4.</b>				

**Table 2: Measurement of Radius of bad conductor disc (S)**

S. No.	Main Scale reading (cm)	Vernier scale reading (cm)	Total (cm)	Average $r_0$ (cm)
1.				
2.				
3.				
4.				

**Table 3: Measurement of thickness of brass disc (B)**

S. No.	Main Scale reading (cm)	Vernier scale reading (cm)	Total (cm)	Average $d$ (cm)
1.				
2.				
3.				
4.				

**Table 4: Measurement of thickness of bad conductor disc (S)**

S. No.	Main Scale reading (cm)	Vernier scale reading (cm)	Total (cm)	Average $d_0$ (cm)
1.				
2.				
3.				
4.				

**(i) Measurements of brass disc (B)**

- Mass of disc (m) = .....gm

- Radius of brass disc ( $r$ ) = .....cm
  - Thickness of disc ( $d$ ) = .....cm
  - Specific heat of brass disc ( $c$ )= ..... kal/gm
- (ii) Measurements of bad conductor disc (S)**
- Radius of disc ( $r_0$ ) = .....cm
  - Thickness of disc ( $d_0$ ) = .....cm

**Table 5: To find steady state temperatures  $\theta_{1s}$  and  $\theta_{2s}$**

S. No.	Temperature increasing		
	Time (min.)	Temperature measured by $T_1$ and $T_2$	
		$\theta_1(^{\circ}\text{C})$	$\theta_2(^{\circ}\text{C})$
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			
13.			
14.			
15.			

**Table 2: To determine the cooling rate of brass disc (B)**

S. No.	Temperature decreasing	
	Time (seconds)	Temperature in $T_2$
		$\theta$ ( $^{\circ}\text{C}$ )
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		

### **37.9 Graph**

Model graphs are plotted as shown.

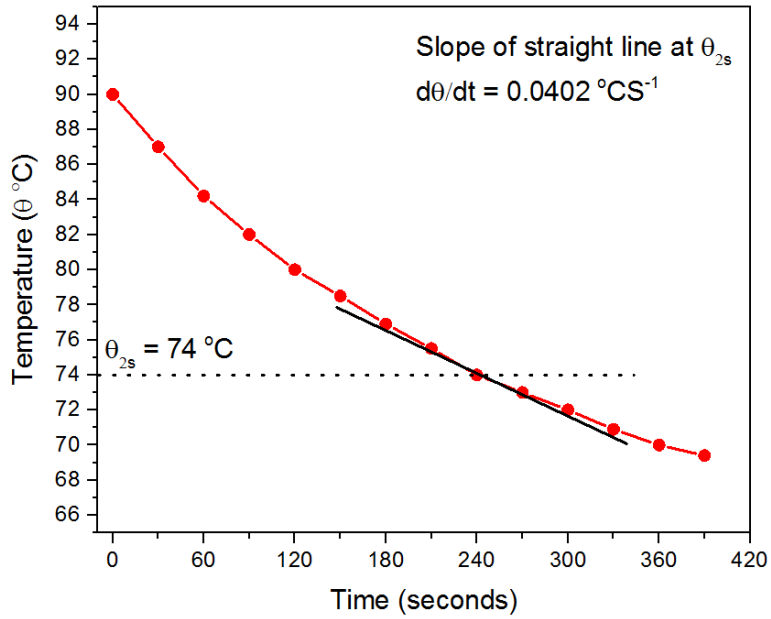


Fig.5: Plot to find out steady state temperatures  $\theta_{1s}$  and  $\theta_{2s}$ .

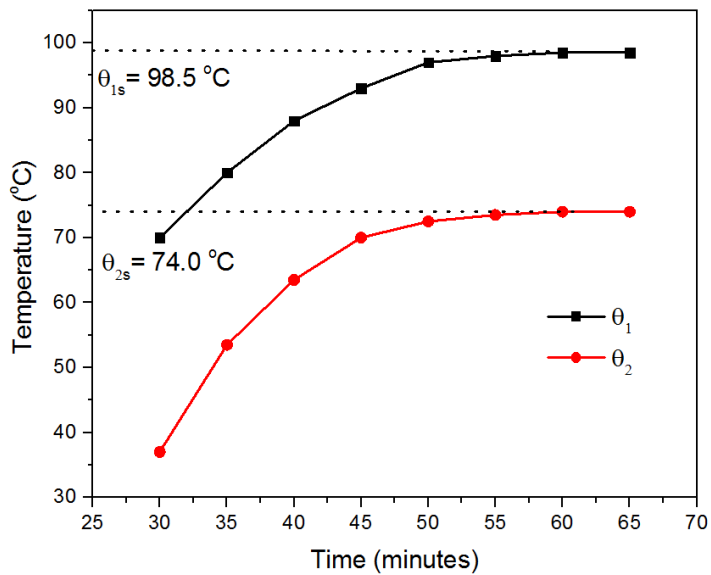


Fig.6: Plot to determine the rate of cooling of brass disc at  $\theta_{2s}$

## 37.10 Calculations

### Model Calculations:

#### (i) Measurements of brass disc (B)

- Mass of disc (m) = 1073 gm
- Radius of disc (r) = 5.75 cm
- Thickness of disc (d) = 1.6 cm
- Specific heat of brass disc (c) = 0.093 Cal/gm/°C

**(ii) Measurements of bad conductor disc (S)**

- Radius of disc ( $r_0$ ) = 6.0 cm
- Thickness of disc ( $d_0$ ) = 0.499 cm

**(iii) Measurements from Graph:**

- Rate of cooling of brass disc  $d\theta/dt = 0.0402 \text{ } ^\circ\text{C s}^{-1}$
- $\theta_{1s} = 98.5 \text{ } ^\circ\text{C}$
- $\theta_{2s} = 74.0 \text{ } ^\circ\text{C}$

Coefficient of thermal conductivity of the given material,

$$K = \frac{mc}{(\theta_{1s} - \theta_{2s})} \left( \frac{d\theta}{dt} \right)_{\theta_2} \left( \frac{r + 2d}{2r + 2d} \right) \frac{d_0}{\pi r^2}$$

$$K = \frac{1073 \times 0.093}{(98.5 - 74.0)} (0.0402)_{\theta_2} \left( \frac{5.75 + 2 \times 1.6}{2 \times 5.75 + 2 \times 1.6} \right) \frac{0.499}{3.14 \times 6 \times 6}$$

$$K = 44.01 \times 10^{-5} \frac{\text{Cal}}{^\circ\text{CcmS}}$$

**37.11 Result**

Coefficient of thermal conductivity of the given material:

K = .....  $\frac{\text{Cal}}{^\circ\text{CcmS}}$

Percentage Error: .....

**37.12 Discussion**

.....

.....

.....

.....

### 37.13 Precautions and Sources of Error

12. Measurements of temperature should be only in steady state.
13. Heat chamber and Disc M should be nickel polished so both can possess the same emission efficiency.
14. Thickness of sample disc of bad conductor should be less.
15. Thermometers  $T_1$  and  $T_2$  should be properly fixed in the grooves of metallic disc M and brass disc B.
16. The tangent should be drawn on steady temperature of brass disc B.

### 37.14 Self Learning Exercise-2

- Q.1 What is thermal conductivity?
- Q.2 How we determine steady state temperatures in Lee's Disc experiment?
- Q.3 What do you understand by cooling rate of object?
- Q.4 Why the lees method is not used to measure heat conductivity of good conductor?
- Q.5 What is unit of thermal conductivity?

### 37.15 Glossary

**Temperature Controller:** Controllers receive sensor signals and control heaters or other devices to maintain a pre-set temperature.

**Brass:** Brass is an alloy of copper and zinc of historical and enduring importance because of its hardness and workability. The metal was chosen because of its good electrical conductivity. The specific heat capacity of brass is  $0.38 \text{ KJ/W}^\circ\text{C}$  or  $0.093 \text{ Cal/gm}^\circ\text{C}$ . The faces were also smoothed for good thermal contact.

**Steady state:** In thermodynamics we know it as equilibrium state too, where temperature is not arising anymore and temperature gradient across the sample is constant.

**Bad conductor:** A poor conductor of heat is any material that cannot transfer heat. Poor conductors have a low density thus the particles in a given volume of the material are not enough to collide and transfer heat.

**Specific heat:** The amount of heat, measured in calories required to raise the temperature of one gram of a substance by one Celsius degree.

**Temperature gradient:** A temperature gradient is a physical quantity that describes in which direction and at what rate the temperature changes the most rapidly around a particular location. The temperature gradient is a dimensional quantity expressed in units of degrees (on a particular temperature scale) per unit length.

### 37.16 Answers to Self Learning Exercises

#### *Answers To Self-Learning Exercise 1*

**Ans.1:** To measure the coefficient of conductivity of a bad conductor

**Ans.2:** Conduct the heat.

**Ans.3:** Specific heat

**Ans.4:** 4.2

**Ans.5:** 1000

#### *Answers to Self-Learning Exercise- 2*

**Ans.1:** It is defined as the quantity of heat conducted per second normally across unit area of cross section of the material per unit temperature difference. It denotes the heat conducting power.

**Ans.2:** With waiting in time, temperature rise in metallic disc will stop. this time the assembly will be in steady state or equilibrium.

**Ans.3:** The temperature reduction of object per unit of time.

**Ans.4:** Lee's disc method uses the temperature difference between the faces of a disc made from the insulating material. For a good thermal conductor, the temperature difference would be too small to measure.

**Ans.5:** Cal/gm/sec/°C and Watt meter<sup>-1</sup> kelvin<sup>-1</sup>

### 37.17 Viva Questions

**Q.1** What is heat?

**Q.2** What are the units of temperature?

**Q.3** What are the method of heat transfer?

**Q.4** How can we prevent the heat loss through conduction?



- Q.5** How can we prevent the heat loss through radiation?
- Q.6** By which process(conduction/convection/radiation), heat loss is possible in vacuum?
- Q.7** Why our hands by touching do not feel a wood as cold as metal?
- Q.8** What is specific heat?
- Q.9** What do you understand by thermal conductivity?
- Q.10** What is SI unit of thermal conductivity?
- Q.11** What do you understand by coefficient of heat conductivity?
- Q.12** Is thermal conductivity scalar, vector or a tensor quantity?
- Q.13** What is the difference between bad conductor and good conductor of heat?
- Q.14** Give the name of poor conductors of heat.
- Q.15** Give the name of good conductor of heat.
- Q.16** Does the value of thermal conductivity depend on the dimension of the specimen?
- Q.17** Can Lee's Disc method be used for good conductors?
- Q.18** Is there any reason totake the specimen in the form of a disc?
- Q.19** Why should the cross section area large in comparison to the thickness of disc?
- Q.20** Why the slope is measured only at  $\theta_{2s}$ ?

### **37.18 Answers of Viva Questions**

- Ans.1:** Heat is simply the transfer of energy from a hot object to a colder object.
- Ans.2:** Kelvin, Degree Celsius and Degree Fahrenheit.
- Ans.3:**The fundamental modes of heat transfer are conduction or diffusion, convection and radiation.
- Ans.4:** It can be reduced using a layer of bad conductor of heat.
- Ans.5:** By using a metal reflecting surface.
- Ans.6:** Radiation.

**Ans.7:** Metal is a good conductor of heat so it takes away heat from hand while wood not.

**Ans.8:** The specific heat is the amount of heat per unit mass required to raise the temperature by one degree Celsius.

**Ans.9:** It is defined as the quantity of heat conducted per second normally across unit area of cross section of the material per unit temperature difference. It denotes the heat conducting power.

**Ans.10:** Its unit is Watt meter<sup>-1</sup> Kelvin<sup>-1</sup>

**Ans.11:** The rate at which heat is transferred by conduction through a unit cross-sectional area of material when a temperature gradient exists perpendicular to the area.

**Ans.12:** It is a tensor quantity.

**Ans.13:** Good conductors transfer the heat from end to other end with minimum loss while the ability of transfer of heat of bad conductors is very less.

**Ans.14:** Metals, mica, etc.

**Ans.15:** Wood, air, rubber, glass wool, etc.

**Ans.16:** No, it depends only on the material of the specimen.

**Ans.17:** No, Lee's disc method uses the temperature difference between the faces of a disc made from the insulating material. For a good thermal conductor, the temperature difference would be too small to measure.

**Ans.18:** A thin disc is taken because its area of cross section is large, while thickness is small. It increases the quantity of heat conducted across its faces.

**Ans.19:** Disc is taken with large cross sectional area ( $A = \pi r^2$ ) compared to the area exposed to the air at the edge ( $a = 2\pi r d$ ) in order to reduce the energy loss.

**Ans.20:** In steady state, the rate of heat flow through specimen plate is equal to the radiated heat from disk B which is at temperature  $\Theta_{2s}$ .

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# **UNIT-38**

## **To Study Absorption of Particles and Determination of Range**

### **Structure of the Unit**

- 38.1 Aim
  - 38.2 Apparatus
  - 38.3 Diagram
  - 38.4 Formula
  - 38.5 Model Graph
  - 38.6 Theory and Description
  - 38.7 Self Learning Exercise-I
  - 38.8 Procedure
  - 38.9 Observation
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  - 38.11 Calculations
  - 38.12 Result
  - 38.13 Discussion
  - 38.14 Precautions and Sources of Error
  - 38.15 Self Learning Exercise-II
  - 38.16 Glossary
  - 38.17 Answers to Self Learning Exercises
  - 38.18 Viva Questions
  - 38.19 Answers to Viva Questions
- References and Suggested Readings

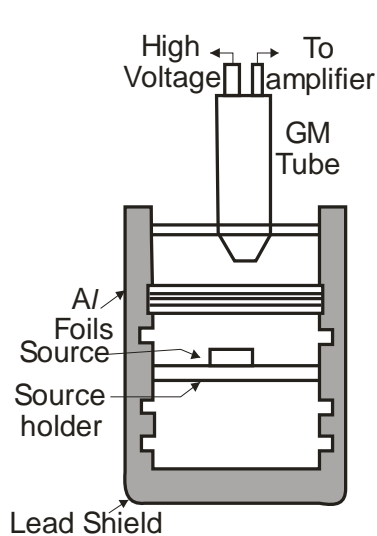
## 38.1 Aim

- (i) Determination of end point energy of  $\beta$  particles by using Geiger Muller counter and study of their absorption by aluminum.
- (ii) To study absorption of particles and determine range using at least two sources.

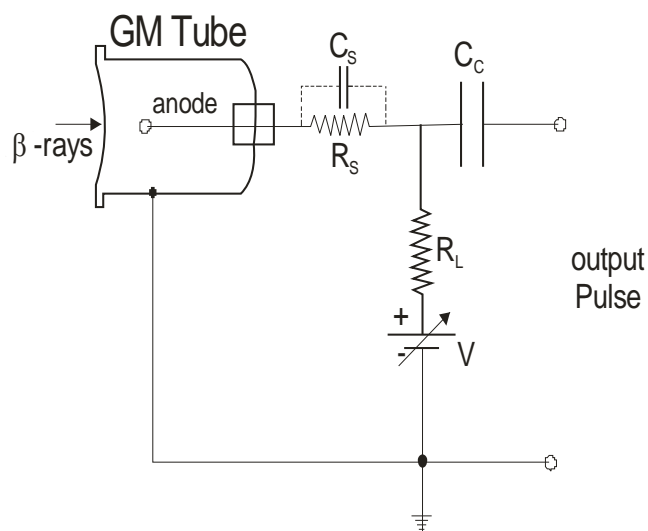
## 38.2 Apparatus

Geiger Muller counter, lead container, source holder, radioactive sample, voltage source having time sensor, rectangular.

