Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-05
Mechanics
Section – C
(Long Answers Questions)

Note :- Each question carries 16 marks and maximum word limit will be 800 words.

1. Show that the motion of a body about its centre of Inertia is the same as it would be if the centre of Inertia were fixed and the same forces acted on the body.
   (Pg. 14)
2. A rod of length 2a is suspended by a string of length l, attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that :
   $$\frac{3l}{a} = \frac{(4\tan\theta - 3tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$$
   (Pg. 18)
3. Two uniform spheres, each of mass M and radius a, are firmly fixed to the ends of two uniform thin rods, each of mass m and length l, and the other ends of the rods are freely hinged at a point O. The whole system revolves, as in the governor of a steam-engine, about a vertical line through O with angular velocity w. Show that when the motion is steady, the rods are inclined to the vertical at an angle θ given by the equation
   $$Cos\,\theta = \frac{g}{w^2}.\frac{M(1+a)+\frac{1}{2}lm}{M(1+a)^2+\frac{1}{3}ml^2}$$
   (Pg. 22)
4. A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizontal, and a man of mass M', starting from the upper end, walks down the plank, so that is does not move; show that he gets to the other end in time $\sqrt{\frac{2\,M\prime a}{(M+M')g\sin\alpha}}$
   (Pg. 24)
5. A thin circular disc of mass M and radius a can turn freely about a thin axis OA, which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the inclination θ to the

vertical of the radius of the disc through O is $cos^{-1}\left(\frac{g}{aw^2}\right)$ $unless\, w^2 < \frac{g}{a}$ and then θ is zero.
(Pg. 25)

6. A uniform rod, of mass m and length 2a, can turn freely about one end which is fixed, it is started with angular velocity w from the position in which is hangs vertically find its angular velocity at any instant.
(Pg. 34)
7. A uniform vertical circular plate of radius a, is capable of revolving about a smooth horizontal axis through its centre; a rough perfectly flexible chain, whose mass is equal to that of the plate and whose length is equal to its circumference, hangs over its rim in equilibrium, if one end be slightly displaced, show that the velocity of chain, when the end reaches the late is $\sqrt{\frac{\pi\, ag}{6}}$.
(Pg. 39)
8. A sphere of radius a is suspended by a fine wire from a fixed point at a distance l from its centre, show that the time of a small oscillation is given by $2\pi\sqrt{\left(\frac{5l^2+2a^2}{5gl}\right)}\left\{1+\frac{1}{4}sin^2\frac{\alpha}{2}\right\}$
(Pg. 58)
9. A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position. Show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle 45^0 to the vertical, and that the vertical strain is then $\frac{11}{8}$ times the weight of the rod.
(Pg. 67)
10. A right cone of vertical angle 2α, can turn freely about an axis passing through the centre of its base and perpendicular to the axis. If the cone starts from rest with its axis horizontal, show that when the axis is vertical, the thrust on the fixed axis is to weight of the cone as:
$1+\frac{1}{2}\ cos^2\alpha\ to\ 1-\frac{1}{3}cos^2\alpha$
(Pg. 72)
11. Discuss the motion of a uniform sphere which rolls down inclined plane rough enough to prevent any slipping.
(Pg. 90)
12. A uniform rod if placed with one end in contact with a horizontal table and is then at an inclination α to the horizon and is allowed to fall. When it becomes horizontal show that its angular velocity is $\sqrt{\left(\frac{3g\sin\theta}{2a}\right)}$, whether the plane be perfectly smooth or perfectly rough. Show also that the end of the rod will not leave the plane in either case.
(Pg. 101)
13. A sphere is projected with an under twist down a rough inclined plane, show that it will turn back in the course of its motion if $2aw\ (\mu - tan\alpha) >$

$5\mu u$, where u is the initial linear velocity and w the initial angular velocity of the sphere is the coefficient of friction and α is the inclination of the plane.
(Pg. 116)

