Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final)
Paper Code:MT-08
Numerical Analysis
Section – A
(Very Short Answers Questions)

1. Write two examples of transcendental equations.

A (i) $x^3 + \sin x = 0$ (ii) $e^x \log x + \tan x + x^2 = 0$

2. Write the formula of Newton-Raphson method.

A $x_{n+1} = x_n = \dfrac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \dots \dots$

3. Write the formula of chebysehv method of third order.

A $x_{n+1} = x_n = \dfrac{f(x_n)}{f'(x_n)} = \dfrac{1}{2}\left[\dfrac{f(x_n)}{f'(x_n)}\right]^2 \dfrac{f''(x_n)}{f'(x_n)}$

4. Write the definition of generalized Newton-Raphson method.

A The multiple root of the equation $f(x) = 0$ with multiplicity m can be obtained using the formula.

$$x_{n+1} = x_n - m\,\frac{f(x_n)}{f'(x_n)}$$

This is called generalized Newton-Raphson method.

5. Write the polynomial equation of degree n.

A $P_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots \dots + a_{n-1}x + a_n$

6. Write any two direct methods to solve system of simultaneous equations.

A (i) Gauss Jordan method   (ii) Method of determinants

7. Write the definite on of eigen values and rigen vectors.

A Let $A = [a_{ij}]$ be a given $n \ x \ n$ matrix and consider the vector equation of the form $AX = \lambda X$ ----------(1)

Where A is given square matrix x is unknown vector and λ is an unknown scalar. A value of λ for which (1) has a solution $x \neq 0$ is called an eigen value or characteristic value and corresponding solution $x \neq 0$ of the same equation is called the eigen vectors.

8. What is spectrum?

A The set of eigen values is called the spectrum of A.

9. What is spectral radius?

A The largest eigen value is called the spectral radius of A.

10. Write two properties of eigen values and eigen vectors?

A (i) For a given square matrix the eigen values are unique but the eigen vectors are not unique.

(ii) A square matrix and its transpose have the same eigen values.

11. What are eigenvalues of a triangular matrix?

A Diagonal elements.

12. If $\lambda_1, \lambda_2, \lambda_3 \ldots \ldots$ are the eigen values of square matrix A then what are the eigenvalues of A^{-1}.

A $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3} \ldots \ldots$

13. If $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ldots$ are the eigen values of square matrix A then what are the eigenvalues of KA.

A $k\lambda_1, k\lambda_2, k\lambda_3, k\lambda_4 \ldots$

14. If $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ldots$ are the eigen values of square matrix A then What are the eigenvalues of A^k.

A $\lambda_1^k, \lambda_2^k, \lambda_3^k$

15. What is the largest eigenvalue of matrix A called?

A Spectral radius of A.

16. What is the set of all the eigenvalues of Matrix A called?

A Spectrum

17. What is trace of matrix?

A The sum of all the eigen values of a square matrix is equal to the trace of matrix.

18. Write the statement of Cayley Hamilton theorem?

A Every square matrix satisfies its own characteristic equation.

19. If $XX^T = 1$ then eigen vector X is said to be _____.

A Orthonormal or normalized

20. What is complex matrix?

A The elements of a matrix can be complex numbers also, such matrix is known as complex matrix.

21. What is conjugate matrix of A.

A If the elements of matrix $A = [a_{rs}]$ are complex numbers

$\alpha_{rs} + \beta_{rs}, \alpha_{rs}$ and β_{rs} being real, then matrix $\bar{A} = [\overline{\alpha_{rs}}] = [\alpha_{rs} - i\beta_{rs}]$ is called conjugate matrix of A.

22. What is Hermitian matrix?

A A square matrix A such that $A' = \bar{A}$ (transporse of the matrix is equal to the conjugate of the same) is said to be a Hermitian matrix.

23. What is unitary matrix?

A A square matrix U such that $\overline{U'} = U^{-1}$ is called unitary matrix.

24. Write Taylor series expansion of a function $f(x)$ about a pint x_0.

A A Taylor series expansion for a function $f(x)$ about a point x_0 is given by

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{1}{21}(x - x_0)^2 f''(x_0) \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots + \frac{1}{n_1}(x - x_0)f^{(n)}(x_0) + \frac{1}{(n+1)}(x - x_0)^{n+1}f^{(n+1)}(x_0) + \cdots.$$

25. What is the remainder term of Taylor series?

A $R_n = \frac{1}{(n+1)!}(x - x_0)f^{(n+1)}(\theta), x_0 < \theta < x$

It is the truncation term

26. When a set of functions $[\phi, (x)]$ is said to be orthogonal on an interval [a, b], with respect to the weight function $w(x)$?

A $\int_a^b w(x)\phi_j(x)\phi_k(x) = 0, \quad if\ j \neq k$

27. When a set of function $[\phi, (x)]$ is said to be orthogonal over a set of points $\{x_i\}$ wth respect to the weight function $w(x)$?

A For descrete data

$\sum_{i=1}^m Q(x_i)\,\phi_j(x_i)\phi_k(x_i) = 0 \quad if\ j \neq k$

28. What polynomial we have if we take weight function w(x)=1 and interval [-1, 1] in orthogonal polynomials and least square approximation?

A Legendre polynomials

29. What polynomial we have if we take weight function $w(x) = (1 - x^2)^{1/2}$ on the interval [-1, 1] in orthogonal polynomials and least square approximation?

A Chegyshev polynomials.