## 38.3 Diagram



**Fig. (38.1) Arrangement of Source and Absorber**



**Fig. (38.2) : Geiger Muller Counter.**

## 38.4 Formula

The change in the count rate along the entire range of plateau region is represented in terms of slope of plateau region.

$$\text{Slope of the plateau} = \frac{N_2 - N_1}{N_{av}} \times \frac{100}{V_2 - V_1} \text{ \% per volt}$$

where  $N_1$  and  $N_2$  are the count rate at voltages  $V_1$  and  $V_2$ .  $N_{av}$  is the average value of  $V_1$  and  $V_2$ .

To use GM counter we will take the operating voltage at the mid-point of the plateau region

$$\text{Operating voltage } V_{op} = \frac{V_1 + V_2}{2}.$$

where  $V_1$  = Starting voltage of Plateau.

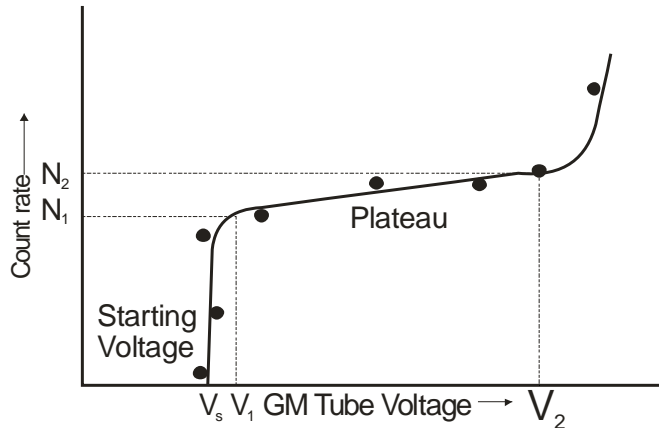
$V_2$  = End voltage of Plateau.

The value of linear absorption coefficient of the medium (Aluminium Foil)

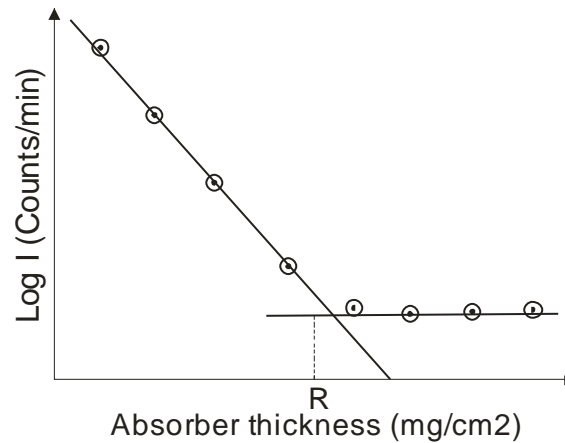
$$\mu = \frac{0.693}{\sqrt{x}}$$

where  $\mu$  is the linear absorption coefficient of the medium and  $\sqrt{x}$  is the thickness of half value of corrected count.

### 38.5 Model Graph



**Fig. (38.3) :- Characteristics Curve of Count rate with applied voltage for a GM tube.**



**Fig. (38.4) : Graph between thickness of the absorber and count rate.**

## 38.6 Theory And Description

The spectrum of  $\beta$  particles is continuous in nature or we can say that the energy of emitted particles lie in between a certain maximum and minimum value. Several times we require the study of maximum emitted energy in such spectrum, which is also known as end point energy.

It is important to note following points:

- (i) The absorption of particles by material is exponential rather than linear.
- (ii) If we are using material having different atomic numbers then that material having higher atomic numbers will have higher absorption rate of particles.
- (iii) If we express thickness of sample in  $\text{gm/cm}^3$  then absorption will not depend on the atomic number of the material.
- (iv) If the  $\text{gm/cm}^3$  thickness of different material is same then the amount of absorption of them will also be same.

When  $\beta$  particles pass through the material, their absorption takes place. The length of the sample for their complete absorption depends on the density of the material therefore if density or atomic number is large, then the length of the sample required for complete absorption of  $\beta$  particles will be less. Hence the length of absorber is inversely proportional to the density of the absorbing material. Except some fluctuations, the ratio of density of sample and length of absorber is generally constant for all the samples.

"For the absorption of  $\beta$  particles having higher energy, we will require longer ( $\text{gm/cm}^3$ ) absorbing material and for the absorption of  $\beta$  particles having lower energy, we will require small ( $\text{gm/cm}^3$ ) absorbing material."

On putting the radioactive source little away (about 2 to 3 centimeters) from the GM tube and applying absorbing medium (aluminum pieces) between the source and the G.M. tube, we will find that count rate will depend on the thickness of the absorber. On increasing the thickness of the absorbing material, the absorption of  $\beta$  particles increases. After a certain thickness limit, we will realize that even on increasing the thickness of absorbing medium, the absorption of  $\beta$  particles becomes constant. Actually these counts are due to  $\gamma$  radiations (

Bremstrahlung) , which are present during emission of  $\beta$  particles. On extrapolating the curves obtained due to Bremstrahlung and  $\beta$  particles; we will obtain the point of intersection of these curves that will correspond to range of absorption of  $\beta$  particles bearing maximum energy.

There range of  $\beta$  particles may be obtained with the help of absorption experiment and hence maximum or endpoint energy of  $\beta$  particles may be obtained by applying range energy relation. The range energy relation for aluminum absorber is well established and can be expressed in the form of empirical relation. One of the commonly used empirical relations for range energy relation may be expressed as:

$$R = 412T_0^{1.265-0.09651nT_0} ; \quad 0.01MeV < T_0 < 3MeV \quad (1)$$

$T_0$  is the end point energy and R is the range of  $\beta$  particles and

$$R = 530T_0 - 106; \quad 1MeV < T_0 < 20MeV \quad (2)$$

In these relations, the units of  $T_0$  is MeV and unit of range is mg/cm<sup>2</sup>. Several scientists developed empirical relations for range and energy of  $\beta$  particles but the relation developed by Katz and Pen fold is more widely used to obtain endpoint energy of  $\beta$  particles. The main limitation associated with these relations is that all the particles in the beam of  $\beta$  particles must have same energy i.e. all the  $\beta$  particles are monoenergetic.

When radioactive radiations pass through matter it under goes absorption primarily by Compton, photoelectric effect and pair production interactions. The intensity of the rays passing through a medium is given by

$$I(x) = I(0)e^{-\mu x} \quad (3)$$

Where  $I(x)$  is the intensity through absorber of thickness  $x$ ,  $I(0)$  is the original intensity of beam (without an absorber) and  $\mu$  is the linear absorption coefficient of the medium.

Half value layer of absorbing medium is defined as the thickness  $\sqrt{x}$  which will cut the initial intensity of half.



$$\text{i. e. } \frac{I}{I_0} = 0.5$$

From equation (1) we have

$$\ln(0.5) = -\mu\sqrt{x}$$

$$-0.693 = -\mu\sqrt{x}$$

$$\mu = \frac{0.693}{\sqrt{x}} \quad (4)$$

### 38.7 Self Learning Exercise-I

- Q.1 What is meant by radioactivity?
- Q.2 What is the Rutherford Soddy law regarding disintegration of radioactive substance?
- Q.3 Define half life of radioactive substance.
- Q.4 What do you mean by Efficiency of the tube?
- Q.5 What is low voltage region?

### 38.8 Procedure

Every GM tube has a characteristic response of counting rate versus applied voltage to the tube. A curve representing the variation of counting rate with voltage is called as "Plateau curve" because of its appearance. The plateau curve of every tube that is to be used for the first time should be drawn in order to determine the optimum operating voltage. Find the plateau curve for your tube using the procedure outlined below.

- (i) Check that the applied voltage as indicated by the meter on the instrument is at its minimum value.
- (ii) Without placing the radioactive source, increase the applied voltage in suitable steps and note the counts through the GM counter panel. These are the background counts.
- (iii) Insert a radioactive source into one of the shelves of the counting chamber. Choose shelf and counting time such that you have at least 1000 counts in the plateau region.

- (iv) Turn on the count switch and slowly increase the high voltage until counts just begins to be recorded by the scalar. The voltage at which counts just begin is called the "starting voltage" of the tube.
- (v) Beginning at the nearest 20-volt mark above the starting voltage, take one-minute (longer time if counting rate is too low) counts every 40 volts until a voltage is reached where a rapid increase in counts is observed. Reset scalar to zero before each count. For each value of applied voltage, subtract the background counts from the obtained counts to find actual counts received from the radioactive source. Tabulate counts versus voltage.
- (vi) Plot the data of (v). A plateau should be observed in the curve. Note the counts after 30 seconds without placing any absorber between the source and the counter calculate the count rate.
- (vii) Now apply one by one aluminum plates having different thickness between the source and the counter and each time note the counts after 30 seconds and hence obtain the rate count rate.
- (viii) Draw a graph between count rate and thickness of the absorber and find the value of R. obtain the end point energy of  $\beta$  Particles.

### 38.9 Observation

- (i) Radio active source =  $\text{Co}^{60}$   
 Half life of radioactive source = ..... Year  
 $E_0$  Standard = ..... MeV

**Observation Table**

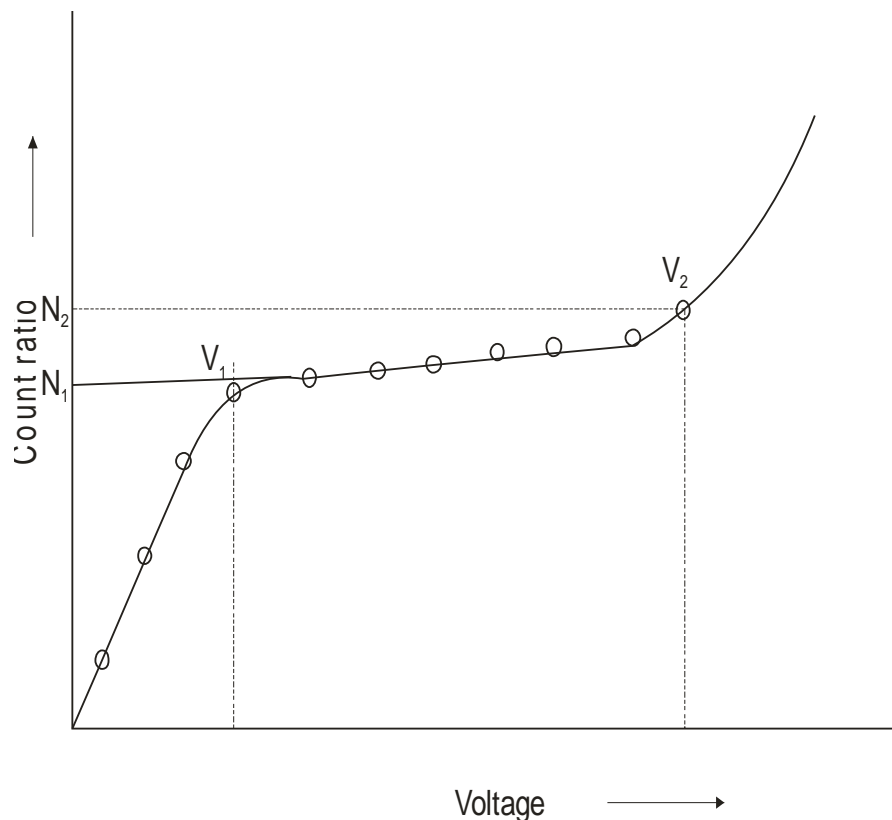
S.No.	Voltage (Volts)	Counts in 30 Seconds (A)	Background Counts in 30 Seconds (B)	Net Counts (A-B)











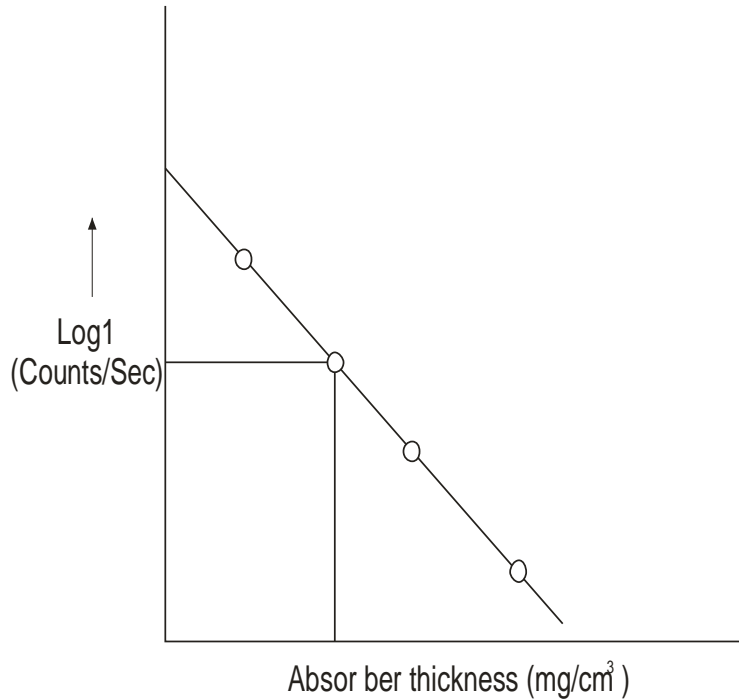
Graph between Absorber thickness and Log I.

Log I (Count/Sec.)								
Absorber Thickness Mg/Cm <sup>3</sup>								

Scale :-

ON x - axis      1 cm = ..... mg/ cm<sup>2</sup>

ON y - axis      1 cm = ..... of Log I



### 38.11 Calculations

1. Plot a graph between applied voltages to the G.M. tube and count rate. The center point of the plateau region will give the operating voltage for the experiment.

For Source Co<sup>60</sup>

(i)  $V_1 =$  Starting Voltage of Plateau = ..... V

(ii)  $V_2 =$  End Voltage of Plateau = ..... V

Threshold voltage = ..... Volt.

Operating voltage

$$V_{op} = \frac{V_1 + V_2}{2} = \text{..... Volt.}$$

For Source Cs137

(i)  $V_1 =$  Starting voltage of plateau = ..... V

(ii)  $V_2 =$  End voltage of plateau = ..... V

Threshold voltage = ..... Volt.

Operating Voltage



$$V_{op} = \frac{V_1 + V_2}{2} = \dots\dots\dots \text{ Volt.}$$

2. Plot a graph between Absorber thickness and count rate and obtain range of  $\beta$  Particles.

3. The value of linear absorption coefficient

$$\mu = \frac{0.693}{\sqrt{x}}$$

For source  $\text{Co}^{60}$

$$\sqrt{x} = \dots\dots\dots$$

$$\mu = \dots\dots\dots$$

For source  $\text{Cs}^{137}$

$$\sqrt{x} = \dots\dots\dots$$

$$\mu = \dots\dots\dots$$

### 38.12 Result

1. Operating voltage of G. M. tube =  $\dots\dots\dots$  volts.
2. Range of  $\beta$  particles in aluminum =  $\dots\dots\dots$   $\text{mg/cm}^2$
3. Absorption coefficient for  $\text{Co}^{60}$   $\beta = \dots\dots\dots$
4. Absorption coefficient for  $\text{CS}^{137}$   $\beta = \dots\dots\dots$
5. End point energy of  $\beta$  particles =  $\dots\dots\dots$  MeV.  
 Theoretical value of (i) Range =  $\dots\dots\dots$   $\text{mg/cm}^2$   
 (ii) End point energy =  $\dots\dots$  MeV  
 Percentage error =  $\dots\dots\dots\%$

### 38.13 Discussion

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### 38.14 Precautions and Sources of Error

1. The radioactive source should not be touched.
2. After the experiment, the source should be placed at an isolated space as it continuously radiates.
3. The applied voltage in the experiment is sufficiently high therefore no terminal should be touched.
4. The applied voltage during the plot of characteristics curve should not exceed Geiger region.
5. It is necessary to determine the operating voltage once and repeat that operating voltage once and repeat that operating voltage in other experiment if you have only one tube. If more than one tube are available in the lab, each time operating voltage of the tube should be determined by plotting its characteristics curve.
6. The background counts should always be subtracted from the obtained counts to get actual counts for each value of applied voltage.

