Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final)
Paper Code:MT-08
Numerical Analysis
Section – C
(Long Answers Questions)

1. (i) Find the real root of the equation $x^3 - 2x - 5 = 0$ using regula-falsi method?
   Ans. (Pg. No. 6)
   (ii) Find square root of 10 suing Newton-Raphson method.
   Ans. (Pg. No. 11)
2. (i) Find a root of the equation
   $3x - \sqrt{1 + \sin x} = 0$ using iteration method.
   Ans. (Pg. No. 13)
   (ii) Apply Aitken's Δ^2 method to find a root of the equation $sin^2 x = x^2 - 1$.
   Ans. (Pg. No. 14)
3. Solve the following system of equation by Newton-Raphson method.
   $y - \sin(x + y) = 0$
   $x - \cos(y - x) = 0$
   Taking initial approximation $x_0 = 1, y_0 = 1$.
   Ans. (Pg. No. 16, 17, 18)
4. (i) Find a real solution of the equations:
   $x^2 - 5x + 4 = 0$
   $3xy^2 - 10y + 7 = 0$
   Taking initial approximation as (0.5, 0.5)
   Ans. (Pg. No. 19, 20)
   (ii) Find the root of the equation $x^4 - x - 10 = 0$ using chebyshev method.
   Ans. (Pg. No. 24, 25)
5. (i) Find the root of the equation $x^3 - 2x - 5 = 0$ by Miller's method take 1, 2 and 3 as initial approximation.
   Ans. (Pg. No. 31,32)
   (ii) Find double root of the equation:
   $x^3 - 0.75x + 0.25 = 0$
   Taking initial approximation $x_0 = 0.3$
   Ans. (Pg. No. 35)
6. (i) Using synthetic division and chebysehv method find a root of the equation $x^3 + x^2 + x + 4 = 0$ Perform two iterations.
   Ans. (Pg. No. 47)

(ii) Extract quadratic factor from the equation $x^3 - 2x + x - 2 = 0$ using Bairston method and hence find the roots of the equation. Perform only two iterations and use (-0.5, 1) as initial approximation.
Ans. (Pg. No. 54)

7. Explain Bairston method?
Ans. (Pg. No. 48,49, 50, 51)
8. Perform two iterations of Briston method to find two roots of the equation.
$x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$
Use (2, 2) as initial approximation
Ans. (Pg. No. 52, 53, 54)
9. Find all the roots of the equation $x^4 - 3x + 1 = 0$ using Graeffe's root squaring method. Use four squaring to estimate roots.
Ans. (Pg. No. 59, 60, 61)
10. (i) Using the Gauss-Jordan method solve the following linear equations.
$10x + y + z = 12$
$2x + 10y + z = 13$
$x + y + 5z = 7$
Ans. (Pg. No. 68, 69, 70)
(ii) Explain method of determinants?
Ans. (Pg. No. 65, 66)
11. Solve the system of equations by LU factorization method:
$2x + 3y + z = 9$
$x + 2y + 3z = 6$
$3x + y + 2z = 8$
Ans. (Pg. No. 71, 72, 73)
12. (i) Using cholesky method solve the system of equation.
$4x - y = 1$
$-x + 4y - z = 0$
$-y + 4z = 0$
Ans. (Pg. No. 73,74)
13. Solve the system of the following equations using relaxation method.
$2x + y - 8z = 15$
$x - 7y + z = 10$
$6x - 3y + z = 11$
Ans. (Pg. No. 81,82)
14. Find the eigenvalues and vectors of the following matrix A.
$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$
Ans. (Pg. No. 88, 89, 90)
15. Suing Jacobi's method to find all the eigenvalues and eigenvectors of the following matrix A (perform three iterations).

$$A = \begin{bmatrix} 1 & 1 & 0.5 \\ 1 & 1 & 0.25 \\ 0.5 & 0.25 & 2 \end{bmatrix}$$

Ans. (Pg. No. 97, 98, 99)

16. Using the Given's method reduce the following matrix to tridiagonal form and use sturm sequence to find eigenvalues.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

Ans. (Pg. No. 104, 105, 106)

17. Find all the eigenvalues and eigenvectors of the following matrix using Given's method.

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix}$$

Ans. (Pg. No. 106, 107, 108, 109)

18. (i) Using the Rutis hauser method to compute all the eigen values of the matrix.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Ans. (Pg. No. 112, 113)

(ii) Fit a curve of the form $y = ax + bx^2$ to the given data:

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.95	3.2	5	8.1	11.9	16.4

Ans. (Pg. No. 124, 125)

19. Population of a city in different years are given in the following table :

x	1970	1980	1990	2000	2010
Y (in thousands)	1450	1600	1850	2150	2500

Fit a parabola to the given data using least squares principle. Also estimate the population of the city in 2005.

