Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final) Question Bank-2015
Paper Code:MT-07
Section –A(Very Short Answer Questions)

Q.1 Define the function of class A.

Q.2 If $L[f(t)] = \bar{f}(p)$ then $[f(at)] =$ _______________ .
Ans: $\frac{1}{a}\bar{f}\left(\frac{p}{a}\right)$

Q.3 If $L[f(t)] = \bar{f}(p)$ then $[e^{at}f(t)] =$ _____________ .
Where a is real or complex number.
Ans: $\bar{f}(p-a)$

Q.4 If $L[f(t)] = \bar{f}(p)$ then $[t^n f(t)] =$ _____________ .
Ans: $(-1)^n \frac{d^n}{dp^n}\bar{f}(p)$

Q.5 If $L[f(t)] = \bar{f}(p)$ then $\left[\frac{f(t)}{t}\right] =$ _____________ .
Provided that the integral exist.
Ans: $\int_p^\infty \bar{f}(u)du$

Q.6 If $L^{-1}[\bar{f}(p)] = f(t)$ then $L^{-1}[\bar{f}(p-a)] =$ ____________ .
Ans: $e^{at}f(t)$

Q.7 Find $L^{-1}\left[\frac{1}{(p+a)^n}\right]$
Ans: $e^{-at}\frac{t^{n-1}}{(n-1)!}$

Q.8 If $\bar{f}(p)$ and $\bar{g}(p)$ are fourier transform of $f(x)$ and $g(x)$ respectively then $f[af(x) + bg(x)] =$ _____________________ .
Ans: $a\bar{f}(p) + b\bar{g}(p)$

Q.9 If $F[f(x)] = \bar{f}(p)$ then $[f(x-a)] =$ __________________ .
Ans: $e^{ipa}\bar{f}(p)$

Q.10 If $M[f(x)] = f^*(p)$ then $[f(ax)] =$ ___________________ .
Ans: $a^{-p}f^*(p)$

Q,11 If $M[f(x)] = f^*(p)$ then $M[x^0 f(x)] =$ ___________________ .
Ans: $f^*(p+a)$

Q.12 If $M[f(x)] = f^*(p)$ then $\left[\frac{1}{X}f\left(\frac{1}{X}\right)\right] =$ ___________________ .
Ans: $f^*(1-p)$

Q.13 If $H[f(x)] = \bar{f}(p)$ then $[f(ax)] =$ ___________________ .
Where a being a constant.
Ans: $\frac{1}{a^2}\bar{f}\left(\frac{p}{a}\right)$

Q.14 If If $M[f(x)] = f^*(p)$ then $[x^a] =$ ___________________ .
Ans: $a^{-1}f^*\left(\frac{p}{a}\right), a > 0$

Q.15 If $M[f(x)] = f^*(p)$ then $[logxf(x)] =$ ___________________ .

Ans: $\frac{d}{dp} f^*(p)$

Q.16 Find $M[e^{-x}]$

Ans: $\Gamma(p)$

Find $L[\sin t \cos t]$.

Ans: $\frac{1}{p^2+4}$, $p > 0$

Q.17 Evaluate $L[2t^2 + 6t + 8]$

Ans: $\frac{12}{p^4} + \frac{6}{p^2} + \frac{8}{p}$, $p > 0$

Q.18 Find $L^{-1}\left[\frac{1}{p^{7/2}}\right]$

Ans: $\frac{8}{15}\sqrt{\frac{t}{\pi}}$

Q.19 Find $L^{-1}\left[\frac{1}{\sqrt{p}}\right]$

Ans: $\frac{1}{\sqrt{\pi t}}$

Q.20 Find the Hankel transform of e^{-x} taking $xJ_0(px)$ as the kernel of the transform.

Ans: $(1+p^2)^{-3/2}$

Q.21 Find the Hankel transform of e^{-ax} taking $xJ_0(px)$ as the kernel of the transform.

Ans: $a(a^2+p^2)^{-3/2}$

Q.22 Find $M\{e^{-x}\}$

Ans: $\Gamma(p)$, $Re(p) > 0$

Q.23 If $\overline{f_s}(p)$ is the Fourier Sine transform of $f(x)$ then Fourier Sine transform of $f(ax)$ is

Ans: $\frac{1}{a}\bar{f_s}\left(\frac{p}{a}\right)$

Section –B(Short Answer Questions)

Q.1 Solve $g(x) = e^x + \lambda \int_0^{10} xtg(t)\, dt$

Ans: $g(x) = e^x + \frac{3\lambda x}{3-10^3\lambda} = (1+9e^{10})$

Q.2 Find $L\left[\frac{\cos hat}{\sqrt{t}}\right]$

Ans: $\frac{\sqrt{\pi}}{2}\left[\frac{1}{(p-a)^{1/2}} + \frac{1}{(p+a)^{1/2}}\right]$ See MT-09, p. 12

Q.3 Find $L[\sin^2 3t]$.