### 38.15 Self Learning Exercise-II

- Q.1 What is proportional region?
- Q.2 What is Geiger region?
- Q.3 What are continuous discharge region?
- Q.4 What do you understand by Dead time of the tube?
- Q.5 Define recovery time.

### 38.16 Glossary

**$\beta$  Particle** : The electron emitter by a nucleus during beta decay.

**Geiger Muller counter**: It is an instrument used for measuring ionizing radiation.

**Detector**: A detector is a device or instrument designed designed to measure whether radioactive material is present.

**Threshold**: Threshold is the point out which radiation first produces an observable effect from acute exposures.

## 38.17 Answers To Self Learning Exercises

### *Answers To Self Learning Exercise-I*

**Ans.1:** Some substances have the tendency of self- disintegration. These materials are known as radioactive materials and this behavior of material is known as radioactivity.

**Ans.2:** (i) Radioactive materials decay themselves and radiate  $\alpha$ ,  $\beta$  particles in addition to  $\gamma$ - rays.

(ii) The disintegration phenomena is random. We cannot predict which particle will disintegrate at what time. However the number of Particles decaying in a fixed time is fixed.

(iii) The number of atoms disintegrating per second is proportional to the total number of atoms present at that time.

**Ans.3:** The time interval during which total number of undecayed particles becomes half of the number of particles initially present is known as half- life.

**Ans.4:** The Probability of recognition of the charged particle entering inside the tube has a finite value. This value of probability is termed as the efficiency of the tube.

**Ans.5:** This region is applied in the ionization counter. The voltage in this region is low but has an increasing trend.

### *Answers To Self Learning Exercise-II*

**Ans.1:** This region has an almost linear behavior of pulse voltage with applied voltage. This region is applied is proportional counter.

**Ans.2:** After the "almost proportional region" the pulse height increases sharply. This region is called Geiger region. This region is applied in G. M. counter.

**Ans.3:** The region after the Geiger region, which is not applicable in any particular application, is called continuous discharge region.

**Ans.4:** After the start of discharge inside the tube, the time interval during which tube becomes non-sensitive towards the charged particles entering the tube is termed as dead time.

**Ans.5:** The system takes some time to return to its normal state after every disintegration. This time is known as recovery time.

### 38.18 Viva Questions

- Q.1** Define decay constant of radioactive substance.
- Q.2** Define activity of radioactive substance.
- Q.3** Mention the unit of radio activity.
- Q.4** Mention the nature of spectrum for  $\alpha$ ,  $\beta$  particles and  $\gamma$  -rays.
- Q.5** What is the source of  $\beta$  particles inside the nucleus?
- Q.6** What is the mass of neutrino?
- Q.7** What is the rotational angular momentum of neutrino?
- Q.8** Compare the penetration power of  $\alpha$ ,  $\beta$  particles and  $\gamma$  -ray.
- Q.9** Compare the range for  $\alpha$ ,  $\beta$  Particles and  $\gamma$  - rays.
- Q.10** Mention the fields where radioactive isotopes mainly find application.
- Q.11** Define Background counts.
- Q.12** What is the plateau region in the characteristics curve?
- Q.13** What do you mean by length of plateau region?
- Q.14** Define slope of plateau region.
- Q.15** State the name of some gaseous detector.
- Q.16** Write formula of operating voltage.
- Q.17** Define threshold voltage.
- Q.18** What is operating voltage?
- Q.19** What type of  $\beta$  particles sources are used?
- Q.20** Define nuclear radiation detector?

### 38.18 Viva Questions

**Ans.1:** The inverse of that time in which number of un - decayed particles becomes Parallel  $e$  times the number of particles initial present is known as decay constant.

**Ans.2:** The number of particles radiated by radioactive substance per second is known as activity of the substance.

**Ans.3:** Curie and Rutherford.

$$1 \text{ Curie} = 3.7 \times 10^4 \text{ Rutherford} \\ = 3.7 \times 10^{10} \text{ Disintegrations/second.}$$

$$1 \text{ Rutherford} = 10^6 \text{ disintegrations/Second}$$

**Ans.4:**  $\alpha$  Particles - Line spectrum

$\beta$  Particles - Continuous spectrum

$\gamma$  rays - Line Spectrum

**Ans.5:** The emission of  $\beta$  Particles is due to transformation of neutrons into protons or protons into neutrons.

**Ans.6:** The mass of neutrino is almost zero.

**Ans.7:** The rotational angular momentum of neutrino is  $\frac{1}{2} \frac{h}{2\pi}$ .

**Ans.8:** The penetration power of  $\beta$  particles is about 100 times more than that of  $\alpha$  particles while it is 100 times less than that of  $\gamma$  rays.

**Ans.9:** The penetration power of  $\beta$  Particles is about 100 times more than that of  $\alpha$  particle while it is 100 times less than that of  $\alpha$  ray.

**Ans.10:** The radioactive isotopes find application mainly in agriculture, medicine, industry and in determination of age of old buried object.

**Ans.11:** The counts even in the absence of the radioactive source are termed as back ground counts. These may be due to any unseen source like. Cosmic radiations etc. To obtain actual counts during the experiment, these counts may be subtracted from the counts obtained in the presence of radioactive source.

**Ans.12:** That region in the characteristics curve where the count rate remains almost constant even on increasing the applied voltage is known as plateau region in the characteristics curve.

**Ans.13:** The voltage interval during which the count rate remains almost constant is known as length of plateau region.

**Ans.14:** The change in the count rate along the entire range of plateau region is represented in terms of slope of plateau region.

$$\text{Slope of plates} = \frac{N_2 - N_1}{V_2 - V_1} \times \frac{100}{N_{av}} \% \text{ per volt.}$$

Where  $N_1$  and  $N_2$  are the count rate at voltages  $V_1$  and  $V_2$ ,  $N_{av}$  is the average value of  $N_1$  and  $N_2$

**Ans.15:** (i) ionization chamber (ii) Proportional counter, (iii) GM Counter.

**Ans.16:**  $V_{op} = \frac{V_1 + V_2}{2}$ .

**Ans.17:** When we increases applied voltage further, the counts increases rapidly due to rapid ionization of the gas filled inside the tube. This particular voltage is termed as threshold voltage.

**Ans.18:** The operating point of the tube may be determined by middle point of the plateau region. Voltage corresponding to this point is termed as operating voltage of the GM tube.

**Ans.19:**  $\text{Cs}^{137}$  &  $\text{CO}^{60}$

**Ans.20:** The instrument used to detect the nuclear particles or radiation such as  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\rho$ , n particles are called nuclear radiation detectors.

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# UNIT-39

## Lattice Dynamics

### Structure of the Unit

- 39.1 Aim
  - 39.2 Apparatus
  - 39.3 Diagram
  - 39.4 Formula
  - 39.5 Model Graph
  - 39.6 Theory and Description
  - 39.7 Self Learning Exercise-1
  - 39.8 Procedure
  - 39.9 Observation
  - 39.10 Graph
  - 39.11 Calculations
  - 39.12 Result
  - 39.13 Discussion
  - 39.14 Precautions and Source of Error
  - 39.15 Self Learning Exercise-2
  - 39.16 Glossary
  - 39.17 Answers to Self Learning Exercises
  - 39.18 Viva Questions
  - 39.19 Answers to Viva Questions
- References and Suggested Readings

### 39.1 Aim

- (i) To study the dynamics of monoatomic and diatomic lattice.

(ii) Draw the dispersion curves and find out the dispersion relation and forbidden energy gap of diatomic lattice.

### 39.2 Apparatus

Cathode ray oscilloscope, lattice dynamics kit, connecting wire.

### 39.3 Diagram

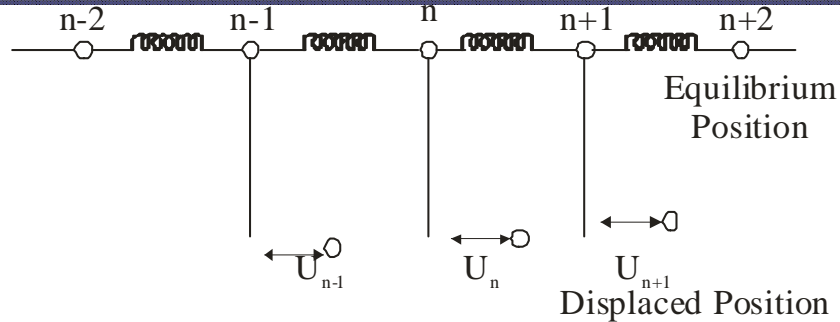


Fig. 39.1: one dimensional linear mono atomic lattice

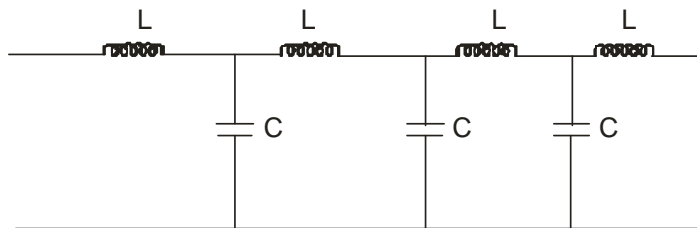


Fig. 39.2: Electrical analogue of linear mono-atomic lattice.

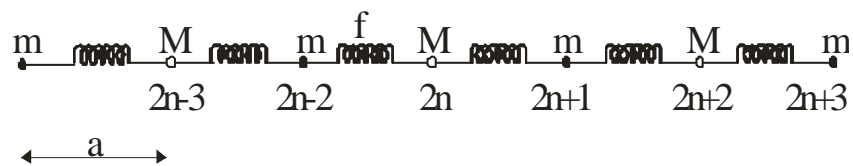


Fig. 39.3: Linear diatomic lattice of lattice parameter 'a' mass 'm' and 'M' and force constant 'f'.

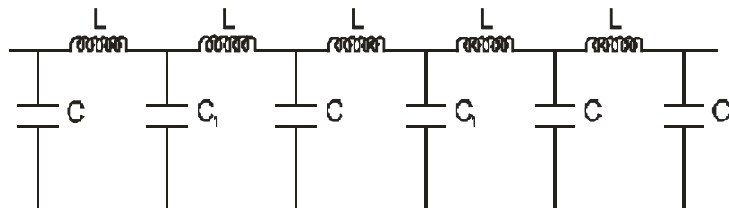


Fig. 39.4: Linear diatomic lattice- electrical analogue.



## 39.4 Formula

For monoatomic lattice and diatomic lattice the angular frequency is given by

$$\omega^2 = \frac{2}{LC}(1 - \cos\phi) \text{ where } L \text{ is the inductance in mH and } C \text{ is the capacitance in } \mu\text{F.}$$

## 39.5 Model Graph

### (i) Mono-atomic Lattice

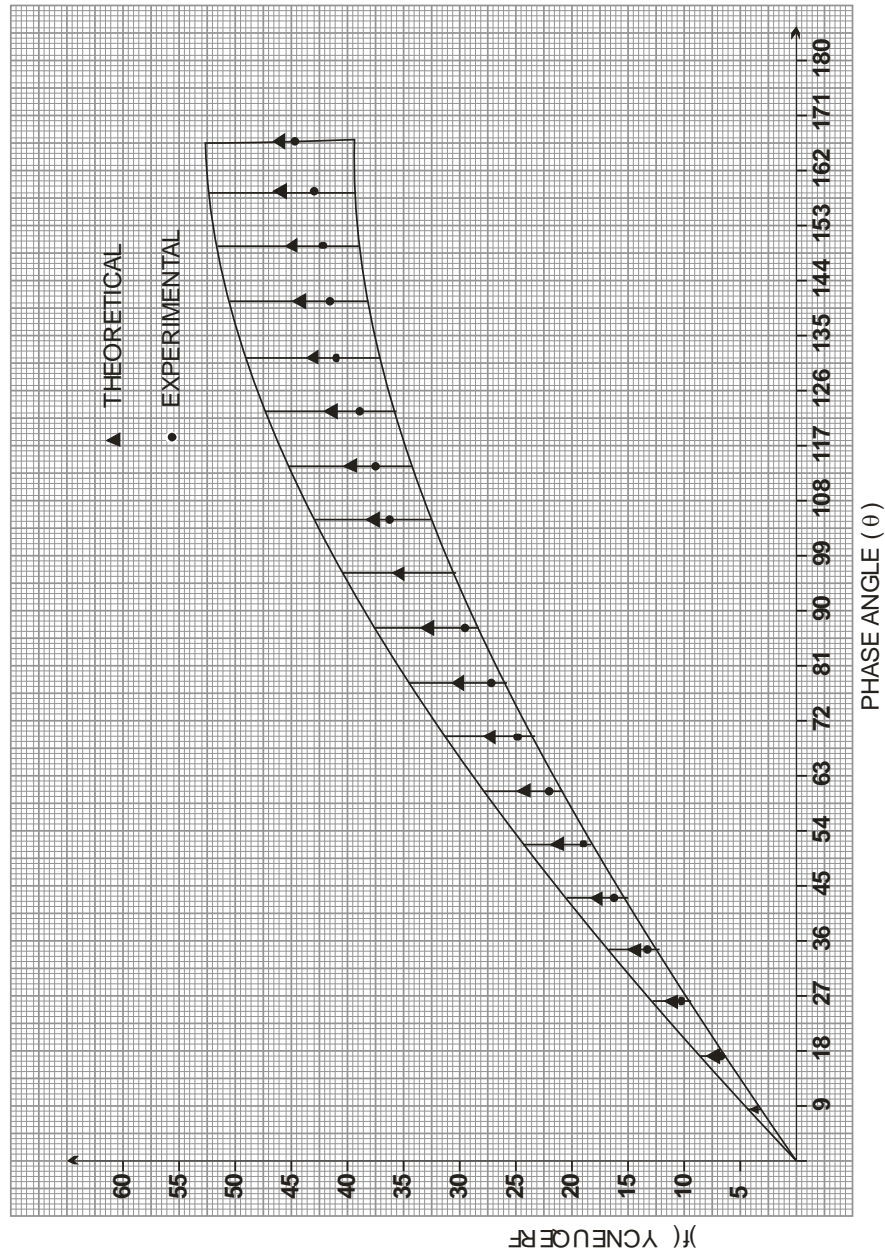


Fig. 39.5: Dispersion curve between frequency (f) and phase angle (θ)