Ans. (Pg. No. 128, 129)

20. (i) Fit a curve of the form $y = ax^b$ to the given data :

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

Ans. (Pg. No. 130, 131)

(ii) Fit a straight line to the following data :

x	1	2	3	4	5	6
y	2.4	3	3.6	4	5	6

Also find y at $x = 3.5$

Ans. (Pg. No. 138. Q.3)

21. Obtain a second degree polynomial approximation to the function $f(x) = x^3$, on the interval [0, 1] using least-squares principle. Take weight function $w(x) = 1$.
Ans. (Pg. No. 135, 136, 137)
22. Explain Gram-Schmidt Orthogonalizing process.
Ans. (Pg. No. 144, 145, 146)
23. (i) Obtain polynomial approximation $P_n(x)$ for the function $f(x) = e^{-x}$ using Taylor series expansion about $x_0 = 0$ and find the value of x when the error in $P_n(x)$ obtained from the first five terms only is to be less than 10^{-7} after rounding.
Ans. (Pg. No. 142, 143)
(ii) Explain chebyshev series expansion.
Ans. (Pg. No. 149, 150)
24. Using the chebyshev polynomials obtain the least squares approximations of second degree for the function $f(x) = x^3 + x^2 + 3$, where $x \in [-1, 1]$
Ans. (Pg. No. 151, 152, 153)
25. (i) Obtain the chebyshev polynomial approximation of second degree (best minimax approximation) to $f(x) = x^3$ on the interval [0, 1]
Ans. (Pg. No. 153, 154)
(ii) Determine the best minimax approximation to the function $f(x) = x^2$ on [0, 1] with a straight line.
Ans. (Pg. No. 154, 155)
26. (i) Find a uniform polynomial approximation of degree four or less to $f(x) = sin^{-1}(x)$ on the interval [-1 1] using Lanczas economization with error tolerance of 0.05.
Ans. (Pg. No. 155, 156,)
(ii) Find a uniform polynomial approximation of degree four or less to the function $f(x) = e^x$ on the interval [-1, 1] using Lanczos economization with error tolerance 0.02.
Ans. (Pg. No. 156, 157)
27. Explain Taylor's series method for Initial value problem (IVP)?
Ans. (Pg. No. 161, 162)
28. (i) Compute y (0, 2) by Taylor's series, where y(t) is the solution of the IVP.
$$\frac{dy}{dt} = t + y \quad , \; y(1) = 1$$
Ans. (Pg. No. 164, 165)
(ii) Explain Picard's method?
Ans. (Pg. No. 166)
29. Explain Range kutta metod of order two?
Ans. (Pg. No. 169, 170, 171)
30. Compute y(14) using fourth order Runge-Kutta method given that

$$\frac{dy}{dt} = \frac{t}{y}, \quad y(1) = 2$$

Ans. (Pg. No. 173, 174, 175)

31. Compute y(1.2) by using Runge-kutta fourt order method where y(t) is the solution of the IVP.

$$\frac{dy}{dt} = ty, \quad y(1) = 2$$

Ans. (Pg. No. 175, 176, 177)

32. Solve the following initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = y = 0, \qquad t \in [0, 0.1], \; y(0) = 0, \qquad y'(0) = 1$$

Ans. (Pg. No. 178, 179, 180)

33. (i) Compute x(0.1), y(0.1) by Taylor's series method where x(t), u(t) satisfy the following system of initial value problems

$$\frac{dy}{dt} = xy + 2y, \quad \frac{dy}{dt} = 2ty + x, \; x(0) = 1, \qquad y(0) = 2$$

Ans. (Pg. No. 180, 181)

(ii) Explain Numerical solution to Higher order Differential equations.

