Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-04
Differential Geometry & Tensors
Section – C
(Long Answers Questions)

1. Prove that if the circle $lx + my + n = 0, x^2 + y^2 + z^2 = 2cz$ has three point contact at the origin with the paraboloid $ax^2 + by^2 = 2z$ then C = $(l^2 + m^2)\ (bl^2 + \ am^2)$

A M.T. -04, Pg. 10

2. Find the inflexional tangents at (x, y, z) on the surface $y^2 z = 4ax$

A M.T. -04, Pg. 12

3. Find the osculating plane at a point of a space curve given by the intersection of two surfaces.

A M.T. -04, Pg. 17

4. Find the osculating plane at the point 't' on the helix $x = a\cos t, y = a\sin f, z=ct$

A M.T. -04, Pg. 19

5. Prove that the osculating plane at (x_1, y_1, z_1) on the curve of intersection of the cylinders in $x^2 + y^2 + z^2 = \ a^2, y^2 + z^2 = b^2$ is given by : $\frac{xx_1^3 - zz_1^3 - a^4}{a^2} = \frac{yy_1^3 - zz_1^3 - b^4}{b^2}$

A M.T. -04, Pg. 19

6. State and prove serrect-frenet formulae.

A M.T. -04, Pg. 32

7. Prove that :

(i) $k \ = \frac{|\overrightarrow{r'} \times \overrightarrow{r''}|}{|\vec{r}|^3}$

(ii) $\tau = \frac{|\overrightarrow{r'}\ \overrightarrow{r''}\ \overrightarrow{r'''}|}{|\overrightarrow{r'} \times \overrightarrow{r''}|^2}$

A M.T. -04, Pg. 37

8. For the curve $x = 3u\ , y = 3u^2, z = 2u^2$ then prove that :

$$\delta = -\sigma = \frac{3}{2}\,(1 + 2u^2)^2$$

A M.T. -04, Pg. 38

9. For the curve $x = a\,(3u - u^3), y = 3au^2\,, z = a\,(3u + u^3)$. Show that the curvature and torsion are equal.

A M.T. -04, Pg. 39

10. Find the radii of curvature and torsion of the halix $x = a\cos\theta, y = a\sin v, z = av\tan\alpha$

A M.T. -04, Pg. 41

11. Find the radii of curvature and torsion at a point of the curve $x^2 + y^2 = a^2, x^2 - y^2 = az$

A M.T. -04, Pg. 42

12. If the curvature K if a curve C is constant, then the curvature k_1 of c_1 is also constant and If torsion τ_1 varies inversely as τ of the curve c.

A M.T. -04, Pg. 47

13. Prove that $\widehat{t_1}\,, \widehat{n_1}\,, \widehat{b_1}$ of c_1 are parallel respectively to $\hat{b}\,, \hat{n}\,, \hat{t}$ of c.

A M.T. -04, Pg. 50

14. If a curve lies on a sphere show that $\delta\ and\ \sigma$ are related by $\frac{d}{ds}\,(\sigma\delta^1) + \frac{\delta}{\sigma} = 0$ Show that a necessary and sufficient condition that a cirve lies ona sphere is that $\frac{\delta}{\sigma} + \frac{d}{ds}\left(\frac{\delta^1}{\tau}\right)$=0

A M.T. -04, Pg. 52

15. State and prove uniquennces theorem for space curves.

A M.T. -04, Pg. 59

16. Find the followings:
    - (i) Curvature of the involutes.
    - (ii) Torsion of the involutes

A M.T. -04, Pg. 64

17. Find the formulae for curvature of the evolute.

A M.T. -04, Pg. 67

18. State and prove Existence theorem for space curves.

A M.T. -04, Pg. 57

19. Prove that Each characteristic touch the edge of regression.

A M.T. -04, Pg. 83

20. Suppose that a tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meetsthe coordinate axes in points P, Q, R. Prove that the envelope of the sphere OPQR is $(ax)^{2/3}+(by)^{2/3}+(CZ)^{2/3}= (x^2+y^2+z^2)^{2/3}$ Where 0 is the origin.

A M.T. -04, Pg. 85

21. Find the equation of the developable surface whose generating line posses through the curve $y^2=4ax, z=0, x^2=4ay, z=c$ and show that ots edge of regression is given by $(x^2-2ay^2=0, cy^2-3ax\ (c-z)$

A M.T. -04, Pg. 87

22. Find the developable surface which passes through the curves $x^2 = 4ax, z=0\ and\ y^2=4bz, x=0$

A M.T. -04, Pg. 90

23. Show that the edge of regression of the developable that passes through the parabolas $x=0, z^2=4ay, y^2=4az, x=a$ is given by :

$$\frac{3x}{y}=\frac{y}{z}=\frac{z}{3(a-x)}$$

A M.T. -04, Pg. 42

24. Find the necessary and sufficient condition that a surface $\zeta = F(\xi, \eta)$ should represent a developable surface.