(ii) Diatomic Lattice

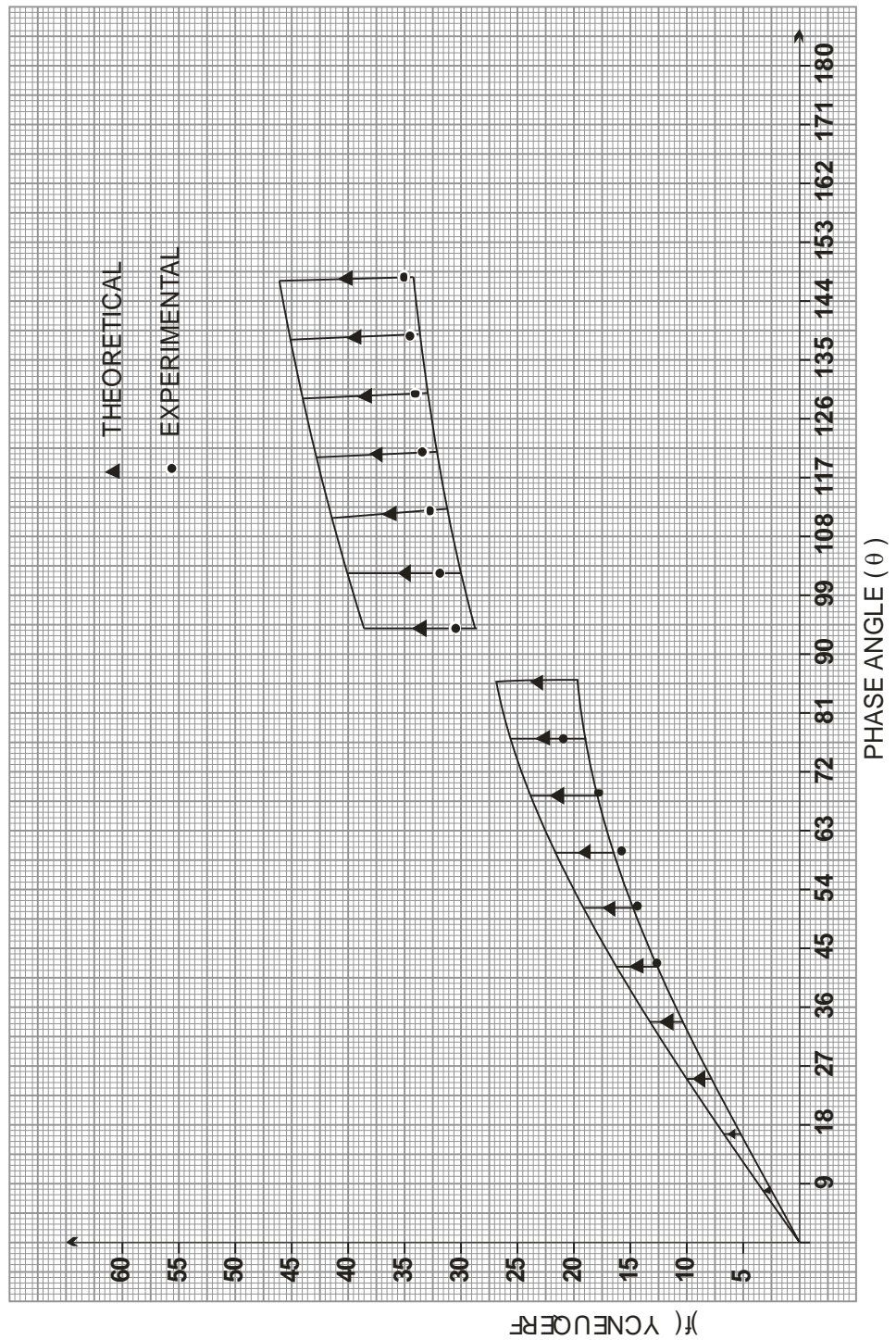


Fig. 39.6: Dispersion curve between frequency (f) and Phase angle ( $\theta$ ) for Diatomic lattice.

<b>DIAL V/S FREQUENCY (KHz)</b>			
<b>LODial Number</b>	<b>Range Observed Number</b>	<b>HI Dial Number</b>	<b>Range Observed Number</b>
0.00	0.842	0.00	8.590
.20	0.856	.20	8.754
.40	0.872	.40	8.916
.60	0.880	.60	9.088
.80	0.906	.80	9.276
1.00	0.924	1.00	9.440
.20	0.944	.20	9.636
.40	0.962	0.40	9.838
.60	0.982	.60	10.046
.80	1.002	.80	10.250
2.00	1.026	2.00	10.492
.20	1.050	.20	10.722
.40	1.072	.40	10.962
.60	10.098	.60	11.224
.80	1.124	.80	11.484
3.00	1.154	3.00	11.788
.20	1.182	.20	12.076
.40	1.212	.40	12.392
.60	1.246	.60	12.736
.80	1.280	.80	13.076
4.00	1.318	4.00	13.470
.20	1.356	.20	13.854
.40	1.398	.40	14.281
.60	1.440	.60	14.717
.80	1.486	.80	15.202

5.00	1.536	5.00	15.712
.20	1.590	.20	16.282
.40	1.644	.40	16.838
.60	1.704	.60	17.462
.80	1.770	.80	18.148
6.00	1.840	6.00	18.856
.20	1.923	.20	19.704
.40	2.000	.40	20.510
.60	2.092	.60	21.416
.80	2.186	.80	22.386
7.00	2.300	7.00	23.486
.20	2.418	.20	24.760
.40	2.554	.40	26.080
.60	2.702	.60	27.592
.80	2.862	.80	29.268
8.00	3.058	8.00	31.356
.20	3.276	.20	33.504
.40	3.526	.40	36.064
.60	3.818	.60	39.064
.80	4.170	.80	42.510
9.00	4.596	9.00	48.804
.20	5.112	.20	58.230
.40	5.750	.40	58.758
.60	6.594	.60	66.750
.80	7.3676	.80	66.750
10.00	9.178	10.00	92.500

## 39.6 Theory And Description

The lattice dynamics kit consist of the following parts:-

(i) Audio oscillator with amplitude control and facility to vary the frequency in the following range.

Switch position	Frequency Range	Output voltage
LO	1.3- 11.6 KHZ	4-6 V
HI	9.64-94 KHZ	4-6V

(ii) This kit consists of electrical transmission line which stimulates one dimensional mono-diatomic lattice.

The smallest possible unit of a crystal which can maintain the identity of a crystal is called unit cell.

### Mono - Atomic Lattice:-

One dimensional monatomic lattice can be shown by mass spring model. One dimensional chain of identical atoms each of mass 'm' are connected by ideal Hooke's law springs. At equilibrium the atoms will be situated on equally spaced equilibrium sites represented by ..... n-1, n, n + 1, ..... with x - co-ordinates as ..... (n-1) a, na, (n+1) a ..... However when the vibrational motion is excited. They will execute periodic motion about their equilibrium position.

Let the displacement of the nth atom from its equilibrium position is given  $u_n$ . To set up a force equation, assume that the force between neighbouring atoms is the linear forces. So the force equation can be written

$$F_n = \beta(u_{n+1} - u_n) - \beta(u_n - u_{n-1}) = \beta(u_{n+1} - 2u_n + u_{n-1}) \quad (1)$$

where  $\beta$  is spring constant

Applying Newton's second law, we can obtain the equation of motion

$$m \frac{d^2 u_n}{dt^2} = \beta(u_{n+1} - 2u_n + u_{n-1}) \quad (2)$$

where  $\frac{d^2 u_n}{dt^2}$  is the acceleration of n<sup>th</sup> atom.

Periodic solution of this wave equation can be given by

$$u_n = u_0 e^{i(\omega t - k n a)} \quad (3)$$

According to this scheme we should expect that

$$u_{n+1} = u_0 e^{i(\omega t - k(n+1)a)}, \quad u_{n-1} = u_0 e^{i[\omega t - k(n-1)a]}$$

Solving the equation we get the final result or final equation as

$$\omega^2 = \frac{4\beta}{m} \sin^2 \frac{ka}{2} \quad \Rightarrow \quad \omega = \pm \sqrt{\frac{4\beta}{m} \sin^2 \left( \frac{ka}{2} \right)} \quad (4)$$

where  $k$  is the wave vector ( $2\pi/\lambda$ ) or  $\left(\frac{\omega}{c}\right)$  and  $c$  is the velocity of propagation and  $\theta = ka$  is the phase change per unit cell. This relation shows that the velocity of propagation is dependent on frequency i.e. dispersion is indicated. It also shows that there is a maximum frequency.

$$v_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{1}{2\pi} 2\sqrt{\frac{\beta}{m}} = \frac{1}{\pi} \sqrt{\frac{\beta}{m}} \quad (5)$$

Beyond which no transmission occurs. The array may thus be considered as a low pass filter which transmits only in the range 0 to  $v_{\max}$ .

The electrical analogue of one dimensional array is shown in fig. (38.2). The

dispersion relation is 
$$\omega^2 = \frac{2}{LC}(1 - \cos \theta) \quad (6)$$

Where  $\theta$  is the phase change introduced by each section (unit cell) of the filter. Thus one has a precise analogy with the one dimensional mono-atomic lattice. With  $C \leftrightarrow M$  and  $\left(\frac{1}{L}\right) \leftrightarrow F$ . By measuring the phase difference between the input and output voltage of the circuit as shown in Fig. (38.2) as a function of frequency. The dispersion relation may be verified.

### Di - Atomic Lattice :-

The di - atomic lattice with alternative masses 'm' and 'M' as shown in fig (38.3) can be stimulated by the transmission line with alternative capacitors C and  $C_1$  as shown in fig. (38.4).

A linear lattice of two different kinds of atoms arranged alternatively is known as diatomic lattice. A lattice with a basis of two atoms in the primitive cell.

Let each atom be represented from its nearest neighbours by a distance 'a' and that the mass of the lighter atom is 'm' and of heavier atom is 'M'.

Equation of force can be written as separately

$$F_{2n} M \frac{d^2 u_{2n}}{dt^2} = \beta (u_{2n+1} - 2u_{2n} + u_{2n-1}) \quad (1)$$

$$F_{2n+1} m \frac{d^2 u_{2n+1}}{dt^2} = \beta (u_{2n+2} - 2u_{2n+1} + u_{2n}) \quad (2)$$

The assumption is that each atom will interact only with its nearest neighbour and that the force constant are identical between all pairs of nearest neighbour.

On solving and rearranging the above equation we get a dispersion relation for the linear di-atomic lattice.

The dispersion relation for mechanical and electrical analogous are given below :-

$$\omega^2(\text{mechanical}) = \beta \left( \frac{1}{m} + \frac{1}{M} \right) \pm \beta \sqrt{\left( \frac{1}{m} + \frac{1}{M} \right)^2 - \frac{4 \sin^2 ka}{mM}} \quad (3)$$

$$\omega^2(\text{Electrical}) = \frac{1}{L} \left( \frac{1}{C} + \frac{1}{C_1} \right) + \frac{1}{L} \left[ \left( \frac{1}{C} + \frac{1}{C_1} \right)^2 - \frac{4 \sin^2 \theta}{CC_1} \right]^{1/2}$$

In contrast to monoatomic lattice, there are two frequencies  $\omega_+$  and  $\omega_-$  corresponding to a particular value of the wave vector K.

In a plot of  $\nu$  versus  $\theta$ , this leads to two branches one corresponding to  $\nu_-$  is called the acoustical branch and the one corresponding to  $\nu_+$  is called the optical branch.

The frequency gap between the two branches depends upon (M/m). By increasing (M/m) the width of optical band can be made narrower which results in decrease of frequency gap.

### Lissajou's Figure:-

When two simple harmonic motions in perpendicular directions are superimposed on a particle. The resultant path of particle is a closed curve and the figure so obtained is called a Lissajous figure.

They depend on the following properties:-

- (i) The frequencies of the two simple harmonic motion.
- (ii) The Phase difference between the simple harmonic motion.
- (iii) The amplitude of simple harmonic motion

### Composition of two simple harmonic motion :-

The shape of the closed curve is given by the equation  $\frac{y^2}{b^2} + \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \phi = -\sin^2 \phi$

the shape depends upon the amplitude 'a' and 'b' and the phase difference  $\phi$ .

For different values a & b &  $\phi$  we get the following figures.

- (i) When  $\phi = \pi$  or  $2\pi, 4\pi, \dots$ , then  $\left[\frac{y-x}{b-a}\right]^2 = 0$  represents two coincident straight lines passing through the origin and having slope  $b/a$ .
- (ii) When  $\phi = 45^\circ$  or  $\pi/4$ ,  $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$   
 Then  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab\sqrt{2}} = \frac{1}{\sqrt{2}}$  This represents an oblique ellipse.
- (iii) When  $\phi = 90^\circ$ , or  $\pi/2$ ,  $\cos\theta = 0$  and  $\sin\theta = 1$ , then  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
 This represents an ellipse. In this case if the amplitudes  $a = b$  then  $x^2 + y^2 = a^2$  This represents a circle having radius 'a'.
- (iv) When  $\phi = 135^\circ$  or  $3\pi/4$ , then the cycle of these figures is completed when the phase difference comes  $\phi = 2\pi$ .

### Cathode ray oscilloscope :-

A cathode ray oscilloscope is a usual display of any time varying electrical signal so it may be regarded as a fast X-Y recorder where electron beam of the cathode ray tube is the pencil and the phosphor coating on its screen is the sheet on which plot gets drawn.

## 39.7 Self Learning Exercise -1

- Q.1 What do you mean by lattice?
- Q.2 What is the dispersion relation for the linear monoatomic lattice?
- Q.3 What is phase velocity?
- Q.4 What is group velocity?
- Q.5 What is frequency?

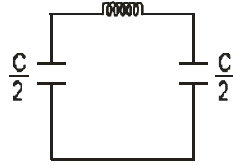
## 39.8 Procedure

### (A) Mono-Atomic Lattice:

- (i) Attach the lattice dynamics kit to the audio oscillator and plug in the power lead of the audio oscillator.
- (ii) Feed the output of the lattice dynamics kit to a general purpose CRO as described in section 2. Operate the CRO Horizontal input on the External Mode.



(iii) Switch the slide switch towards the 'mono-atomic' side. The transmission line (lattice) now consists of ten sections (unit cells) and it is excited by a constant current source (audio oscillator in series with  $R_1$ ). Each unit of the line may be thought of as consisting of



The line is terminated by resistance  $R_2$  equal in value to the characteristic impedance  $\sqrt{L/C}$ . This eliminates the reflected wave and thus simulates an infinite line.

(iv) Starting with the lowest frequency available ( $L_0$ ) vary the frequency of the audio oscillator and determine the frequencies at which the phase difference between the input and output voltages of the simulated lattice are  $\frac{n\pi}{2}$  where

$n = 1, 2, \dots$  etc. At low frequencies the phase difference will be practically zero and the CRO pattern shows  $\mathcal{O}$ . As the frequency is increased, at a particular frequency the pattern shows a circle. The phase difference is now  $\pi/2$ . At this point adjust the amplitude of the oscillator  $R_1$  and  $R_2$  such that you get a reasonably sized figure without distortion. At this frequency, the total phase difference is  $90^\circ$  and the phase difference per unit cell is  $9^\circ$  vary the frequency still for then and determine the phase difference as a function of  $\theta$ .

(v) Represent the observed and calculated value of  $V$  as a function of  $\theta$ . Comment on the agreement between the theoretical and experimental values.

(vi) Find out the maximum frequency of transmission and compare with the theoretical value of  $\frac{1}{\pi} \sqrt{\frac{1}{LC}}$ .

**(B) Di-atomic lattice:-**

(i) Flip the slide switch on the front panel of the lattice dynamics kit towards the diatomic. The alternate capacitors are now charged to  $C_1 = 0.147 \mu\text{F}$ .

(ii) Repeat the procedure outlined for the mono atomic lattice. Adjust  $R_2$  to get distortion free figures on CRO. Note the existence of the energy gap.