14. A homogeneous sphere of radius rotating with angular velocity w about a horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2aw}{7\mu g}$ and then the sphere will roll with angular velocity $\frac{2w}{7}$.
(Pg. 123)
15. Two equal uniform rods AB and AC are freely hinged at A and rest in a straight line on as smooth table. A blow if struck at it perpendicular to the rods; show that the K.E. generated is $\frac{7}{4}$ times what it would be if the rods were rigidly fastened together at A.
(Pg. 130)
16. Deduce the Euler's Dynamical Equations of motion.
(Pg. 141)
17. An uniaxial body is supported at its centre of mass and is rotating initially with angular velocity w about an axis perpendicular to the axis of symmetry. Prove that if a couple of constant moment l is applied above the axis of symmetry, the instantaneous axis will describe a cone whose equation referred to the axis fixed in the body of which that of coincides with the axis of symmetry, is 2 $Al\frac{(x^2+y^2)tan^{-1}y}{x} = C\ (C - A)w^e z^2$.
(Pg. 146)
18. Deduce the Euler's Geometrical equations of motion.
(Pg. 149)
19. Deduce the Euler's equations from Lagrange's equations.
(Pg. 150)
20. A lamina rotating with uniform angular velocity n about an axis through its centre of gravity perpendicular to its plane has an additional angular velocity λn impressed upon it about the axis of least moments : (A<B<C) where $\lambda^2 = \frac{B+A}{B-A}$, Prove that at time t its angular velocities are $\lambda n \sec h\ nt$, λn tan nt and n sec h nt. Also show that it will ultimately revolve about the axis of mean moment.
(Pg. 159)
21. A rectangular parallelepiped whose edges are a, 2a, 3a and turn freely about is centre and is set rotating about a line perpendicular to the mean axis and making an angle $cos^{-1}\frac{5}{8}$ with the least axis Prove that ultimately the body will rotate about mean axis.
(Pg. 162)
22. A rectangular lamina ABCD in which $BC = \sqrt{2}\,AB$ can turn freely about the middle point O of AD. Initially it is set rotating with angular velocity π

about a line through O perpendicular to AD and making a angle 30^0 with the plane of the rectangle. Show that after time t, the components of angular velocity of the rectangle about the principal axis at 0 are $\frac{1}{2}\pi \sqrt{3} \sec h \frac{1}{2}\pi t, \frac{1}{2}\pi\sqrt{3} \tan h \frac{1}{2}\pi\ t\ and \frac{1}{2}\pi \sec h \frac{1}{2}\pi t.$

(Pg. 164)

23. A uniform elliptic disc is free to move about a focus and in set rotating with initial angular velocity π about an axis perpendicular to the corresponding latus rectum and making an angle θ with the plane of the disc. If $\cos s\theta = \frac{A}{3}$, where A, B are moments of inertia of the disc about the major axis and latus rectum respectively. Prove that after time t the component angular velocity of the disc about the major axis will be

$$\pi \sqrt{\left(\frac{B-A}{2B}\right)} \sec h \left[\pi + \sqrt{\frac{B-A}{2B}}\right]$$

(Pg. 166)

24. Discuss the motion of symmetrical bodies under no forces.

(Pg. 171)

25. State and prove the principles of conservation of linear momentum and angular momentum of rigid body under the action of finite forces.

(Pg. 181)

26. A uniform straight rod of length 2a, has two small rings at its ends which can respectively slide on thin smooth horizontal and vertical wires OX and OY. The rod starts at an angle α to the horizon with angular velocity $\sqrt{\{3g\ (1-\sin\alpha)/2a\}}$ and moves downwords. Show that it will strike the horizontal wire at the end of time.

$$2\sqrt{\left(\frac{a}{3g}\right)}\ log\ \left\{\left(\frac{\pi}{8}-\frac{\alpha}{4}\right) tan \frac{\pi}{8}\right\}$$

(Pg. 183)

27. State and prove the principles of conservation of linear momentum and angular momentum of rigid body under the action of impulsive forces.

(Pg. 189)

28. A particle of mass m within a rough circular tube of mass M lying on a horizontal plane and initially the tube is at rest while particles has an angular velocity round the tube. Show that by the time relative motion ceases the fraction $\frac{M}{M+2m}$ of the initial kinetic energy has been dissipated by friction.

(Pg. 186)

29. Derive the Lagrange's equations of motion in generalized coordinates for a halonomic dynamical system under finite forces.

(Pg. 205)

30. Deduce the principle of energy from the Lagrange's equations.

(Pg. 209)