Ans: $\frac{18}{p(p^2+36)}$

Q.4 Find $L[\sin kt \sin hkt]$.

$\frac{2k^2p}{p^4+4k^4}$

Q.5 Find the Laplace transform of $f(t)$ where

$$f(t) = \begin{cases} 0, 0 < t < 1 \\ t, 1 < t < 2 \\ 0, \quad t > 2 \end{cases}$$

Ans: See JPH (Integral transform) p.15

Q.6 If $L[f(t)] = \frac{p^2-p+1}{(2p+1)^2(p-1)}$, show that $L[f(2t)] = \frac{p^2-2p+4}{4(p+1)^2(p-2)}$

Ans: See JPH (Integral transform) p.20

Q.7 If $L\left[2\sqrt{\frac{t}{\pi}}\right]=\frac{1}{p^{3/2}}$ then show that $L\left[\frac{1}{\sqrt{(pt)}}\right]=\frac{1}{p^{1/2}}$

Ans: See JPH (Integral transform) p.27

Q.8 Show that $L[E_i(t)]=\frac{1}{p}log(1+p)$ where $E_i(t)$ is exponential integral defined as $E_i(t)=\int_t^{\infty}\frac{e^{-u}}{u}du$.

Ans: See JPH (Integral transform) p.42

Q.9 Find the Hankel transform of order zero of $1/_x$ and then apply the inversion formula to get the original function.

Ans: See JPH (Integral transform) p.217

Q.10 Show that $L[logt]=\frac{\Gamma'(1)-logp}{p}$

Ans: See JPH (Integral transform) p. 51

Q.11 Find $L^{-1}\left[\frac{3}{p^2-3}+\frac{6-30\sqrt{p}}{p^4}+\frac{2}{2p-3}-\frac{3+4p}{9p^2-16}\right]$

Ans: See JPH (Integral transform) p. 56

Q.12 Find $L^{-1}\left[\frac{(p+1)e^{-\pi p}}{p^2+p+1}\right]$

Ans: See JPH (Integral transform) p. 63

Q.13 Find $L^{-1}\left[\frac{3p+7}{p^2-2p-3}\right]$

Ans: See JPH (Integral transform) p. 60

Q.14 Show that 1*1*1*------------ *1 (n times) = $\frac{t^{n-1}}{(n-1)!}$ Where n=1,2,3,------------

Ans: See JPH (Integral transform) p. 80

Q.15 If $\bar{f}(p)$ is the complex fourier transform of $f(x)$, then the fourier transform of $f(x)\cos ax$ is $\frac{1}{2}\left[\bar{f}(p-a)+\bar{f}(p+a)\right]$

Ans: See JPH (Integral transform) p. 160

Q.16 If $M[f(x)]=\bar{f}(p)$ then show that $M\left[\int_0^x f(u)du\right]=-\frac{1}{p}\bar{f}(p+1)$

Ans: See JPH ( ) p. 151

Q.17 Find $H^{-1}\left[\frac{e^{-ap}}{p}\right]$ when n=1

Ans: See JPH (Integral transform) p. 217

Find $L^{-1}\left\{\frac{2p^2-4}{(p+1)(p-2)(p-3)}\right\}$

Ans: $-\frac{1}{6}e^{-t}-\frac{4}{3}e^{2t}+\frac{7}{2}e^{3t}$

Q.18 Find the Fourier Sine and Cosine transform of:

$$f(x)=\begin{cases} x & for\ 0<x\le 1 \\ 2-x & for\ 1<x<2 \\ 0 & for\ x\ge 2 \end{cases}$$

Ans: $2\sqrt{\frac{2}{\pi}}\frac{cosp}{p^2}(1-\cos p)$

Q.19 Find the Hankel transform of :

$$f(x) = \begin{cases} 1, & 0 < x < a, \quad n = 0 \\ 0 & \quad x > a, \quad n = 0 \end{cases}$$

Ans: $\frac{a}{p} J_{1(pa)}$

Q.20 Solve the Integral equation

$F(t) = t^2 + \int_0^t f(u) \sin(t-u)\, du$

Ans: JPH (Integral Transform) p.132

Q.21 Solve $F'(t) = \sin t + \int_0^t F(t-u) \cos u du,\ F(0) = 0$

Ans: JPH (Integral Transform) p.136

Q.22 Show that the function $g(x) = (1+x^2)^{-3/2}$ is a solution of the volterra integral equation