## 39.9 Observation

(i) For mono-atomic lattice:-

$$L = 1 \text{ mH}, C = 0.047 \mu\text{F}, \quad n = 10$$

S.R. No.	CRO Pattern	In degree		Dial Reading	Frequency in KHz	
		$n\theta$	$\theta$		Observed	Calculated
1	/					
2	0					
3	\					
4	0					
5	/					
6	0					
7	\					
8	0					
9	/					
10	0					
11	\					
12	0					
13	/					
14	0					
15	\					
16	0					
17	/					
18	0					
19	\					
20	0					
21	/					
22	0					

23	\					
24	0					
25	/					

(ii) **For di-atomic lattice :-**

$$L = 1 \text{ mH}$$

$$C = 0.047 \mu\text{F}$$

$$n = 10$$

S.R. No.	CRO Pattern	In degree		Dial Reading	Frequency in KHz	
		N $\theta$	$\theta$		Observed	Calculated
1	0					
2	/					
3	0					
4	\					
5	0					
6	/					
7	0					
8	\					
9	0					
10	/					
11	gap	gap	gap	gap	gap	gap
12	-	-	-	-	-	-
13	0					
14	\					
15	0					

### 39.10 Graph

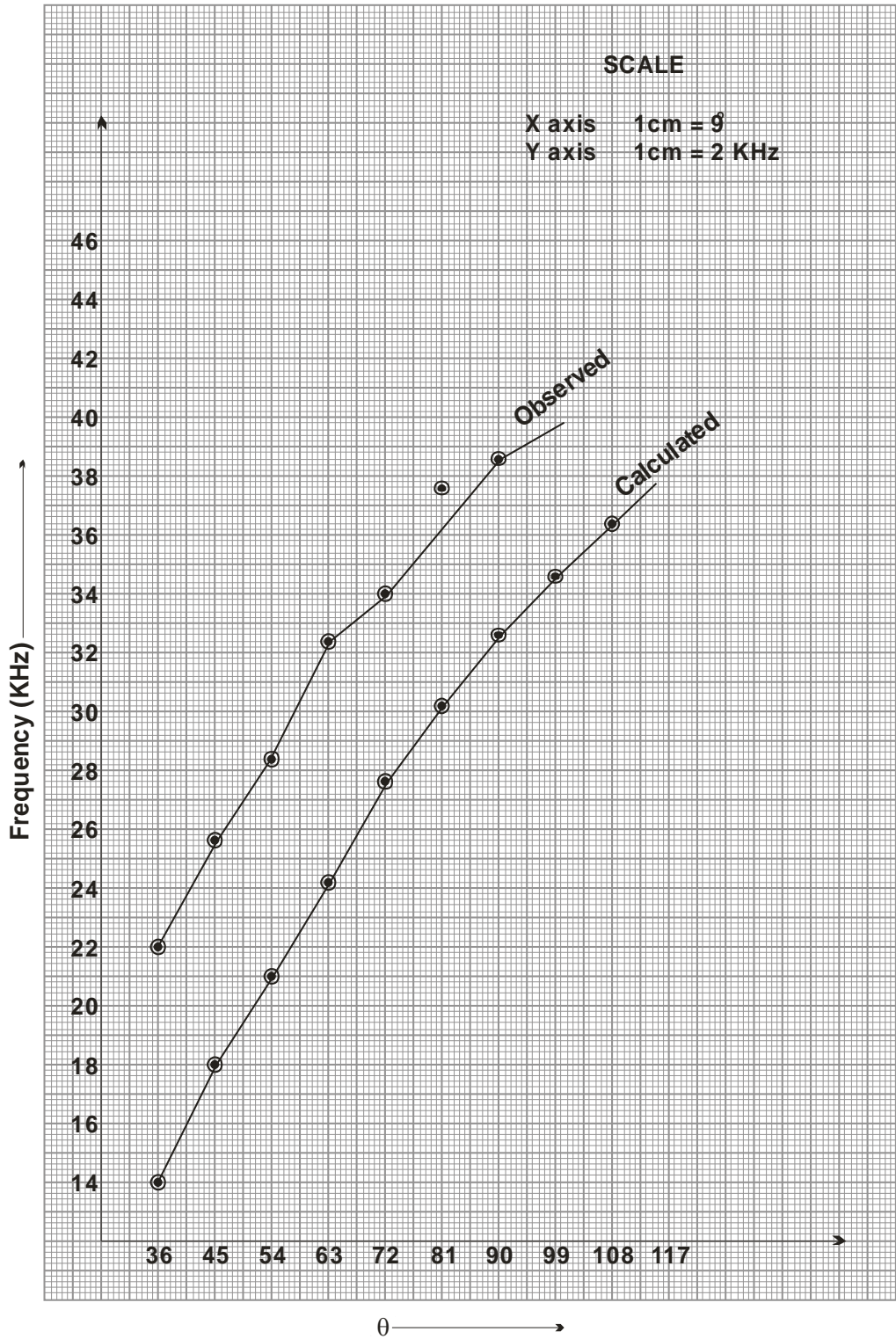
Graph between Frequency (KHz) and  $\theta$ .

$\theta$									
Frequency (KHz)									

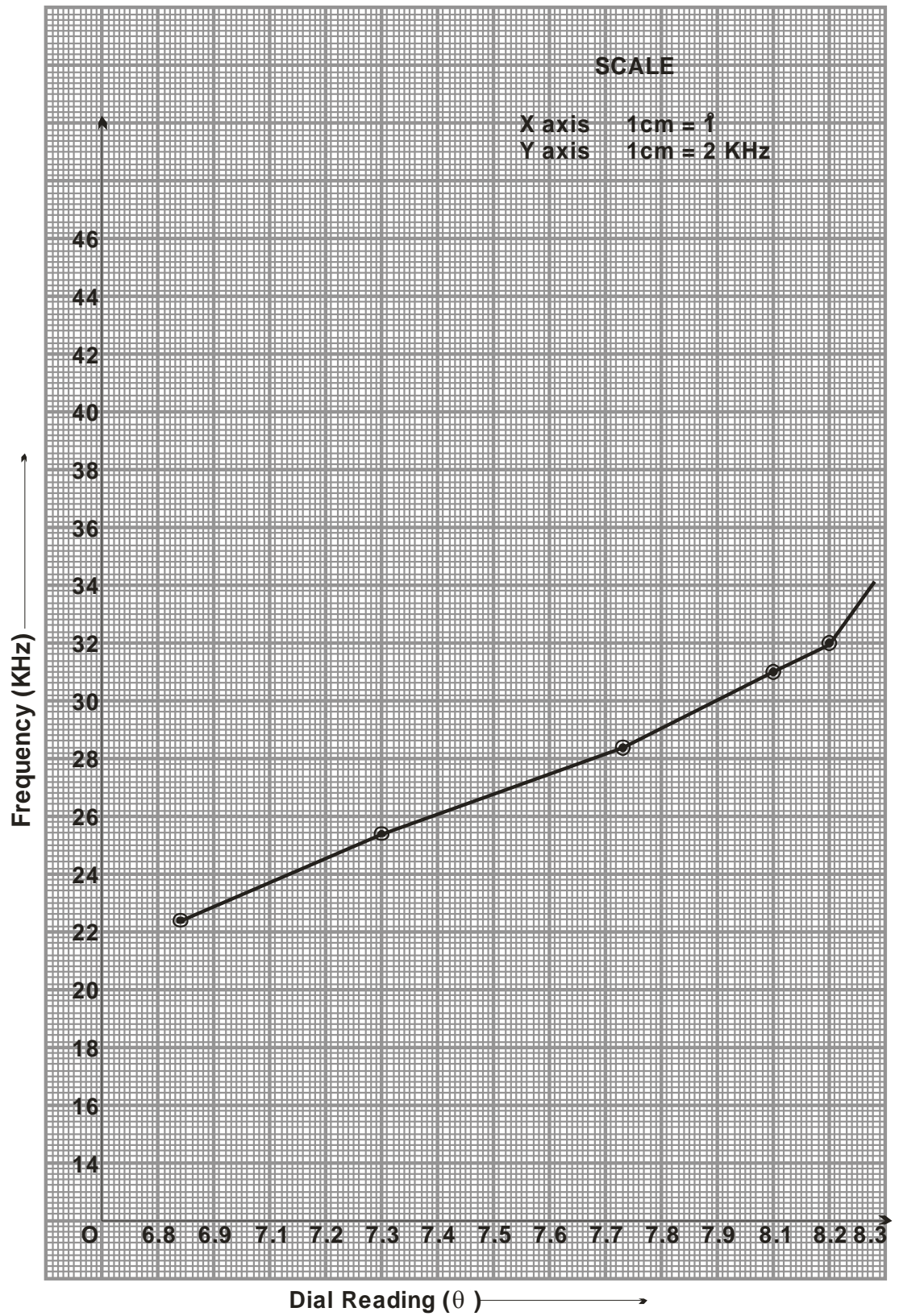
Scale

On x - axis 1 Cm = ....., On y - axis 1 Cm = .....

FOR MONO ATOMIC LATTICE



# FOR DIATONIC LATTICE



### 39.11 Calculations

For Mono Atomic Lattice :-

$$\omega^2 = \frac{2}{LC}(1 - \cos\theta) \quad \& \quad \omega = 2\pi f \quad \& \quad f = \frac{1}{2\pi} \omega = \frac{1}{2\pi} \sqrt{\frac{2}{LC}(1 - \cos\theta)}$$

$$f = \frac{1}{2 \times 3.14} \left[ \frac{2(1 - \cos\theta)}{1 \times 10^{-3} \times 0.047 \times 10^{-6}} \right]^{1/2}$$

$$\Rightarrow f = 32.84 \sqrt{(1 - \cos\theta)} \times 10^3 \text{ Hz}$$

- (i) for  $\theta = \dots\dots\dots$  ,  $f = \dots\dots\dots$  KHz
- (ii) for  $\theta = \dots\dots\dots$  ,  $f = \dots\dots\dots$  KHz
- (iii) for  $\theta = \dots\dots\dots$  ,  $f = \dots\dots\dots$  KHz

### 39.12 Result

For monoatomic lattice dispersion curve (between frequency and phase difference  $\theta$ ) is almost same for theoretical and practical values. For diatomic lattice, the dispersion curve consist of two branches optical branch which is a upper part and the acoustical branch which is lower part the difference between these two branches is termed as forbidden frequency gap. The graph for experimental & theoretical value is almost same.

### 39.13 Discussion

.....

.....

.....

.....

### 39.14 Precautions And Sources Of Error

1. Lissajous figure may be slightly distorted due to CRO used for minimized by  $R_1$  and  $R_L$  controls in the kit.
2. CRO connections should be proper and tight REo wire is to be connected to horizontal input of CRO. The vertical input to the ground.
3. The amplitude  $R_1$ ,  $R_2$  knobs are properly sit, should not be altered throughout the experiment.

4. Phase angle must be given properly.
5. Materials present in the vicinity of the experimental set-up should be absorbing ones. It's advised to perform this experiment in open space.

### 39.15 Self Learning Exercise- II

- Q.1 What is the dispersion relation of linear diatomic lattice for the mechanical?
- Q.2 What is the dispersion relation of linear diatomic lattice for the electrical analogues?
- Q.3 What is the acoustical branch?
- Q.4 What is the optical branch?
- Q.5 What is Lissajous Figures? and how it create and change with frequency?

### 39.16 Glossary

**Acoustic** :- The branch of physics dealing with the production, transmission and effects produced by sound.

**Dispersion** :- The separation of white light into colours or of any radiation according to wavelength.

**Lattice** :- Regular periodic arrangement of ions or molecules in a crystalline solid.

### 39.17 Answer To Self Learning Exercises

#### *Answer To Self Learning Exercise-1*

**Ans.1:** A regular, periodic configuration of particles in a crystalline solid.

**Ans.2:**  $\omega^2 = \frac{2}{LC}(1 - \cos\theta)$

**Ans.3:** The phase velocity of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels.

**Ans.4:** The group velocity of a wave is the velocity with which the overall space of the waves amplitude propagates through space.

**Ans.5:** Frequency is the measurement of the number of times that a repeated event occurs per unit of time.

### ***Answer To Self Learning Exercise-2***

$$\text{Ans.1: } \omega^2 = F \left( \frac{1}{m} + \frac{1}{M} \right) + F \left[ \left( \frac{1}{m} + \frac{1}{M} \right)^2 - \frac{4 \sin^2 \theta}{mM} \right]^{1/2}$$

$$\text{Ans.2: } \omega^2 = \frac{1}{L} \left( \frac{1}{C} + \frac{1}{C_1} \right) + \frac{1}{L} \left[ \left( \frac{1}{C} + \frac{1}{C_1} \right)^2 - \frac{4 \sin^2 \theta}{CC_1} \right]^{1/2}$$

**Ans.3:** In an acoustical branch the atoms of different masses moves in the same direction with the same amplitude.

**Ans.4:** In an optical branch the atoms of different masses move in opposite direction and their amplitudes are inversely proportional to their masses in such a way that the centre of mass of unit cell remains at rest.

**Ans.5:** Lissajous pattern is a phenomenon of wave superposition. Wave super position can be happen in four manners. There are:-

- (i) When two waves with same frequency moving in same direction and super impose then these phenomena known as interference.
- (ii) When two waves with slight different frequency moving in same direction super impose them this phenomena known as beats.
- (iii) When two waves with same frequency moving in opposite direction superimpose then it as create standing waves.
- (iv) When two waves with same or different frequency moving perpendicular to each other then it create Lissajous pattern.

### **39.18 Viva Questions**

- Q.1** What is a primitive cell?
- Q.2** What is symmetry operation in crystals?
- Q.3** Define a Brillouin Zone?
- Q.4** What is the volume of the first Brillouin zone of a simple cubic lattice of lattice constant  $a$ ?
- Q.5** What do you mean by monoatomic linear lattice vibrations?
- Q.6** What is the angular frequency of vibration of monoatomic linear lattice?



- Q.7** What is the angular frequency of vibration of diatomic linear lattice?
- Q.8** What is the range of K-values defining first Brillouin Zone of the linear lattice?
- Q.9** What is the maximum angular frequency of monoatomic linear lattice?
- Q.10** A mono-atomic linear lattice behaves as a low pass filter. How?
- Q.11** What is the cut-off frequency for the monoatomic linear lattice?
- Q.12** What are optical and acoustical branches in diatomic linear lattice vibrations?
- Q.13** Which branch corresponds to a higher frequency?
- Q.14** What is the frequency gap in diatomic lattice vibrations?
- Q.15** What is the relation between the ratio of the masses of the two kinds of atoms in diatomic lattice, and the frequency gap between the two branches?
- Q.16** What is a phonon?
- Q.17** What is phonon Momentum?
- Q.18** What is the cut-off frequency for a mono-atomic linear lattice lies in the region of electromagnetic spectrum?
- Q.19** Which statistics obeys phonons?
- Q.20** What is the quantum of lattice vibration energy?

### 39.19 Answers To Viva Questions

**Ans.1:** It is a unit cell that contains lattice points at the corners only.

**Ans.2:** A symmetry operation is one which leaves the crystal and its environment invariant.

**Ans.3:** A Brillouin zone is defined as a Wigner-Seitz cell in the reciprocal lattice.

**Ans.4:**  $(2\pi/a)^3$

**Ans.5:** A mono-atomic linear crystal is an array of identical equidistant atoms. When elastic waves passes through the crystal, the atoms are set in vibration. These are lattice vibrations.

**Ans.6:**  $\omega = 2\sqrt{\frac{F}{m}} \sin \frac{ka}{2}$

**Ans.7:**  $\omega_+ = \sqrt{\frac{2F}{M_1}}$        $\omega_- = \sqrt{\frac{2F}{M_2}}$

**Ans.8:**  $-\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}$

**Ans.9:**  $\omega_{\max} = 2\sqrt{\frac{F}{m}}$

**Ans.10:** A mono-atomic linear lattice transmits only in the frequency range  $0 - \nu_{\max}$  where  $0 - \nu_{\max} = \nu/2a$ ,  $\nu$  being the velocity of sound wave in the solid lattice.

