Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Final) Question Bank-2015
Paper Code:MT-06

Section A (Very short answer type Questions)

1. Define n-dimensional unitary space.

Ans.- P.N.-4

2. Define graph of a mapping.

Ans. P.N.-53

3. Define Schwartz inequality.

Ans. P.N.-78

4. State Pythagorean theorem.

Ans. P.N. 92

5. State Bessel's inquality.

Ans. P.N. – 100

6. Let H be a given Hilbert space and T^* be adjoint of the operator T, then T^* is a unbounded linear transformation. Is this statement true?

Ans. No. T^* must be bounded. (P.N.-117)

7. State any one property of eigenvalue and eigenvector.

Ans. P.N. 141

8. What is the statement of spectral theorem.

Ans. P.N. 147

Section B (Short answer type questions)

1. Suppose that X any Y be Banach spaces over the same field K of scalars and V be an open subset of X. Let $f; V \to Y$ is differentiable at $x \in V$. Prove that, all the directional derivative of f exists at x and

$D_v f(x) = Df(x).v$, where $v \in V$ is an unit vector.

Ans. P.N. – 157

2. Let I be an interval in R, W be a subset of a Banach space X and $g: I \times W \to X$ be a locally Lipschitz function. If there are two exact solutions f_1 and $f_2 : I \to X$ of the differential equation $\frac{dx}{dt} = g(t,x)$ and if they are equal for one value $t_0 \in I$ then, show that they are identical in the entire I.

Ans. P.N. 202

3. If N be an normed linear space and $x, y \in N$, then show that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

Ans. P.N.- 4

4. It T be a linear transformation from a normed linear space N into normed space N', then show that T is continuous either at every point or at no point of N.

Ans. P.No. 34

5. It X is an inner product space, then show that $\|x\|=(x,x)^{1/2}$ is a norm on X.

Ans. P.N.- 78

6. Show that a closed linear subspace M of a Hilbert space H reduces an operator T iff M is invariant under both T and T^*.

Ans. P.N. – 139

7. Prove that an operator T on a finite-dimensional Hilbert space H is singular iff there exists a non-zero vector x in H such that $Tx=0$.

Ans. P.N. – 144

8. Show that every Hilbert space is reflexive.

Ans. P.N. – 112

Section C(Long – Answer Type Questions)

1. (i) Show that every normed linear space is a metric space.

(ii) Show that the limit of a convergent sequence is unique.

Ans. P.N. – 4, 5, 6

2. (i) Show that the weak limit of sequence is unique.

(ii) Show that on a finite dimensional linear space X, all norms are equivalent.

Ans. P.N. – 37-39

3. If B is a complex Banach space whose norm obeys the parallelogram law, and if an inner product is defined on B by

$$4(x,y)=\|x+y\|^2-\|x-y\|^2+i\|x+iy\|^2-\|x-iy\|^2$$

Show that, then B is a Hilbert space.

Ans. P.N. – 82

4. Prove that an operator T on a Hilbert space H is unitary iff it is an isometric isomorphism of H onto itself.

Ans. P.N.-130