

Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous) Question Bank-2015
Paper Code:MT-05

Section-A (Very Short Answer type Questions)

- 1). Define moment of Inertia. (p.2)
- 2). Define compound pendulum (p.43)
- 3). Define instantaneous axis of rotation. (p.145)
- 4). Define degree of freedom. (p.204)
- 5). What is principle of energy? (p.210)
- 6). Define steady motion of a Top. (p.242)
- 7). Define compressibility. (p.272)
- 8). Define unsteady fluid flow. (p.273)

- 9) Write moment of Inertia of a solid ellipsoid of mass M and length of axes 2a, 2b and 2c about the axis OX. (p.8)

- 10). Write formula for time period of simple pendulum of length l. (p.44)

- 11). What is the locus of invariable line? (p.156)

- 12). Write principle of conservation of Angular Momentum under Impulsive forces. (p.189)

- 13). In which system, all generalized co-ordinates are independent to each other? (p.204)

- 14). Define density. (p.271)

- 15). Define steady flow of fluid. (p.273)

- 16). What is Barotropic fluid flow? (p.274)

(Section-B)

- 1). Find M.I. of a uniform rectangular lamina of mass M and sides of length 2a, and 2b about a line through its centre and parallel to sides 2a, 2b and perpendicular to its plane. (p.5)

- 2). Find the length of simple equivalent pendulum in case of circular disc when the axis is horizontal and a tangent to it. (p.49)

- 3). A body moves under no forces about a point O, the principal moments of inertia at O being 6A, 3A and A. Initially the angular velocity of the body has components $w_1=n$, $w_2=0$, $w_3=3n$ about the principal axes. Show that at any later time $w_2 = -\sqrt{5}n \tanh(\sqrt{5}nt)$ and ultimately the body rotates about mean axis. (p.158)

- 4). State and prove principle of conservation of angular momentum under finite forces. (p,181)

- 5). Two equal rods AB and BC, each of length l, smoothly jointed at B, are suspended

from A and oscillates in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$, where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right) \frac{g}{l}$. (p.219)

6). Find the stream lines and path lines of the particles of the velocity field:

$$u = \frac{x}{1+t}, v = y \text{ and } w=0. \quad (\text{p.290})$$

7). Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2} \phi(t) + \frac{z^2}{c^2} \psi(t) = 1$, where $f(t) \cdot \phi(t) \cdot \psi(t) = 1$ is a possible form of the boundary surface. (p.334)

8). In the case of the two-dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O. Show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the line joining S to the ends of a diameter at right angles to OS cut the circle and prove that its magnitude at these points is $\frac{2m \cdot OS}{(OS^2 - a^2)}$. (p.415)

9) Derive the equation of motion of translation. (p.13)

10). Find the change in kinetic energy due to action of impulse. (p.133)

11). If the earth be regarded as a solid of revolution, whose principal moments of inertia at its centre of gravity are A, A, C. Show that its axis of rotation describes a cone of very small angle about the axis of the figure in period $\frac{A}{C-A}$ sidereal days. (p.175)

12). Show that when a body moves under the action of a system of conservative forces, the sum of its kinetic and potential energies is constant throughout the motion. (p.195)

13). When the axis of a symmetrical top is stationary and the spin is large and equal to n , a blow J is applied perpendicular to the axis at a distance d from the fixed point. Prove that the maximum angular deflection of the axis is approximately $2 \tan^{-1} \left\{ \frac{Jd}{Cn} \right\}$, C being the moment of inertia of the top about its axis of symmetry. (p.252)

14). The velocity field at a point in fluid is given as $\vec{q} = \frac{x}{y} \hat{i} + y\hat{j} + 0 \cdot \hat{k}$, obtain path lines. (p.288)

15). Derive the equation of continuity in vector form by Euler's method. (p.299)

16). Show that ellipsoid $\frac{x^2}{a^2 k^2 t^4} + k t^2 \left(\frac{y^2}{b^2} + \frac{z^2}{c^2} \right) = 1$ is a possible form of the boundary surface of a liquid at time t .

(Section-C)

1). A solid homogeneous cone of height h and vertical angle 2α , oscillates about a horizontal axis through its centre. Show that the length of the simple equivalent pendulum is

$$\frac{h}{5} (4 + \tan^2 \alpha). \quad (\text{p.52})$$

2). Two equal cylinders each of mass m are bound together by an elastic string whose tension is T and roll with their axes horizontal down a rough plane of inclination α . Show that their acceleration is

$$\frac{2}{3} g \sin \alpha \left[1 - \frac{2\mu T}{mg \sin \alpha} \right], \text{ where } \mu \text{ is the coefficient of friction between the cylinders.}$$

(p.98)

3). A uniform thin circular disc is set rotating with an angular velocity ω about an axis through the centre making an angle i with the normal. Prove that the semi-vertical angle θ of the cone described by the axis of disc is given by $\tan \theta = \frac{1}{2} \tan i$.

If ω be angular velocity, prove that above cone is described in period $\frac{2\pi}{\omega\sqrt{1+3\cos^2 i}}$. (p.173)

4). Determine the stream line if the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$, where $r^2=x^2+y^2+z^2$. (p.288)

5). Two unequal masses M and M' rest on two rough planes inclined at angles α and β to the horizon, they are connected by a fine string passing over a small pulley of mass m and radius a , which is placed at the common vertex of two planes, show that the acceleration of either mass is $g[M(\sin \alpha - \mu \cos \alpha) - M'(\sin \beta + \mu' \cos \beta)] \div \left[M + M' + m \frac{k^2}{a^2}\right]$, where μ and μ' are the coefficients of friction of two planes, k is the radius of gyration of the pulley about its axis and mass M moves downwards.

(p.33)

6). A rigid body, symmetrical about an axis, so that $B=A$, is supported at its centre of gravity which is fixed and only forces that have moments about the C.G. are equivalent to a retarding frictional couple, proportional to the angular velocity and acting in a plane at right angles to the instantaneous axis. Assuming $C>A$, prove that equations of motion can be integrated in the form $\omega_1 = \alpha e^{-c\lambda t/A} \sin\left(\frac{\sigma}{\lambda} e^{-\lambda t} + \epsilon\right)$,

$$\omega_2 = \alpha e^{-c\lambda t/A} \cos\left(\frac{\sigma}{\lambda} e^{-\lambda t} + \epsilon\right), \omega_3 = n e^{-\lambda t}$$

where n, α, σ are constants, $\sigma = \frac{n(C-A)}{A}$ and λ is a constant defined by a constant couple.

(p.145)

7). If initially the axis of the top is horizontal and it is set spinning with angular velocity ω in a horizontal plane prove that the axis will start to rise if $nC\omega > mgh$ and that, when $nC\omega = 2mgh$, the axis will rise to an angular distance $\cos^{-1} \frac{A\omega}{nC}$, provided that $A\omega < nC$ and will there be at instantaneous rest. A, C and n have their usual meanings. (p.248)

8). Show that if the velocity potential of an irrotational fluid motion is equal to

$$A(x^2 + y^2 + z^2)^{-3/2} z \tan^{-1} \left(\frac{y}{x}\right), \text{ the lines of flow will be on the series of the surface } (x^2 + y^2 + z^2) = C^{2/3}(x^2 + y^2)^{2/3}$$

(P.311)