**Ans.11:**  $\nu_{\max} \cong 10^{13} \text{ Hz}$

**Ans.12:** The  $\omega - k$  (dispersion) relation for the linear diatomic lattice gives two angular frequencies,  $\omega_+$  and  $\omega_-$  corresponding to a single value of  $k$ . Therefore, in a  $\omega - k$  plot, two branches are obtained. The branch for  $\omega_+$  is called 'optical' branch, and that for  $\omega_-$  is called 'acoustical' branch.

**Ans.13:** The optical branch is at higher frequency.

**Ans.14:** The frequency band lying between frequencies  $\sqrt{\frac{2f}{M_1}}$  and  $\sqrt{\frac{2f}{M_2}}$  is called frequency gap.

**Ans.15:** The larger the mass ratio, the wider is the frequency gap.

**Ans.16:** A phonon is a quantum of energy in an elastic wave.

**Ans.17:** A phonon of wave vector  $\bar{k}$  interacts with other particles as if it has a momentum  $\left(\frac{h}{2\pi}\right)\bar{k}$ .

**Ans.18:** Infra-red region.

**Ans.19:** Bose-Einstein Statistics.

**Ans.20:** Phonon

## References And Suggested Readings

1. M.A. Omar, Elementary solid state Physics, Addison Wesley, 1993.
2. Steven H. Simon, The oxford solid state Basics, oxford university press, 2013.
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# UNIT-40

## Programme in FORTRAN 77 to determine the Integration

### Structure of the Unit

- 40.1 Objectives
  - 40.2 Simpson's 1/3 and 3/8 Rules:
  - 40.3 Apparatus
  - 40.4 Introduction Fortran Programming Language
  - 40.5 Algorithm of Simpson's 1/3 rd and 3/8 rd Rules
  - 40.6 Glossary
  - 40.7 Viva Questions
  - 40.8 Answers to Viva Questions
- References and Suggested Readings

### 40.1 Objectives

To set a programme in FORTRAN 77 to determine the integration of a given function between given limits using the method of (a) Trapezoidal rule (b) Simpson's 1/3 rule and Simpson's 3/8 rule

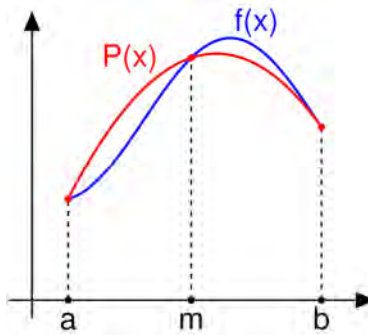
### 40.2 Simpson's 1/3 and 3/8 Rules

#### Simpson's 1/3 Rule

Simpson's 1/3 Rule is used to estimate the value of a definite integral. It is a method for numerical integration. It works by creating an even number of intervals and fitting a parabola in each pair of intervals. Simpson's rule provides the exact result for a quadratic function or parabola.

$$\int_b^a f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + \dots + y_{n-2}) + 4(y_1 + \dots + y_{n-1})]$$

Where 'x' is an equation 'a' refers Upper Limit and 'b' refers Lower Limit.



### Simpson's 3/8 Rule

An odd number of segments with an even number of points formula (use a 3<sup>rd</sup> order polynomial to approximate  $f(x)$ ).

$$\int_a^b f(x) dx = \frac{3}{8}h(f_0 + 3f_1 + 3f_2 + 2f_3 + 3f_4 + 3f_5 + \dots + 2f_{N-3} + 3f_{N-2} + 3f_{N-1} + f_N)$$

### 40.3 Apparatus

256Ram, Windows/Linux Os, Fortran language

### 40.4 Introduction Fortran Programming Language

The Fortran programming language was one of the first (if not the first) "high level" languages developed for computers. It is referred to as a high level language to contrast it with machine language or assembly language which communicate directly with the computer's processor with very primitive instructions. Since all that a computer can really understand are these primitive machine language instructions, a Fortran program must be translated into machine language by a special program called a Fortran compiler before it can be executed. Since the processors in various computers are not all the same, their machine languages are not all the same. For a variety of reasons, not all Fortran compilers are the same. For example, more recent Fortran compilers allow operations not allowed by earlier versions. In this chapter, we will only describe features that one can expect to have available with whatever compiler one may have available. Fortran was initially developed almost exclusively for performing numeric computations (Fortran is an acronym for "Formula Translation"), and a host of other languages (Pascal, Ada, Cobol, C, etc.) have been developed that are more suited to nonnumerical operations such as searching databases for information.

Fortran has managed to adapt itself to the changing nature of computing and has survived, despite repeated predictions of its death. It is still the major language of science and is heavily used in statistical computing. The most standard version of Fortran is referred to as Fortran 77 since it is based on a standard established in 1977. A new standard was developed in 1990 that incorporates some of the useful ideas from other languages but we will restrict ourselves to Fortran 77.

## Significant Language Features

Some of the more significant features of the language are as listed below:

- **Simple to learn** - when FORTRAN was design one of the objectives was to write a language that was easy to learn and understand.
- **Machine Independent** - allows for easy transportation of a program from one machine to another.
- **More natural ways to express mathematical functions** - FORTRAN permits even severely complex mathematical functions to be expressed similarly to regular algebraic notation.
- **Problem orientated language**
- **Remains close to and exploits the available hardware**
- **Efficient execution** - there is only an approximate 20% decrease in efficiency as compared to assembly/machine code.
- **Ability to control storage allocation** -programmers were able to easily control the allocation of storage (although this is considered to be a dangerous practice today, it was quite important some time ago due to limited memory.
- **More freedom in code layout** - unlike assembly/machine language, code does not need to be laid out in rigidly defined columns, (though it still must remain within the parameters of the FORTRAN source code form).

## Areas of Application

FORTTRAN is useful for a wide variety of applications, some of the more outstanding ones are as follows:

- **Number crunching** - due to the more natural (like it's true algebraic form) way of expressing complex mathematical functions and it's quick execution time, FORTRAN is easy and efficient at processing mathematical equations.
- **Scientific, mathematical, statistical, and engineering type procedures** -due to it's rapid number-crunching ability FORTRAN is a good choice for these type of applications.

Basically FORTRAN is most useful for applications that are "*computational-bound*" rather than "*I/O bound*".

## Fortran Program Structure

The basic program structure used in this guide is:

```
PROGRAM A_Program_Name
! Comment explaining the purpose of the program
IMPLICIT NONE
REAL :: Var1, Var2 a declaration part...
INTEGER :: Var3, Var4
Var1 = 0. an initialisation part ...
Var2 = 0.
Var3 = 0.
Var4 = 0.
... some operations ...
PRINT *, some output
END PROGRAM A_Program_Name
```

You are free to indent with spaces and add empty lines as you wish, the aim is to improve the readability of the program source.

## Data Types and Constants

A key component of a program is the use of objects that store data. There are five data types: REAL, INTEGER, COMPLEX, CHARACTER, LOGICAL. Most commonly used in numerical work are type REAL and type INTEGER. In the following example program we have objects named A, V, and Momentum that are

declared to store type real data (numbers with decimal points), and objects named Count, Missed, and Decay, that are declared to store type integer data, and an object named Month declared to store type character data. All these objects are called variables as their values can be changed (varied) during program execution

## Arithmetic Operations

### Operators

The symbols ( ) \* / + - \*\* are used in arithmetic operations. They represent parenthesis, multiplication, division, addition, subtraction and exponentiation, respectively.

### Priority Rules

Arithmetic operations follow the normal priority; proceeding left to right, with exponentiation performed first, followed by multiplication and division, and finally addition and subtraction. Parenthesis can be used to control priority.

### Mixed-mode, and integer operations

If integers and reals are mixed in arithmetic operations the result is a real. Operations involving only reals yield a type real result. Operations involving only integers yield a type integer result. Be especially careful when dividing two integers -the result is truncated to an integer; for example, 3/2 = 1, and 1/2 = 0. This is illustrated in the program below

### Long arithmetic expressions

When writing long arithmetic expressions it can be useful to break them down into constituent parts. For example the expression:

$$Z = ((X^{**2} + 2.*X + 3.)/(5.+Y)^{**0.5}-((15.-.*X^{**3})/Y^{**1.5})^{**0.5}) / (X^{**2} - 4.*X*Y - 5.*X^{**(-0.8)})$$

can be written more clearly (and carefully)as

$$A = (X^{**2} + 2.*X + 3.) / (5.+ Y)^{**0.5}$$

$$B = (15.-77.*X^{**3}) / Y^{**1.5}$$

$$C = X^{**2} - 4.*X*Y - 5.*X^{**(-0.8)}$$

$$Z = (A - B^{**0.5}) / C$$

This is implemented in the program below:

```

PROGRAM Equation
IMPLICIT NONE
REAL :: X = 0.2, Y = 1.9
REAL :: A, B, C, Z
A = (X**2+2.*X+3.) / (5.+Y)**0.5
B = (15.-77.*X**3) / Y**1.5
C = X**2 -4.*X*Y -5.*X**(-0.8)
Z = (A -B**0.5) / C
PRINT *, Z
END PROGRAM Equation

```

## 40.5 Algorithm of Simpson's 1/3 rd and 3/8 rd Rules

### Simpson's 1/3 rd Rule.

1. *derive the formula for Simpson's 1/3 rule of integration,*
2. *use Simpson's 1/3 rule it to solve integrals,*
3. *develop the formula for multiple-segment Simpson's 1/3 rule of integration,*
4. *use multiple-segment Simpson's 1/3 rule of integration to solve integrals.*

### What is integration?

Integration is the process of measuring the area under a function plotted on a graph.

Sometimes, the evaluation of expressions involving these integrals can become daunting, if not indeterminate. For this reason, a wide variety of numerical methods has been developed to simplify the integral. Here, we will discuss Simpson's 1/3 rule of integral approximation, which improves upon the accuracy of the trapezoidal rule.

Here, we will discuss the Simpson's 1/3 rule of approximating integrals of the

form 
$$I = \int_a^b f(x)dx$$

where  $f(x)$  is called the integrand,

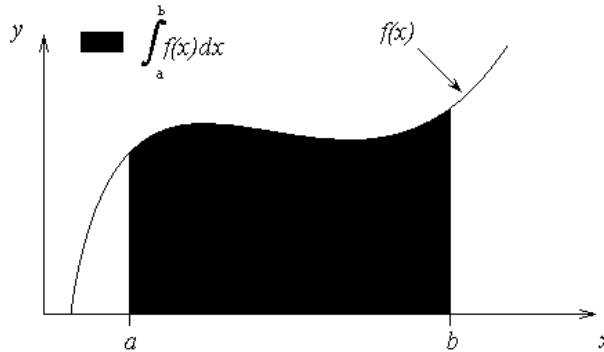
$a$  = lower limit of integration



$b =$  upper limit of integration

### Simpson's 1/3 Rule

The trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial over interval of integration. Simpson's 1/3 rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.



**Figure 1** Integration of a function

#### Method 1:

Hence

$$I = \int_a^b f(x) dx \approx \int_a^b f_2(x) dx$$

where  $f_2(x)$  is a second order polynomial given by

$$f_2(x) = a_0 + a_1x + a_2x^2$$

Since for Simpson 1/3 rule, the interval  $[a, b]$  is broken into 2 segments, the segment width

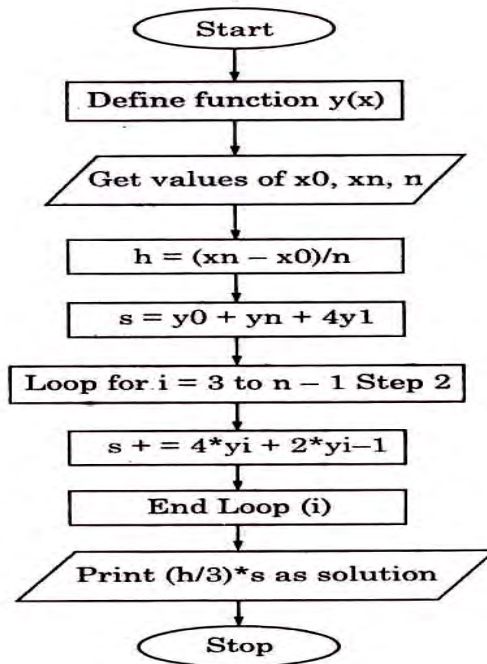
$$h = \frac{b - a}{2}$$

Hence the Simpson's 1/3 rule is given by

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

Since the above form has 1/3 in its formula, it is called Simpson's 1/3 rule.

## Flow Chart of Simpson's 1/3



### Algorithm of Simpson's 1/3 Rd Rule

Step-1. Start of the program.

Step-2. Input Lower limit a

Step-3. Input Upper limit b

Step-4. Input number of subintervals n

Step-5.  $h=(b-a)/n$

Step-6. sum=0

Step-7.  $sum=fun(a)+4*fun(a+h)+fun(b)$

Step-8. for  $i=3; i<n; i += 2$

Step-9.  $sum += 2*fun(a+(i - 1)*h) + 4*fun(a+i*h)$

Step-10. End of loop i

Step-11.  $result=sum*h/3$

Step-12. Print Output result

Step-13. End of Program

Step-14. Start of Section fun

Step-15. temp = 1/(1+(x\*x))

Step-16. Return temp

Step-17. End of Section fun

### Simpson's 3/8 rule

1. derive the formula for Simpson's 3/8 rule of integration,
2. use Simpson's 3/8 rule to solve integrals,
3. develop the formula for multiple-segment Simpson's 3/8 rule of integration,
4. use multiple-segment Simpson's 3/8 rule of integration to solve integrals,

### Introduction

The main objective of this chapter is to develop appropriate formulas for approximating the integral of the form

$$I = \int_a^b f(x)dx \quad (1)$$

Most (if not all) of the developed formulas for integration are based on a simple concept of approximating a given function  $f(x)$  by a simpler function (usually a polynomial function)  $f_i(x)$ , where  $i$  represents the order of the polynomial function. In Chapter 07.03, Simpson's 1/3 rule for integration was derived by approximating the integrand  $f(x)$  with a 2<sup>nd</sup> order (quadratic) polynomial function.  $f_2(x)$

$$f_2(x) = a_0 + a_1x + a_2x^2 \quad (2)$$

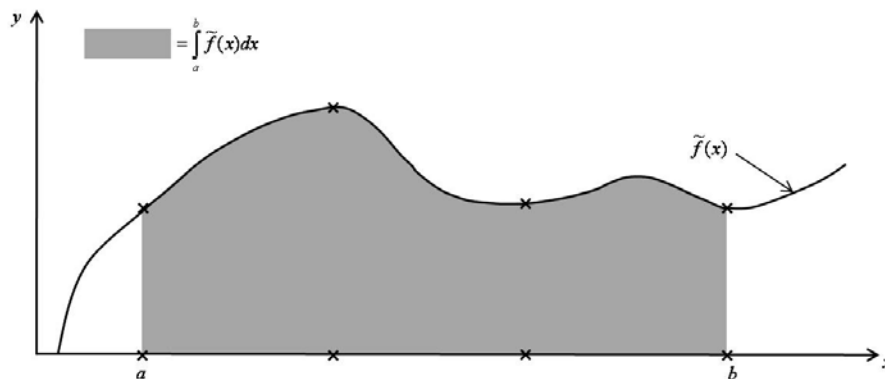


Figure 1  $\tilde{f}(x)$  Cubic function.