31. A bead of mass M, slides on a smooth fixed wire whose inclination to the vertical is α and has hinged to it a rod of mass m and length 2l, which can move freely in the vertical plane in the wire. If the system starts from rest with the rod hanging vertically show that
$\{4M + m\,(1 + 3\,cos^2\theta)\}\, l\theta^2 = 6\,(M + m)g \sin\alpha\ (\sin\theta - \sin\alpha)$
Where θ is the angle between the rod and the lower part of the wire.
(Pg. 211)
32. A uniform rod of mass 3m and length 2l has its middle point fixed and a mass m attached at one octremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{\frac{2\,ng}{l}}$ show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $cos^{-1}\{\sqrt{n^2+1} - n\}$ and will then rise again.
(Pg. 214)
33. Two equal rods AB and BC each of length l, smoothly joined at B are suspended from A and oscillates in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$.
(Pg. 219)
34. Three equal uniform AB, BC, CD each of the mass m and length 2a, are at rest in a straight line smoothly joined at B and C, A. blow I is given to the middle rod at distance c from its centre O in a direction perpendicular to it, show that the initial velocity of O is $\frac{2I}{3m}$, and that the initial angular velocities of the rods are:
$\frac{(5a+9c)I}{10\,ma^2}, \quad \frac{6\,CI}{5ma^2} \quad and \quad \frac{(5a-9c)I}{10ma^2}$
(Pg. 232)
35. Obtain the equation of motion of top.
(Pg. 240)
36. Find the stability conditions for the motion of a top, when :
(i) The axis of the top is vertical.
(ii) The axis of top is not vertical.
(Pg. 243)
37. If initially the axis of the top is horizontal and it is set spinning with angular velocity w in a horizontal plane, prove that the axis will start to rise if $ncw > mgh$ and that when $ncw = 2mgh$ the axis will rise to an angular distance $cos^{-1}\left(\frac{AW}{nC}\right)$, provided that $AW < nC$ and will there be at instantaneous rest $A <\ C$ and n have their usual meanings.
(Pg. 248)
38. A symmetrical top is set in motion on a rough horizontal plane with an angular motion about its axis of figure the axis being inclined at an angle I to the vertical show that between the greatest approach to and reciss from the vertical the centre of gravity describe on arc

$h\, tan^{-1}\left(\frac{\sin i}{P-\cos i}\right)$, where o and h have their usual meanings.
(Pg. 250)

39. When the axis of a symmetrical top is stationary and then spin is large and equal to n, a blow J is applied perpendicular to the axis at a distance d from the fixed point. Prove that the maximum angular deflection of the axis is approximately $2\, tan^{-1}\left(\frac{Jd}{cn}\right)$, c being the moment of inertia of the top about its axis of symmetry.
(Pg. 252)
40. Discuss the "Hamilton's Principle" and "Principle of least Action".
(Pg. 256)
41. Show that the Hamilton's principle function S for simple harmonic motion in a straight line is
$$\frac{\sqrt{\mu}\,(x^2 + x_0^2)Cos\,(t - t_0)\,\sqrt{\mu} - \;2xx_0}{2\sin(t - t_0)\,\sqrt{\mu}}$$
Where x, x_0 are the displacements from the centre of force at time t, t_0 respectively.
(Pg. 261)
42. A particle of unit mass moves along OX under a constant force f starting from rest at the origin at time t=0. If T and V are the kinetic and potential energies of the particle calculate
$$\int_0^{t_0} (T - V)dt$$
Evaluate this for the varied motion in which the position of particle is given by $x = \frac{1}{2} f t^2 {+}\in f\; t\;(t - t_0)$ where $\in$ is a constant and show that the result is in agreement with Hamilton's principle. What are the essential features of the varied motion that ensure this agreement.
(Pg. 265)
43. Define and explain:
(i) Density (ii) Pressure (iii) Compressibility (iv) Viscosity
(Pg. 271)
44. Explain all kinds of fluid flow.
(Pg. 273)
45. Establish the relationship between the Lagrangian and Eulerian method.
(Pg. 275)
46. Discuss the "Material Derivative".
(Pg. 276)
47. Obtain the velocity in terms of stream function.
(Pg.281)
48. Discuss the Rotational and Irrotational motion.
(Pg. 296)
49. Obtain the equation of continuity (vector form) by Euler's method.
(Pg. 299)

50. Obtain the equation of continuity by the Lagrangian method.
(Pg. 301)
51. Show that the two forces of equation of continuity obtained through Eulerian approach and Lagrangian approach are equivalent.
(Pg.303)
52. Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the stream line at time "t" are the curves $(x-t)^2 - (y-t)^2 = constant$ and the path of the fluid particles have the equation.
$\log(x-y) = \frac{1}{2}\ [(x+y) - \ a(x-y)^{-1}] + b$
(Pg. 313)
53. Obtain the equation of continuity in Cartesian coordinates system.
(Pg. 316)
54. Obtain the equation of continuity in cylindrical polar coordinates system.
(Pg. 318)
55. Obtain the equation of continuity in spherical polar coordinates system.
(Pg. 320)
56. If the lines of motion are curves on the surface of spheres all touching the plane of xy at the origin O, the equation of continuity is
$$r\sin\theta\frac{\partial P}{\partial t} + \frac{\partial(Pv)}{\partial\phi} + \sin\theta\ \frac{\partial(Pu)}{\partial\theta} + Pu\ (1 + 2\cos\theta) = 0$$
(Pg. 329)
57. Obtain the condition for a surface may be boundary surface.
(Pg. 331)
58. Obtain the Euler's dynamical equations of motion in vector notation.
(Pg. 338)
59. Obtain the "Halmohltz Equation".
(Pg. 345)
60. An infinite mass of fluid is acted on by a force $\mu v^{-3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the surface r = c in it, show that the cavity will be filled up after an interval of time $(2/5\mu)^{1/2}C^{5/4.}$
(Pg. 354)
61. An infinite fluid in which is a spherical hollow shell of radius a is initially at rest under the action of no forces. If a constant pressure p is applied at infinity, show that the time of filling up the cavity is
$\pi^2\left(\frac{\rho}{P}\right)^{1/2}.2^{5/6}\left(\sqrt{1/3}\right)^{-3}$
(Pg. 356)
62. An infinite mass of Homogeneous, incompressible fluid is at rest subject to a confirm pressure p and contains a spherical cavity of radius a filled with a gas at a pressure mP. Prove that if the inertia of the gas be neglected and Boyle's law be supposed to hold throughout the ensuring motion, the radius