$g(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} g(t) dt$

Ans: Advanced Integral Equation by Gokharoo & others, p. 5

Q.23 Solve $g(x) = 1 + \int_0^1 (1 + e^{x+t})\, g(t)\ dt$

Ans: Advanced Integral Equation by Gokharoo & others, p. 50

Q.24 Solve $g(x) = sinx + \lambda \int_4^{10} xg(t)dt$

Ans : $g(x) = \text{sinx} + \frac{2\lambda x \sin 7 \sin 3}{1-42\lambda}$

Section –C(Long Answer Questions)

Q.1 Prove that $L\left[\frac{\sin t}{t}\right] = \tan^{-1}\left(\frac{1}{p}\right)$ and hence $L\left[\frac{\text{sina}\, t}{t}\right]$.

Does the Laplace transform of $\frac{\cos at}{t}$ exists.

Ans: See JPH (Integral transform) p.28

Q.2 Prove that $L[J_0(t)] = \frac{1}{\sqrt{1+p^2}}$

And hence deduce that

(a) $L[tJ_0(at)] = \frac{p}{(p^2+a^2)^{3/2}}$

(b) $L[e^{-at}J_0(at)] = \frac{1}{p^2+2ap+2a^2}$

(c) $\int_0^\infty J_0(t)\, dt = 1$

Ans: See JPH (Integral transform) p.38

Q.3 Evaluate $L^{-1}\left\{\frac{1}{p} log\left(1 + \frac{1}{p^2}\right)\right\}$

Ans: See JPH (Integral transform) p.72

Q.4 Apply convolution theorem to find

$L^{-1}\left[\frac{p}{(p^2+a^2)^3}\right]$

Ans: See JPH (Integral transform) p.79

Q.5 Solve the following integral equation by the method of successive approximations to third order

$g(x) = 2x + \lambda \int_0^1 (x+t) g(t)\, dt$ by taking $g_0(x) = 1$

Ans: $g_3(x) = 2x + \lambda\left(x + {}^2/_3\right) + \lambda^2\left(\frac{7}{6}x + \frac{2}{3}\right) + \lambda^3\left(\frac{13}{12}x + \frac{5}{8}\right)$

Q. 6 Show that

$L^{-1}\left[\frac{p^2}{p^4+4a^4}\right] = \frac{1}{2a}[\cos hat \sin at + \sin hat \cos at]$

Ans: See JPH (Integral transform) p.91

Q.7 Solve $(D^2+9)y = \cos 2t$

Given that $y(0) = 1, y\left({}^{\pi}/_2\right) = -1$

Ans: See JPH (Integral transform) p.101

Q.8 Find fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence find fourier sine transform of $F(x) = \frac{x}{1+x^2}$

Ans: See JPH (Integral transform) p.166

Q.9 Find the fourier cosine transform of $f(x)$, if $f(x) = \begin{cases} 1, 0 < x < a \\ 0,\ x \geq a \end{cases}$

What is the function whose fourier cosine transform is $\frac{\sin ap}{p}$.

Ans: See JPH (Integral transform) p.170.

Q.10 Using Fourier Cosine transform solve

$\frac{\partial \theta}{\partial t} = c^2 \frac{\partial^2 \theta}{\partial x^2}$

Subject to the conditions

(i) $\theta = 0\ \ when\, t = 0,\ x \geq 0$

(ii) $\frac{\partial \theta}{\partial x} = -\mu\ \ a\ constant\ when\, x = 0\ and\, t > 0$

Assume that $\theta(x,t)\ and\ \frac{\partial \theta}{\partial x}$ both to zero as $x \to \infty$.

Q.11 Find the Hankel transform of

$f(x) = \begin{cases} a^2 - x^2 & 0 < x < a & n = 0 \\ 0 & x > a & n = 0 \end{cases}$

Ans: $\frac{2a^2}{p^2} J_2(pa)$

Q.12 Determine the resolvent kernel and hence solve the integral equation

$g(x) = f(x) + \int_0^x e^{x-t} g(t)\, dt$

Ans: MT-9, p. 273

Q.13 Solve the following integral equation by the method of successive approximations to third order

$g(x) = 1 + \lambda \int_0^1 (x+t) g(t)\, dt$ by taking $g_0(x) = 1$

Ans: $g_3(x) = 1 + \lambda\left(x + {}^1/_2\right) + \lambda^2\left(x + {}^7/_{12}\right) + \lambda^3\left(\frac{13}{12}x + \frac{5}{8}\right)$