Ans. (Pg. No. 177, 178)

34. (i) Solve by Milne's method

$$\frac{dy}{dt} = \frac{t}{y}, \quad y(1) = 2 \quad t \in [1, 14]$$

Ans. (Pg. No. 188, 189, 190)

(ii) Compute y(0.5) by Milne's method, given that

$$\frac{dy}{dt} = 2e^t - y$$

And the corresponding values of t and y are given as

t :	0	0.1	0.2	0.3
y :	2	2.01	2.04	2.09

Ans. (Pg. No. 190, 191)

35. Explain Adams-Moulten method write predictor formula or corrector formula.

Ans. (Pg. No. 191, 192, 193, 194)

36. Solve the BVP

$$\frac{d^2y}{dt^2} = y, \quad y(0) = 0, y(1) = 1.1752$$

Ans. (Pg. No. 208, 209, 210)

37. Explain finite difference methods?

Ans. (Pg. No. 211, 212)

38. (i) Solve the boundary value problem

$$\frac{d^2y}{dx^2} = (1 + x^2)y + 1 = 0 \quad x \in [0, 1]$$

By a second order finite difference method with step size $h = \frac{1}{4}$.

Ans. (Pg. No. 215, 216)

(ii) Solve the BVP

$$\frac{d^2y}{dx^2} = \frac{3}{2}y^2, \ \ y(0+4, \ \ y(1) = 1$$

With step size $h = \frac{1}{3}$ using second order method.

Ans. (Pg. No. 219)

39. Solve the BVP by Number method

$x^2 - 2y + x = 0$

$y(2) = y(3) = 0$ with step size h = 0.25

Ans. (Pg. No. 221, Q.4)

40. (i) Find the root of the equation $x^2 - 5x + 2 = 0$ correct to four decimal places by Newton-Raphson method.

Ans. (Pg. No. 22,Q.6)

(ii) Find double root of the equation

$x^2 - x^2 - x + 1 = 0$

Taking initial approximation $x_0 = 0.9$

Ans. (Pg. No. 22, Q.9)

41. Find complex roots of the equation

$z^2 - 2z^2 + z - 2 = 0$ taking initial approximation $z_0 = 0.5 + 0.5i.$

Ans. (Pg. No. 39, Q.5)

42. Explain synthetic division method for n degree polynomial equation by a linear factor?

Ans. (Pg. No. 40, 41, 42, 43)

43. Explain Bairstin method?

Ans. (Pg. No. 48, 48, 50, 51, 52)

44. Using Graeffe's root squaring method, find all the roots of the equation.

$x^4 - 3x^3 - 3x^2 + 11x - 6 = 0$

Ans. (Pg. No. 63, Q.7)

45. Explain Graeffe's root squaring method?

Ans. (Pg. No. 56, 57, 58)

46. Find the Dootittle, crout and cholesky factorizations of the matrix.

$$A = \begin{bmatrix} 60 & 30 & 20 \\ 30 & 20 & 15 \\ 20 & 15 & 12 \end{bmatrix}$$

Ans. (Pg. No. 85 Q.4)

47. Find all the eigenvalues and eigenvectors of the matrix.

$$A = \begin{bmatrix} 1 & \sqrt{3} & 4 \\ \sqrt{3} & 4 & \sqrt{3} \\ 4 & \sqrt{3} & 1 \end{bmatrix}$$

Ans. (Pg. No. 101, Q.6)

48. Compute the eigenvalues using Rutishause method of the following matrix.

$$A = \begin{bmatrix} 6 & 4 & 4 & 1 \\ 4 & 6 & 1 & 4 \\ 4 & 1 & 6 & 4 \\ 1 & 4 & 4 & 6 \end{bmatrix}$$

Ans. (Pg. No. 118, Q.3)

49. (i) Fit a curve of the form $y = ax^b$ to the data given below:

x	2	4	7	10	20	40	60	80
y	43	25	18	13	8	5	3	2

Ans. (Pg. No. 138, Q.5)

(ii) Fit a curve $y = ae^{bx}$ to the following data:

Also estimate y at $x = 7$

Ans. (Pg. No. 138, Q.7)

50. Construct a least squares quadratic approximation to the function $y(x) = \sin x$ on $[0, \frac{\pi}{2}]$ with respect to weight function $w(x) = 1$.