of the sphere will oscillate between the value a and na, where n is determined by the equation $1 + 3\, m \log n - n^3 = 0$

If m be nearly equal to 1, the time of an oscillation will be $2\pi\sqrt{\frac{a^2\rho}{3P}}$

ρ is being the density of the fluid.

(Pg. 359)

63. Obtain the "Cauchy's Integrals."

(Pg. 371)

64. Obtain the equations of motion under impulsive force in catesian form.

(Pg. 378)

65. A portion of Homogeneous fluid is confined between two concentric spheres of radii A and a, and is attracted towards their centre by a force varying inversely as the square of the distance. The inner spherical surface s suddenly annihilated and when the radii of the inner and outer surface of the fluid are r and R the fluid impinges on a solid ball concentric with these surfaces, prove that the impulsive pressure at any point of the ball for different values of R and r varies as $\left[a^2 - r^2 - A^2 + R^2\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{1/2}$

(Pg. 384)

66. A sphere of radius a is surrounded by infinite liquid of density ρ the pressure at infinity being π. The sphere is suddenly annihilated falls to $\pi\left(1 - \frac{a}{r}\right)$. Show further that if the liquid is brought to rest by impinging on a concentric sphere of radius $\frac{a}{2}$, the impulsive pressure sustained by the surface of this sphere is $(7\pi\rho^2/6)^{1/2}$

(Pg. 386)

67. Obtain the complex potential for a doublet.

(Pg. 396)

68. Show that velocity potential

$$\phi = \frac{1}{2}\log\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$$

Gives a possible motion. Determine the stream lines and show also that the curves of equal; (q = constant) are ovals of Cassini given by rr'= constant.

(Pg. 408)

69. An area A is bounded by that part of the x-axis for which x > a and by that branch of $x^2 - y^2 = a^2$ which is in the positive quadrant. There is a two dimensional unit source at (o, o) which sends out liquid uniformly in all directions. Show by means of the transformation $w = \log(z^2 - a^2)$that in steady motion the stream lines of the liquid within the area A are portions of rectangular hyperbola. Find the stream lines corresponding to $\Psi = 0; \frac{\pi}{4}, \frac{\pi}{2}, If\ \rho_1\ and\ \rho_2$ are the distance of a point P within the fluid from the points $(\pm, a, o)$ show that the velocity of the fluid at P is measured by $\frac{2aP}{\rho_1 \cdot \rho_2}$, 0 being the origin.

(Pg. 411)

70. Between the fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = -\frac{\pi}{6}$ there is a two dimensional liquid motion due to a source of strength m at the point $(r = c, \theta = \alpha)$ and a sink at the origin, absorbing water at the same rate as the source produces it. Find the stream function and show that one of the stream lines is a part of the curve

$\gamma^3 \sin 3\alpha = C^3 \sin 3\theta$

(Pg. 413)

71. In the case of the motion of liquid in a part of a plane bounded by a straight line due to a source in the plane prove that if m o is the mass of fluid of density p generated at the sources per unit of time, the pressure on the length 2l of the boundary immediately opposite to the source is less than that on an equal length at a great distance by

$\frac{1}{2}\frac{m^2\rho}{\pi^2}\left\{\frac{1}{c}tan^{-}\frac{1}{c} - \frac{1}{l^2 + c^2}\right\}$

(Pg. 415)

72. In the case of the two dimensional fluid motion produce by a source of strength m place at a point S outside a rigid circular disc of radius a whose centre is O. Show that the velocity of slip of the fkuid in contact with the disc is greatest at the points where the line joining S to the ends diameter at right angles to OS cut the circle and prove that its magnitude at these points is $\frac{2\, m.OS}{(OS^2 - a^2)}$

(Pg. 416)