Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final)
Paper Code:MT-09
Integral Transforms and Integral Equations
Section – A
(Very Short Answers Questions)

1. Define Error function and complementary Error function.
2. Find the Laplace transform of $(t + 2)^2 e^t$
3. Find $L[cesh^2 \mu t\ ; p]$
4. State Existence conditions of Laplace Transform.
5. Define Laplace transform and write the formula.
6. Define Integral transform and give the formula.
7. Define Null function & give an example.
8. Write the change of scale property for Inverse laplace transform.
9. Evaluate : $L^{-1}\left\{\frac{1}{(p-4)^5} + \frac{5}{(p-2)^2+5^2} + \frac{P+3}{(p+3)^2+6^2}\right\}$
10. Evaluate : $L^{-1}\left[\frac{1}{p^{n+1}}\ ; t\right]$
11. Define the convolution of two functions.
12. Write Dirchlet's conditions.
13. If u(x, t) is a function of two independent variables for $a \le x \le b, t > 0$, then under suitable restrictions on $u = u(x, t)$ show that
    $L\frac{\partial u}{\partial t} = p\bar{u}\ (x, p) - \ u(x, 0)$
14. What is a boundary value problem?
15. Write two dimensional heat conduction equation?
16. Write 3-dimensional wave equation?
17. Find $L(x, y'' + \ (x-1)y' - y\ ;\ p)$ with $y(0)\ =\ 5, y(\infty) = 0$.
18. Solve : $L\ \left\{\frac{\partial^2 u}{\partial t^2}; p\right\}$; $u(x, 0) =\ 0, u_t\ (x, o) =\ 5$
19. Define complex fourier transform and Inverse fourier transform.
20. Define fourier sine transfom and inverse fourier sine transform.
21. Define fourier cosine transform and Inverse fouries cosine transform.
22. Derive the relationship between fourier transform and Laplace transform.
23. What is convolution theorem for fourier transform?
24. State the Parseval's Identity for fourier transform.
25. State the mellin inversion theorem.
26. State the convolution theorem for Mellintransfom.
27. Define Inverse Mellin transform.
28. Show that $M\{x^m f(ax^n); p\} = \frac{1}{n} a^{-(p+m)/n}$
    $f\left(\frac{p+m}{n}\right), (a > 0) Here\ M\ \{f(x); p\} = f(p)$
29. If $M\{f(x); p\} =\ f(p)$ then show that $M\{\log x\ f(x)\} = \frac{d}{dp} f(p)$
30. Define Mellin Transform of $f(x)$.
31. Define Hankel Transform.

32. Define Bessel function of first kind & show that $xJ_v'(x) = vJ_v(x) - xJ_{vH}(x)$
33. Write change of scale property for Hankel transform.
34. State the relation between Hankel and Laplace transform.
35. State inversion formula for the Hankel transform.
36. State Parseval's theorem for Hankel transform.
37. If we want to remove the term $\frac{\partial^2 U}{\partial x^2}$ from a p.d.e. then at $x = 0$ what type of condition is required in case of (i) fourier cosine transform and (ii) fourier sine transform.
38. If the differential equation ranges from -∞ to ∞ then which type of fouries transform can be used to solve a boundary value problem?
39. If Hankel transform of zero order is applied w.r. to variable r to p.d.e.

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = 0, \qquad o \le r \le \infty, z \ge 0$$

Then $u(p, z) = ?$

Where $u(p, z) = \int_0^\infty U(r, z) r\, J_0(pr) dr$

40. Derive the formula for $fs\left\{\frac{\partial^2 U}{\partial x^2}\right\}, where\ U = U\ (x, t)$.
41. Find fourier cosine transform of $\frac{\partial^2 U}{\partial x^2}$
42. Write the Hankel transform of the derivative $f'(x)$ of function $f(x)$ where $f'(x) = \frac{df}{dx}\ if\ F_v)p) = H_v\{f(x); p\}$.
43. Define Integral Equation.
44. What is the difference between Linear and Non-linear Integral equation.
45. Define the term singular Integral equation.
46. Define volterra Integral equation of first and second kinds.
47. Define fredhilm integral equation of first nad second kinds.
48. What is the integral equaton of convolution type?
49. Define the terms :
    (i) Separable or Degenerate kernel.
    (ii) Symmetric kernel of an integral equation.
50. Show that the function $g(x) = e^x\left(2x - \frac{2}{3}\right)$ is a solution of the fredholn equation.

$g(x) + 2\int_0^1 e^{x-t}\, g(t) dt = 2x\, e^x$

51. Define eigen values of a kernel in the integral equation.
52. Define eigen values of a kernel in the integral equation.
53. State whether the following statements are true or false.
    (i) The eigenfunctions of a symmetric kernel, corresponding to different eigenvalues are not orthogonal.
    (ii) The eigen values of a symmetric kernel are real.
    (iii) If ϕ(x) is an eigen function, then Cϕ(x) is also an eigen function corresponding to same eigen value.
    (iv) An integral of the type $\int_{-\infty}^{\infty} K(p, t) f(t) dt$ is defined as the integral transform of f(t) provided it is divergent.
    (v) The resolvent kernel of te non homogeneous integral equation cannot be determined by the method of integral transform.

(vi) The convolution of two functions G(t) and H(t), where $-\infty < t < \infty$ is denoted and derived by $G * H = \int_{-\infty}^{\infty} G(t)H\,(x-t)dt$

54. Solve the integral equation:

$$\sin x = \int_0^x J_0(x-t)g(t)dt$$

55. Define the Abel integral equation.
56. Define Intergo-differential equation.
57. Define the term "separable kernel".
58. Define the term "Orthogonal function".
59. State whether the following statements are true or false :
    (i) If the sum of infinite series occurring in the formula of resilvent kernel cannot be determined, then in such cases we may use the method of successive approximation.
    (ii) The nth approximate solution $g_n(x)$ of fredholm integra; equation of second kind $g(x) = f(x) + \int_a^b k_m(x,t)g(t)dt$ is obtained by $g_n(x) = f(x) + \sum_{m=1}^{n} \lambda^m \int_a^b k_m(x,t)f(t)dt$
    (iii) The series $g(x) = f(x) + \sum_{m=1}^{\infty} \int_a^b k_m(x,t)g\,(t)dt$ is known as Neumann series.
    (iv) The iterated kernel of the function $k(x,t) = e^x \cos t, a = 0, b = r$ does not exist.
60. State Regularity conditions.
61. Define the Iterated kernel
62. Define the Resvent kernel
63. Define the Neumann series
64. Define the inner or Scalar product of two functions.
65. Define complex Hilbert space.
66. Define orthogonal system of functions.
67. Define the term "Orthonormal set".
68. Define Schwarz inequality.
69. State Hilbert-Schmidt Theorem
70. State True of false:
    (i) Fredholm's first theorem hold when λ is a root of the equation $D(\lambda) = 0$
    (ii) The resolvent kernel $R(x,t\,;\lambda)$ satisfies the following relations:
    $$R(x,t\,;\lambda) = k(x,t) + \lambda \int_a^b k(x,z)R\,(z,t;\lambda)dz$$
    (iii) The series $D(\lambda)$ is an absolutely and uniformly converging power series in λ.
    (iv) The series $D(x,t\,;\,\lambda)$ is not absolutely but uniformly converging power series in λ.

Define the following terms ☹71-72)

71. Fredholm determinant
72. Fredholm minor
73. State Fredholm's first fundamental theorem.
74. State first and second series for non-homogeneous fredholm integral equation of second kind.