Ans. (Pg. No. 139, Q.11)

51. Obtain least-square approximation of second degree for x^4 on [-1, 1] using chebyshev polynomials.

Ans. (Pg. No. 158, Q.4)

52. (i) Compute six correct to three significant digits, by the economization of the power series

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{840} + \cdots ....$$

Ans. (Pg. No. 158, Q.9)

(ii) Use Chebyshev polynomial to find the best uniform approximation of degree four or less to x^5 on [-1, 1]

Ans. (Pg. No. 158, Q.8)

53. Given that :

$$\frac{dy}{dt} = ty + x \qquad \frac{dx}{dt} = xy + t \ ; \quad y(0) = -1, \ x(0) = 1$$

Compute y(0.2), x(0.2) by fourth order Runge-Kutta method (Take step size h = 0.2)

Ans. (Pg. No. 183, Q.5)

54. Compute y(1) by Adams-Moulton method, given that :

$$\frac{dy}{dt} = y - t^2, \qquad y(0) = 1, \qquad y(0.2) = 1.2857,$$

$$y(0.4) = 1.46813, \qquad y(0.8) = 1.73779$$

Ans. (Pg. No. 202, Q.1)

55. Solve the boundary value problem:

$$\frac{d^2y}{dx^2} = y \quad ; y(0) = 0, \ y(0.4) = 0.4$$

By shooting method (Use Runge-Kutta Method)

Ans. (Pg. No. 210, Q.2)

56. Solve the BVP:

$y'' = -8\,(sin^2\pi\, x)y \quad ; \; y(0) = y(1) = 1$

By second order method with step size $h = \frac{1}{4}$.

Ans. (Pg. No. 221, Q.1)

57. (i) Solve the given system of equation cholesky method:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

Ans. (Pg. No. 84, Q.3)

(ii) Using Gauss Jordan method solve the following system of equations:

$3x + 2y + z = 10$

$2x + 3y + 2z = 14$

$x + 2y + 3z = 14$

Ans. (Pg. No. 84, Q.1(i))

58. Find the eigenvalues and vectors of the following matrices:

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

Ans. (Pg. No. 91, Q.1)

59. Use Jacobi method to estimate the eigen values of the following matrix:

$$\begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$$

Ans. (Pg. No. 99, Q1

60. Reduce the following matrix A to the tridiagonal form using Given's method. Use sturm sequence to locate the eigen values.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Ans. (Pg. No. 109, Q.2)

61. Explain Given's Method?

Ans. (Pg. No. 102, 103)

62. Explain Complex eigen values, Hermitian matrix, Unitary Matrix and solve the eigen values.

Ans. (Pg. No. 113, 114, 115, 116)

Construct a least-squares quadratic approximation to the function $y(x) = \sin x \; on \; [0, \frac{\pi}{2}]$ with respet to weight function $w(x) = 1$.

Ans. (Pg. No. 139)

63. Use Taylor's series method to compute $x\,(0.1) y\,(0.1)$ given that

$$\frac{dx}{dt} = x + y + t \qquad \frac{d^2y}{dt^2} = x - t$$

Ans. (Pg. No. 183, Q.7)

64. Solve the boundary value problem:

$\frac{d^2y}{dx^2} = 64y - 10\,; \qquad y(0) = 0,\; y(1) = 0$ by shooting method.

Ans. (Pg. No. 210, Q.3)

65. (i) Express $T_0(x) + 2T_1(x) + T_2(x)$ as a polynomial in x.

Ans. (Pg. No. 158, Q.2)

(ii) Obtain the best lower degree approximation to $x^3 + 2x^2$.

Ans. (Pg. No. 158, Q.5)

66. (i) Find the root of the equation $x^3 - x^2 - x - 1 = 0$ using Muller's method, taking initial approximation as $x_0 = 0, x_1 = 1, x_2$.

Ans. (Pg. No. 39, Q.3)

(ii) Find a quadratic factor of the polynomial $P_4(x) = x^4 + 5x^3 + 3x^2 - 5x - 9 = 0$ by Bairstow method. Take initial approximation (3, -5).

Ans. (Pg. No. 62, 63, Q.4)

67. (i) Find all roots of the equation :

$x^3 - 2x^2 - 5x + 6 = 0$ by Graeffe's root squaring method.

Ans. (Pg. No. 63, Q.8)

(ii) Solve the following system of equation by the Relaxation method.

$x + 9y - z = 10$

$2x - y + z = 20$

$10x - 2y + z = 12$

Ans. (Pg. No. 85, Q.5)