30. What is the chebyshev polynomials of the first kind of degree n over the interval [-1, 1]?

A $T_n(x) = \cos(n\ \cos^{-1}x)$

31. What is the recurrence relation of chebyshev polynomials?

A $T_{n+1}(x) = 2xT_n - T_{n-1}(x)$

32. What is orthogonal properties of chebyshev polynomials?

A $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \neq 0 \\ \pi & m = n = 0 \end{cases}$

33. What is the minima property of chebyshev polynomials?

A An important property of chebyshev polynomials called minimax property, is that of all polynomials of degree in where the coefficient of leading team x^n is unity, the polynomial $2^{1-n}T_n(x)$ has the smallest least upper bound for its absolute value in the interval vound for its absolute value in the interval [-1, 1] that is

$\max_{-1\leq x\leq 1}|2^{1-n}T_n(s)| \leq \max_{-1\leq x\leq 1}|P_n(x)|$

Here $P_n(x)$ is any monic polynomial of degree n.

34. What is minimax principal or minimax polynomial?

A In chebyshev approximation the maximum error is kept down to minimal. This is called minimax principle and polynomial $T_n(x)$ is referred to as minimax polynomial.

35. Which type of polynomials are chebyshev polynomials?

A Orthogonal

36. What is the important property of chebyshev polynomials?

A Minimax property

37. Write first approximation to y for picard method?

A $y^{(1)} = y_0 + \int_{t_0}^{t} f(t, y_0)dt$

38. Why we use Picard method?

A For solve the differential equation with IVP.

39. Why we use Runge-Kutta method?

A For solve the differential equation with IVP.

40. What is homogenous boundary value problem?

A A homogeneous boundary value problem is that which involves a homogeneous differential equation (i.e. the equation that contains dependent variable and its derivative) and the homogeneous boundary conditions.

41. Which type of solution has a homogeneous BVP?

A Trivial solution.

42. What is eigen value problem?

A   A BVP which involves a parameter μ in the differential equation or in the boundary conditions is called an eigen value problem and the values is take are called eigen values.

43. Write two point boundary value problem?

A $\frac{d^2y}{dx^2} = f(x, y, y')$, $x \in [a, b]$

Where two arbitrary constants which require two end conditions.

44. Why we use shooting method?

A   Shooting method is used to solve a boundary value problem where in the given BVP is converted into a system of initial value problems.

45. What is condition when one root of the equation $f(x) = 0$ lies in the interval (a, b) if $f(x)$ is continuous on the interval [a, b]

A $f(a)$ and $f(b)$ have opposite signs.

46. What is the other name of Regular Falsi method?

A   False position

47. Write the formula for Newton-Raphson method for rearly equal roots.

A $x = a \pm \sqrt{\frac{-2f(a)}{f''(a)}}$

48. Write the formula for Newton-Raphson method for pth rot of a number?

A $x_{n+1} = \frac{(p-1)x_n^p + N}{px_n^{p-1}}$

49. What is Aitken's Δ^2 −method to accelerate the convergence?

A   We know that the convergence of iteration method is linear which can be accelerated by method known as Aitken's method.

50. If α is a simple root of the equation $f(x) = 0$, then $f(x)$ is equal to

A $f(x - \alpha)g(x)$, $g(\alpha) \neq 0$

51. What is the other name of Newton-Raphson method?

A   Tangent method

52. Write the condition for Newton-Raphson method to be convergent.

A   Initial approximation should be very close to the exact root.

53. Write the convergence condition of Regula-falsi method.

A   Regula-falsi method has linear convergence.

54. Which method is faster in Regula-falsi oe secant method.

A Secant method is faster than Regula-falsi.

55. What is the condition for iterative scheme $x = \phi(x)$ so that it becomes convergent?

A $|\phi'(x)| < 1$, in the neighbourhood of the root.

56. What is the convergence of the chebyshev method?

A Three

57. What is the other name of chebyshev method to third order?

A Newton-Raphson extended formula

58. What is error equation of Newton-Raphson extended formula?

A $\in_{n+1} \leq \in_n^3$ where $M = \left[\frac{f''(\alpha)}{f'(\alpha)}\right]^2 - \frac{2f''(\alpha)}{3f'(\alpha)}$

59. What is the rate of convergence of the Muller's method?

A 1.84

60. Write the formula of generalized Newton-Raphson method.

A $x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$

61. Write rot of equation is approximated in muller method.

A Quadratic polynomial.

62. What is the remainder in Bairsto Method?

A. Remainder will be a linear polynomial say, Rx+s.

63. By which method we can obtained all rots of the given polynomials equation at a time.

A Graeffe's root squaring method.

64. Which types of roots we can solve by Graeffe's root squaring method?

A Different rots, complex rot and double root.

65. In which method we do not need any information about initial approximation.

A Graeffe's root squaring method.

66. In least-squares principle sum of errors e_i^n is minimzed. What is the value of n here.

A 2

67. To fit a parabola, unknown parameters can be obtained by schoring how many normal equations consist.

A 3

68. For fitting a straight line ehat is the equation of sum of squares S of the error.

A $S = \sum_{i=1}^{m} [y_i - (a + bx_1)^2]$

69. What is the method of absolutely stable?

A A method is said to be stable if the total effect of all errors is bounded and is independent of the number of mesh points. A method may be absolutely stable.

70. Write Adams-moultan predictor corrector formulae?

A Predictor

$$y_{j+1}^{(p)} = y_j + \frac{h}{24}\left[55y_j' - 59_{j-1}' + 37y_{j-2}' - 9y_{j-3}'\right]$$

Corrector

$$y_{j+1}^{(c)} = y_j + \frac{h}{24}\left[9y_{j+1}' + 19_j' - 5y_{j-1}' + y_{j-2}'\right]$$

71. Write Milne's predictor formula.

A $y_H = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$