In a similar fashion, Simpson 3/8 rule for integration can be derived by approximating the given function  $f(x)$  with the 3<sup>rd</sup> order (cubic) polynomial

$$f_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$= \{1, x, x^2, x^3\} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (3)$$

which can also be symbolically represented in Figure 1.

### Method 1

The unknown coefficients  $a_0, a_1, a_2$  and  $a_3$  in Equation (3) can be obtained by substituting 4 known coordinate data points

$\{x_0, f(x_0)\}, \{x_1, f(x_1)\}, \{x_2, f(x_2)\}$  and  $\{x_3, f(x_3)\}$  into Equation (3) as follows.

$$\begin{cases} f(x_0) = a_0 + a_1x_0 + a_2x_0^2 + a_3x_0^3 \\ f(x_1) = a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 \\ f(x_2) = a_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3 \\ f(x_3) = a_0 + a_1x_3 + a_2x_3^2 + a_3x_3^3 \end{cases} \quad (4)$$

**Equation (4) can be expressed in matrix notation as**

$$I \approx \frac{3h}{8} \times \{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\}$$

### Algorithm Of Simpson's 3/8 th Rule

- Step-1. Start of the program.
- Step-2. Input Lower limit a
- Step-3. Input Upper limit b
- Step-4. Input number of sub intervals n
- Step-5.  $h = (b - a)/n$
- Step-6. sum = 0
- Step-7. sum = fun(a) + fun (b)
- Step-8. for i = 1; i < n; i++

Step-9. if  $i \% 3 = 0$ :  
 Step-10.  $sum += 2 * fun(a + i * h)$   
 Step-11. else:  
 Step-12.  $sum += 3 * fun(a + (i) * h)$   
 Step-13. End of loop i  
 Step-14.  $result = sum * 3 * h / 8$   
 Step-15. Print Output result  
 Step-16. End of Program  
 Step-17. Start of Section fun  
 Step-18.  $temp = 1 / (1 + (x * x))$   
 Step-19. Return temp  
 Step-20. End of section fun

### **Coding Simpson's 1/3**

```

program Simpson_1/3
dimension y(20)
f(t)=1/((t**2+1)*(3*t**2+4))**.5
a=0
b=1
n=6
h=(b-a)/n
s=0
do i=1,n+1
x=a+h*(i-1)
y(i)=f(x)
enddo
do i=1,n,2
s=s+(h/3)*(y(i)+4*y(i+1)+y(i+2))
enddo

```

```
print*, 'Integration by Simpsons 1/3 Rule=', s
end program Simpson_1/3
```

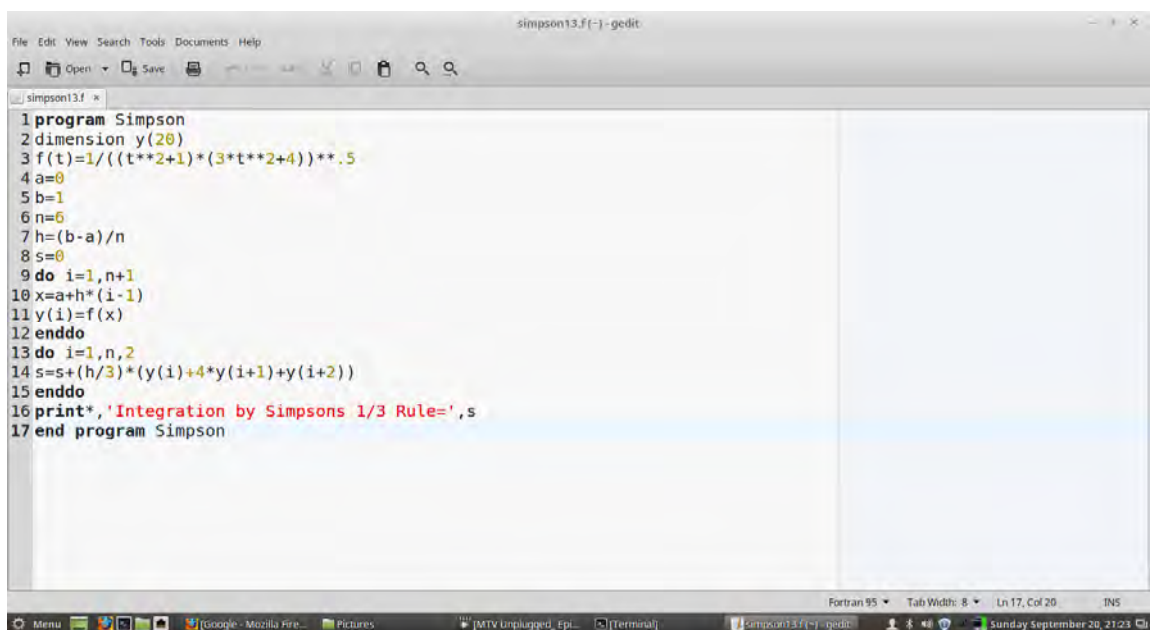
## Screen shot

Step1: Create a File

```
gedit simpson13.f
```

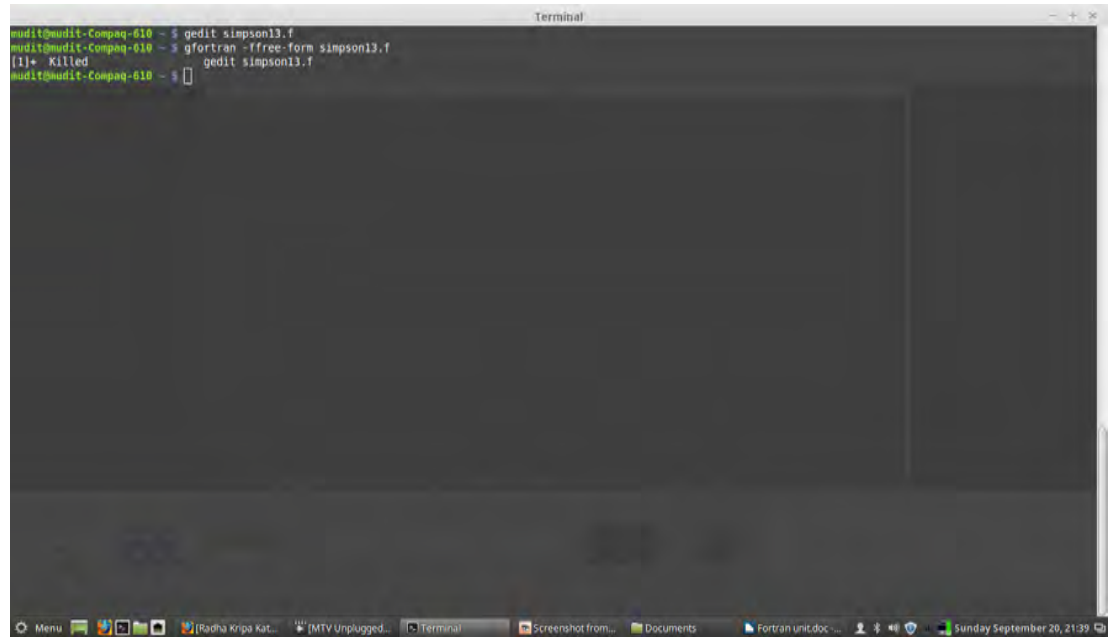


Step 2:  
Write program in editor



### Step3:

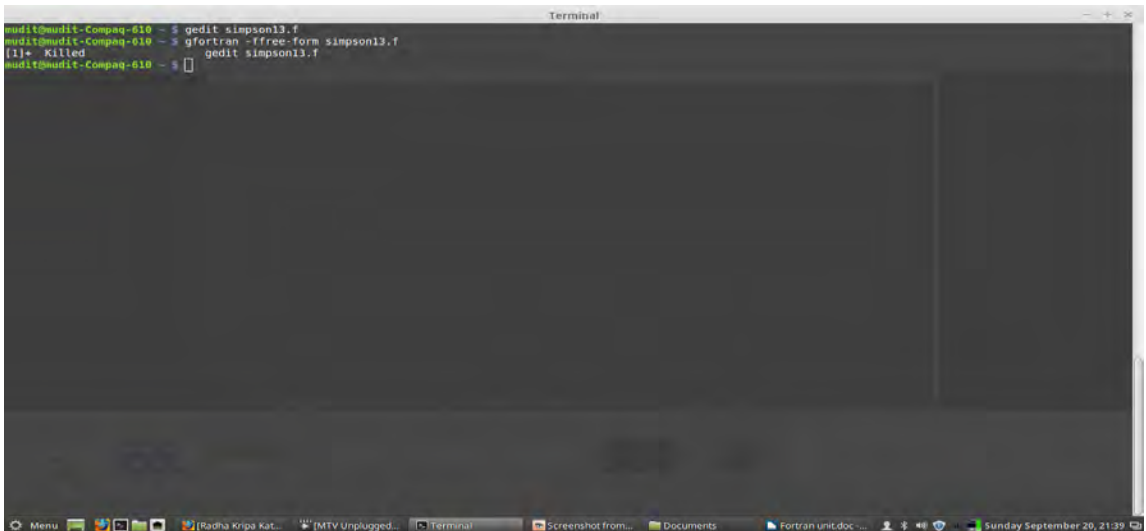
Compile Program: `gfortran -ffree-form simpson13.f`



```
Terminal
audit@audit-Compaq-610 ~ $ gedit simpson13.f
audit@audit-Compaq-610 ~ $ gfortran -ffree-form simpson13.f
[1]- Killed
audit@audit-Compaq-610 ~ $ gedit simpson13.f
audit@audit-Compaq-610 ~ $
```

### Step4:

Run Program: `./a.out`



```
Terminal
audit@audit-Compaq-610 ~ $ gedit simpson13.f
audit@audit-Compaq-610 ~ $ gfortran -ffree-form simpson13.f
[1]- Killed
audit@audit-Compaq-610 ~ $ gedit simpson13.f
audit@audit-Compaq-610 ~ $
```

### Output:

Integration by Simpson's 1/3 Rule= 0.402183354

### Coding Simpson's 3/8

! program Simpson's 3/8 Rule

dimension y(0:10)

```

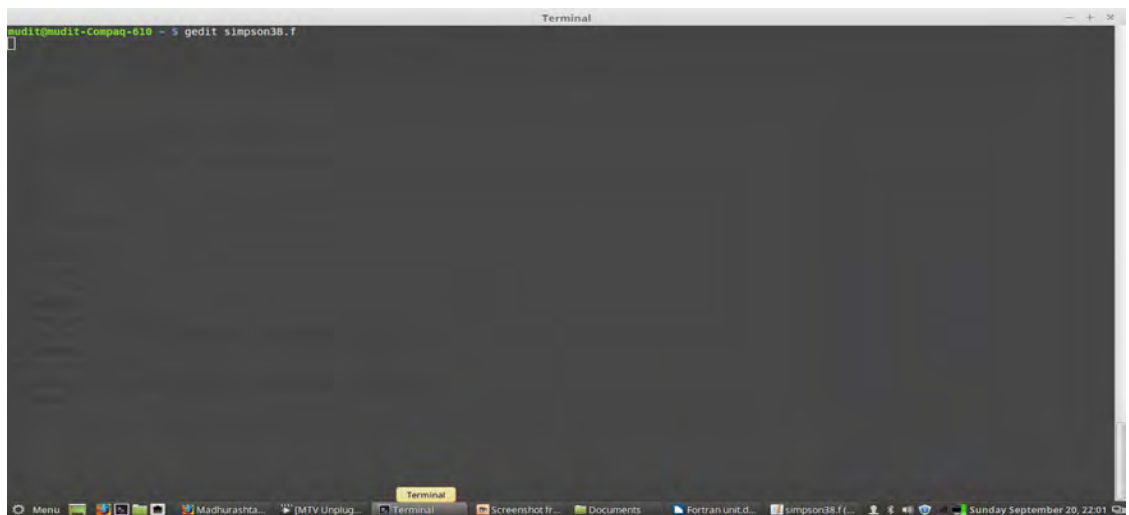
f(t)=1/((t**2+1)*(3*t**2+4))**.5
a=0
b=1
n=6
h=(b-a)/n
s=0
do i=0,n
x=a+h*i
y(i)=f(x)
enddo
do i=1,(n/2-1)
s=s+y(3*i-3)+3*(y(3*i-2)+y(3*i-1))+y(3*i)
enddo
s=(3*h/8)*s
print*, 'Integration by Simpsons 3/8 Rule=',s
end

