MCA 09 Discrete Mathematics

SET: 1

Section-A

(Very Short Answer Questions)

1 (i) What are finite and infinite sets, explain with example?
(ii) Explain partition of set, with suitable example.
(iii) What do you mean by into function?
(iv) Construct truth table of negation.
(v) Write Demorgan's law.
(vi) Simplify the Boolean function
a + ab' + a'b
(vii) Give statement for generalized pigeonhole principle.
(viii) What is COSET?
(ix) 'Every tree is a graph, but every graph is not necessarily a tree'. Specify condition(s), when a graph can also be called as tree.
(x) Specify number of edges in tree having n vertices.

Section-B

(Short Answer Questions)

2. In a survey of 100 students it is found that 40 read Readers' Digest, 32 read India Today, 26 read the Outlook, 10 read both Readers' Digest and India Today, 6 read India Today and the Outlook, 7 read Readers' Digest and the Outlook and 5 read all the three. How many read none of the magazines?

3. Let Z be the set of integers, and n be any fixed positive integer.
Let a, b ∈ Z
a is said to be congruent to b modulo n if and only if a–b is divisible by n.
Symbolically, we can write
$a \equiv b \pmod{n}$
Show that the relation 'congruence modulo n' is an equivalence relation on Z.

4. Explain Identity function and Constant function, with suitable example and diagram.

5. Write down
(a) the converse (b) the opposite (c) the contrapositive of the implication:
If a quadrilateral ABCD is a square, then all the sides of quadrilateral ABCD are equal. Write down the truth value of each resulting statement.

6. A polygon has 44 diagonals. Find the number of sides, for that polygon.

7. Show that following statements are logically valid

(a) $(\forall x)(P(x) \wedge Q(x)) \leftrightarrow ((\forall x)(P(x) \wedge (\forall x)(Q(x))$
(b) $(\exists x)(P(x) \wedge Q(x)) \rightarrow ((\exists x)P(x) \wedge (\exists x)Q(x))$

8. Solve following
   (a) In how many ways can 5 gents and 4 ladies can sit around a round table if no two ladies are allowed to sit together?
   (b) How many four digit even numbers can be formed with the digits 1, 2, 3 and 4?

9. Explain with suitable example, bipartite graph and complete bipartite graph.

Section-C

(Long Answer Questions)

10. Explain following operations on set(s), with the help of Venn diagram and example
   a) Union b) Intersection c) Difference d) Complement
   Also specify various identities associated with these operations.

11. Solve following

   a) Let $f : Z \times Z \rightarrow Z$, $f(a, b) = ab$, $(a, b) \in Z \times Z$
      Examine if f is (i) injective, (ii) surjective.
   b) Let A, B be two non-empty sets.
      Let $f : A \times B \rightarrow B \times A$, $f(a, b) = (b, a)$; $(a, b) \in A \times B$
      Show that f is bijective.
   c) Let $A = R-\{3\}$, $B = R-\{1\}$, show that function $f: A \rightarrow B$, such that
      $f(x) = (x-4)/(x-3)$ is one-one onto.
   d) Let $A = \{-2, 1, 3, 4\}$. A function $f: A \rightarrow A$ is defined such that
      $f(x) = x^2 - 2x + 2$
      Find range of f(x) and pre-image of 5.

12. How many combinations of the letters of the word 'NUMBERS' taken 3 at a time
   i) contain M? ii) do not contain M?
   iii) contain M and R iv) contain neither M nor R?

13. Explain following terms related to graph, with suitable example and diagram
   (a) Edge Connectivity (b) Vertex Connectivity (c) Adjacency Matrix
   (d) Incidence Matrix (e) Complete Graph (f) Regular Graph

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SET: 2

Section-A

(Very Short Answer Questions)

1 (i) Is set A = {1, 2, 3}, then find P^A.
(ii) What is meant by anti-symmetric relation?
(iii) When a function can be called as bijective?
(iv) Construct the truth table for P ^ (~P).
(v) Write the symbolic form of following
He is ambitious or no one is ambitious.
(vi) In Boolean algebra, state Involution law.
(vii) What is value of ${}^nP_r$ and ${}^nC_r$.
(viii) Explain Boolean Ring.
(ix) What is meant by eccentricity of vertex, in a graph.
(x) What do you mean by Complete Graph?

Section-B

(Short Answer Questions)

2. Describe following terms, with suitable example
a) Equivalent Sets b) Equal Sets c) Null Set d) Singleton Set

3. Explain Identity relation and Inverse relation, with suitable example.

4.' Every function is a relation, but every relation is not a function.' Justify the statement with suitable example.

5. Determine whether the conclusion C follows logically from the premises H1 and H2.
(a) H1: P →Q H2: P C: Q
(b) H1: P →Q H2: ~P C: Q
(c) H1: P → Q H2: ~ (P ^ Q) C: ~P
(d) H1: ~P H2: P↔Q C: ~ (P ^ Q)
(e) H1: P →Q H2: Q C: P

6. (a) For all a , b , c in a Boolean algebra B, prove that
(i) (a + c= b + c and a + c' = b + c') ⇒a =b
(ii) (ac = bc and ac' =bc')⇒ a=b.
(b) Write the dual of each of the following Boolean equations:
(i) a (a' +b) =ab.
(ii) (a +1) (a+0) = a
(iii) (a +b) (b+c) = ac + b

7. If R is a ring such that $a^2 = a$, for all $a \in R$. Prove that
   a) $a + b = 0 \Rightarrow a = b$, for all $a, b \in R$
   b) R is commutative ring

8. Laxman has 4 shirts viz Black, White, Blue and Purple. Also he has 3 pants: Blue, black and Brown. In how many ways can Laxman get dressed up using all possible combinations of shirt and pants, write all possible combinations?

9. Explain the following terms (with respect to graph), with suitable example and diagram:
   (a) Open Walk (b) Closed Walk
   (c) Path (d) Circuit

Section-C

(Long Answer Questions)

10. Let $\approx$ be a relation on $A \times A$ defined as follows
   $(a,b) \approx (c,d)$ whenever $ad=bc$
   where A is set of non zero integers. Determine whether $\approx$ is an equivalence relation or not.

11. Let $D_6 = \{1, 2, 3, 6\}$. Define ' + ', ' . ', and ' ' ' in D_6 by
   a + b = L C M of a and b
   a . b = G C D of a and b
   $a' = \frac{6}{a}$ for all $a, b \in D_6$
   Show that (D_6, ' + ', ' . ', ' ' ') is a Boolean algebra.

12. Solve the following
   (a) Show that the set
   $F = \{a + b\sqrt{2}: a, b \in Q\}$
   is a field.
   (b) Prove following theorem
   'The intersection of any two normal subgroup of a group is again a normal subgroup of the group.'

13. Explain EULERIAN and HAMILTONIAN GRAPH, with suitable diagram and example.