```

**Screen Shot:-**

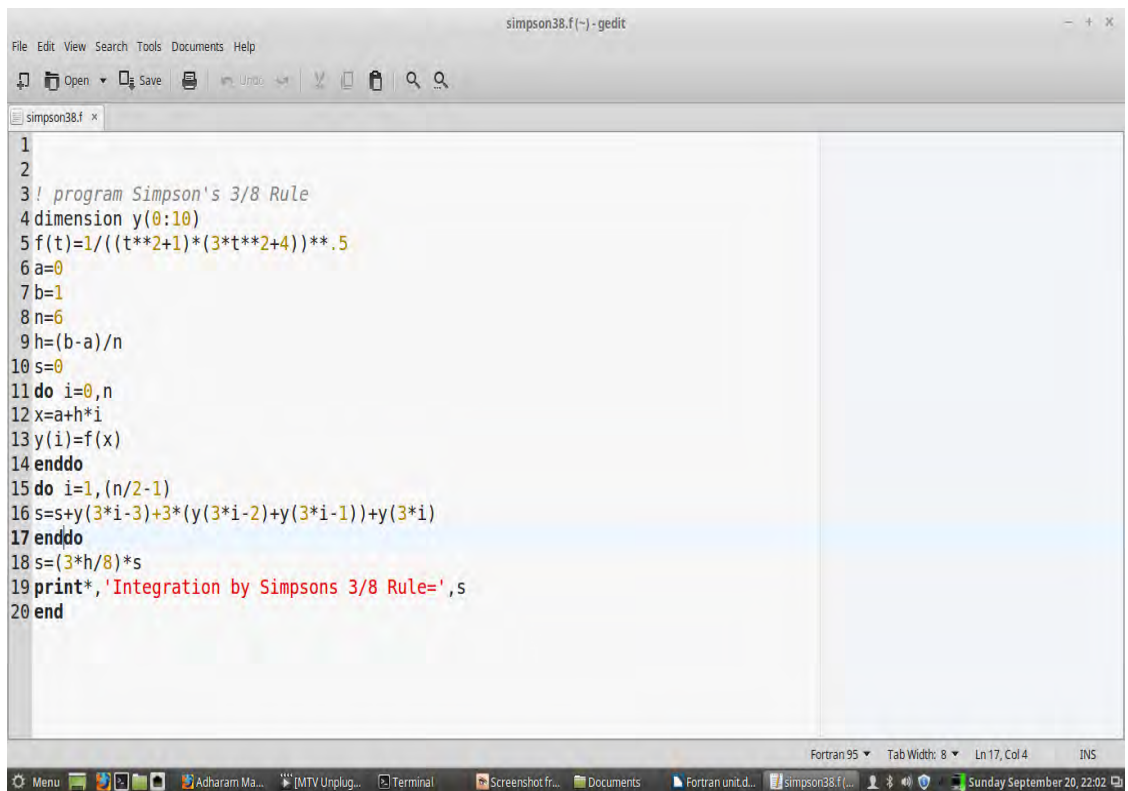
**Step1:**

**Create File: gedit simpsons38.f**





## Step2:- Write Code in editor

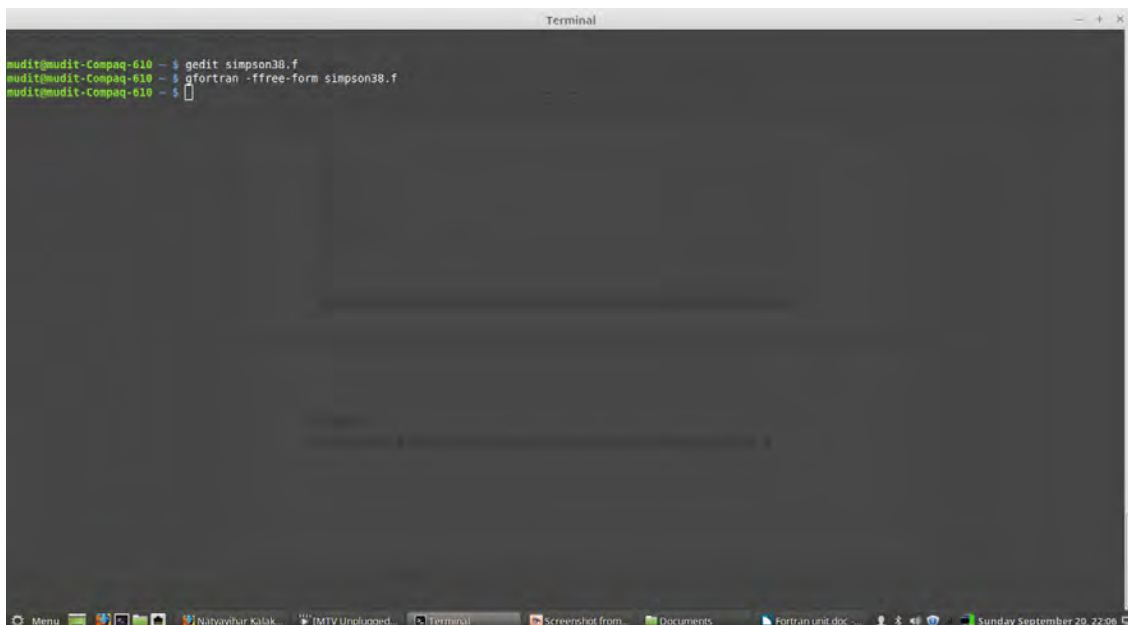


The screenshot shows a gedit editor window titled 'simpson38.f(-) - gedit'. The code is as follows:

```
1
2
3 ! program Simpson's 3/8 Rule
4 dimension y(0:10)
5 f(t)=1/((t**2+1)*(3*t**2+4))**.5
6 a=0
7 b=1
8 n=6
9 h=(b-a)/n
10 s=0
11 do i=0,n
12 x=a+h*i
13 y(i)=f(x)
14 enddo
15 do i=1,(n/2-1)
16 s=s+y(3*i-3)+3*(y(3*i-2)+y(3*i-1))+y(3*i)
17 enddo
18 s=(3*h/8)*s
19 print*, 'Integration by Simpsons 3/8 Rule=',s
20 end
```

The status bar at the bottom indicates 'Fortran 95', 'Tab Width: 8', 'Ln 17, Col 4', and 'INS'. The system tray shows the date and time as 'Sunday September 20, 22:02'.

## Step3: Compile File: gfortran -ffree-form simpson38.f



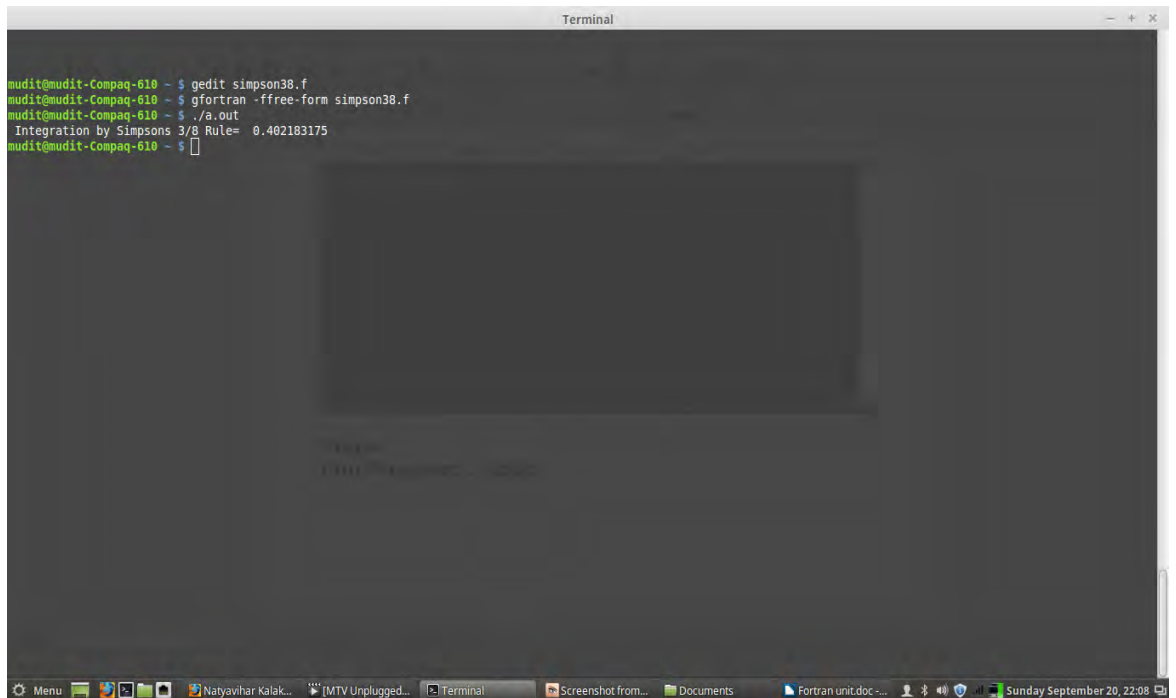
The screenshot shows a terminal window with the following commands and output:

```
audit@audit-Compaq-610 ~$ gedit simpson38.f
audit@audit-Compaq-610 ~$ gfortran -ffree-form simpson38.f
audit@audit-Compaq-610 ~$
```

The terminal window title is 'Terminal'. The system tray at the bottom shows the date and time as 'Sunday September 20, 22:06'.

## Step4:

### Run Program: ./a.out

A terminal window titled "Terminal" showing the execution of a Fortran program. The prompt is "mudit@mudit-Compaq-610 ~". The user enters "gedit simpson38.f", then "gfortran -ffree-form simpson38.f", and finally "./a.out". The output is "Integration by Simpsons 3/8 Rule= 0.402183175". The terminal window has a dark background and a light border. The system tray at the bottom shows various icons and the date "Sunday September 20, 22:08".

```
mudit@mudit-Compaq-610 ~ $ gedit simpson38.f
mudit@mudit-Compaq-610 ~ $ gfortran -ffree-form simpson38.f
mudit@mudit-Compaq-610 ~ $ ./a.out
Integration by Simpsons 3/8 Rule= 0.402183175
mudit@mudit-Compaq-610 ~ $
```

## Output:

Integration by Simpson's 3/8 Rule= 0.402183175

**There are various Fortran tools and libraries. Some are free and some are paid services.**

Following are some free libraries:

- RANDLIB, random number and statistical distribution generators
- BLAS
- EISPACK
- GAMS–NIST Guide to Available Math Software
- Some statistical and other routines from NIST
- LAPACK
- LINPACK

- MINPACK
- MUDPACK
- NCAR Mathematical Library
- The Netlib collection of mathematical software, papers, and databases.
- ODEPACK
- ODERPACK, a set of routines for ranking and ordering.
- Expokit for computing matrix exponentials
- SLATEC
- SPECFUN
- STARPAC
- StatLib statistical library
- TOMS
- Sorting and merging strings

**The following libraries are not free:**

- The NAG Fortran numerical library
- The Visual Numerics IMSL library
- Numerical Recipes

## 40.6 Glossary

**actual argument:** A value (a variable, expression, or procedure) passed from a calling program unit to a subprogram.

**alphanumeric:** Pertaining to letters and digits.

**alternate key:** On OpenVMS systems, an optional key within the data records in an indexed file, which can be used to build an alternate index.

**alternate return:** A subroutine argument that permits control to branch immediately to some position other than the statement following the call. The actual argument in an alternate return is the statement label to which control should be transferred.

**ANSI :**The American National Standards Institute. An organization through which

accredited organizations create and maintain voluntary industry standards.

**argument** : See actual argument and dummy argument.

**argument association**: The relationship (or "matching up") between an actual argument and dummy argument during the execution of a procedure reference.

**argument keyword**: The name of a dummy (formal) argument. The name is used in a subprogram (routine) definition. It can also be used when the subprogram is invoked to associate dummy arguments with actual arguments, so that they can appear in any order.

Argument keywords are supplied for many of the intrinsic procedures.

**array** : A set of scalar data that all have the same type and kind parameters. An array can be referenced by element (using a subscript), by section (using a section subscript list), or as a whole. An array has a rank (up to 7), bounds, size, and a shape. *Contrast with* scalar. *See also* bounds, conformable, shape, size, whole array, and zero-sized array.

**array element** : A scalar item in an array. An array element is identified by the array name followed by one or more subscripts in parentheses, indicating the element's position in the array. For example, B(3) or A(2,5).

**ASCII**: The American Standard Code for Information Interchange. A 7-bit character encoding scheme associating an integer from 0 through 127 with 128 characters. *See also* multinational character set.

**association** : The relationship that allows an entity to be referenced by different names in one scoping unit or by the same or different names in more than one scoping unit. The principal kinds of association are argument association, host association, pointer association, storage association, and use association.

**common block** (n.) a block of physical storage that may be accessed by any of the scoping units in an executable program

**conformable** (adj.) two arrays are said to be conformable if they have the same shape. A scalar is conformable with any array.

**construct** (n.) a sequence of statements starting with a CASE, DO, IF or WHERE statement and ending with the corresponding terminal statement.

**data** (n.) plural of datum - an entity of information.

**datum** (n.) a single quantity that may have any of the set of values specified in its data type.

**definable** (adj.) a variable is definable if its value may be changed by the appearance of its name or designator on the left of an assignment statement. An allocatable array that has not been allocated is an example of a data object that is **not** definable. An example of a subobject that is not definable is C(I) when C is an array that is a constant and I is an integer variable.

**deleted feature** (n.) a feature in Fortran 77 that is considered to be redundant and largely unused. No features from Fortran 77 were deleted in arriving at the Fortran 90 standard. In principle a feature designated as obsolescent in a standard may become a deleted feature in the subsequent revision of the standard. In practice, KAH doubts any features will ever be actually deleted - such is the clout of existing codes.

**disassociated** (adj.) a pointer is disassociated following execution of a DEALLOCATE or NULLIFY statement.

**dummy argument** (n.) an entity whose name appears in the parenthesized list following the procedure name in a FUNCTION statement, a SUBROUTINE statement, an ENTRY statement, or a statement function statement.

**entity** (n.) the term used for any of the following: a program unit; a procedure; an operator; an interface block; a common block; an external unit; a statement function; a type; a named variable; an expression; a component of a structure; a named constant; a statement label; a construct; or a NAMELIST group.

**executable construct** (n.) a CASE, DO, IF or WHERE construct in Fortran 90.

**explicit interface** (n.) for a procedure referenced in a scoping unit, the property of being a module procedure, an intrinsic procedure, an external procedure that has an interface block or is defined by the scoping unit and is recursive, or a dummy procedure that has an interface block.

**external file** (n.) a sequence of records that is stored on a medium external to the executable program.

**file** (n.) an internal file or an external file.

**function** (n.) a procedure that is invoked in an expression.

**generic identifier** (n.) a name, operator or assignment token specified in an INTERFACE statement to provide an alternative means of invoking any of the procedures in the interface block.

**global entity** (n.) an entity identified by a lexical token whose scope is an executable program. It may be a program unit, a common block, or an external procedure.

**host** (n.) a main program or subprogram that contains an internal procedure is called the host of the internal procedure. A module that contains a module procedure is called the host of the module procedure.

**implicit interface** (n.) a procedure referenced in a scoping unit is said to have an implicit interface if the procedure does not have an explicit interface there.

**intent** (n.) a property of a dummy argument that is neither a procedure nor a pointer, indicating whether it is intended to transfer data into the procedure, out of the procedure, or both.

## 40.7 Viva Questions

- Q.1** Explain what is FORTRAN? What is the structure of the main program?
- Q.2** What makes Fortran a suitable language for scientific purpose?
- Q.3** Explain how recursion is used in Fortran?
- Q.4** Mention what are the rules involved in debugging?
- Q.5** What are the advantages that can be taken in the case of strength reduction in FORTRAN?
- Q.6** What is the method of Dead store elimination? - FORTRAN
- Q.7** What are the optimization techniques used for code optimization? - FORTRAN
- Q.8** What are the common pitfalls used for the Data-types? - FORTRAN
- Q.9** What are the rules involved in debugging? - FORTRAN
- Q.10** What is the procedure to test a program in FORTRAN?
- Q.11** What are the methods used to correct the syntax errors in FORTRAN?

- Q.12** What are the options provided by the compiler during programming? -  
FORTRAN
- Q.13** Write a program that shows the dynamic memory allocation? - FORTRAN
- Q.14** What are the features involved in FORTRAN 2008?
- Q.15** What are the features added in FORTRAN 90?
- Q.16** What are the changes being made in FORTRAN 77?
- Q.17** Write a simple program in FORTRAN II to calculate the Area of the Triangle?
- Q.18** What are the main features of FORTRAN II?
- Q.19** How is recursion used in FORTRAN?
- Q.20** What are the functionalities performed by Frequency statements? -  
FORTRAN
- Q.21** What are the different types of statements used in FORTRAN?
- Q.22** What is the use of adding underscore suffix? - FORTRAN
- Q.23** What are the conditions that are required for the underscores to be used? -  
FORTRAN
- Q.24** What is the use of Array storage order in FORTRAN?

## 40.8 Answers to Viva Questions

**Ans.1:** Fortran is referred as a third-generation programming language, built for numeric computation and scientific computing.

The main structure of the program

- Programname
- Declarations
- Statements
- Stop end

**Ans.2:** Built in support for arguments in subroutines

- Available built in support for complex numbers
- Rich set of intrinsic functions
- Support for array notation enabling operations on array sections

- For memory pointers strong aliasing rules, resulting in more efficient code after compilation

**Ans.3:** When one function calls to another function or itself, in a defined area of call is called recursion.

- It is denoted as subroutine as well, and it has the scope of the stack that directly supports the subroutines
- The return location on the stack that uses the subroutine is fixed and is adjacent to the subroutine code
- To call a function within a function recursion is used, and it allows easy calling of the function and minimizes writing of the code

**Ans.4:** For debugging the rules involved are

- Code modification is not done until not sure about the bug by checking all the conditions and criteria
- Modular program is being used which defines the task, and it is placed in the debug statements that can ISOLATE some of the functioning of the code
- Combinations of inputs are carried out to get the desired output if strange results are noticed
- Use the tools provided with the debugger

## References and Suggested Readings

1. Ian C and Malcon C, Interactive FORTRAN 77, Affiliated East West Press Pvt Ltd, New Delhi
2. Rajaraman V, Computer Programming in FORTRAN 77, Prentice-Hall of India Pvt Ltd, New Delhi
3. Stephen J Chapman: Introduction to Fortran 90/95, McGraw-Hill, Boston 1998. ISBN 0-07-011969-4.
4. Thomas F. Coleman and Charles Van Loan: Handbook for Matrix Computations, Frontiers in Applied Mathematics, Vol. 4, SIAM, Philadelphia 1988. ISBN 0-89871-227-0.