A M.T. -04, Pg. 104

25. Prove that the metric of a surface is invariant under parametric transformation.

A M.T. -04, Pg. 112

26. State and prove second fundamental theorem.

A M.T. -04, Pg. 115

27. Find the fundamental magnitude for – Anchor ring (B) conodial surface.

A M.T. -04, Pg. 119

28. Find the angle between two tangential direction on the surface in the terms of direction ratio.

A M.T. -04, Pg. 124

29. Derive the formulae for curvature of normal section.

A M.T. -04, Pg. 135

30. Find the curvature of a normal section of the right helicoids $x = 4\cos\phi, y=u\sin\phi, z=c\phi$

A M.T. -04, Pg. 144

31. Find the equation giving principal directions at a point of surface and to derive the differential equation of the principal section.

A M.T. -04, Pg. 148

32. Find the equations of principal curvature at a point A(u,v) of the surface $\vec{r} = \vec{r}\,(u, v)$

A M.T. -04, Pg. 152

33. Prove that for an umbilic :

$$\frac{1+P^2}{r} = \frac{Pq}{S} = \frac{1+q^2}{t} = \frac{P}{H}$$

A M.T. -04, Pg. 155

34. Find the principal section and principal curvature of the surface $x = a\,(u+v), y = b\,(u-v), z = uv$

A M.T. -04, Pg. 156

35. For the huperboloid $2z = 7x^2 + 6xy - y^2$ prove that the principal radii at the origin are $\frac{1}{3}$ $and -\frac{1}{2}$ and that the principal section are $x = 3y, 3x = -y$

A M.T. -04, Pg. 159

36. Show that the point of intersection of the surface $x^m + y^m + z^m = a^m$ line x = y = z are umbilici and that the radius of curvature at an umbilic is given by $\delta = \frac{a}{m-1} \cdot \delta^{(m-1)/2m}$

A M.T. -04, Pg. 160

37. Prove that the cone $kny = z\left\{(x^2+z^2)^{1/2}(y^2+z^2)^{1/2}\right\}$ passes through a line of curvature of the paraboloid $xy = az$

A M.T. -04, Pg. 169

38. To show that to a given direction there is one and only one conjugate direction. Also derive the condition for the two directions (du, dv) and (Du, Dv) to be conjugate.

A M.T. -04, Pg. 178

39. Derive the principal radii through a point of surface $z = f\,(x, y)$

A M.T. -04, Pg. 175

40. To show that conjugate direction at a appoint P on a surface are parallel to conjugate diameters of the indicalrix at P.

A M.T. -04, Pg. 182

41. State and prove beltran's-Enneper theorem.

A M.T. -04, Pg. 192

42. Define the general differential equations of geodesics on a surface $\vec{r} = \vec{r}\,(u, v)$

A M.T. -04, Pg. 200

43. Derive the canonical equations of a geodesics on the surface $\vec{r} = \vec{r}\,(u, v)$

A M.T. -04, Pg. 202

44. Prove that on a general surface, a necessary and sufficient condition that the curve $v = c\ (Const.)$ be a geodesic is $EF_2 + FE_1 = 2EF_1 = 0$, when v = c for all values of u.

A M.T. -04, Pg. 206

45. Derive the differential equation of a geodesics on the surface $F(x, y, z) = 0$

A M.T. -04, Pg. 210

46. State and prove Claprut's theorem.

A M.T. -04, Pg. 213

47. If parameters S are lengths, then show that geodesic curvature $Kg = [\hat{N}\,.\overrightarrow{r'}\,\overrightarrow{r''}]$ and if we replace parameter s by t then show that:

$$Kg = \frac{1}{HS^3}\left\{\frac{\partial T}{\partial u}\,V(t) - \frac{\partial T}{\partial u}\,U(t)\right\}$$

A M.T. -04, Pg. 219

48. Express the Torsion of a geodesic in the terms of fundamental magnitude and also in the terms of principal curvature.

A M.T. -04, Pg. 226

49. State and prove Gauss bonnet theorem.

A M.T. -04, Pg. 228

50. A geodesic on the ellipsoid of revolution $\frac{x^2+y^2}{a^2} + \frac{z^2}{c^2} = \perp$, crosses a meridian at an angle θ at a distance u from the axis prove that at the point of crossing it makes an angle $cos^{-1}\left(\frac{cu\cos\theta}{\sqrt{(a^4 - u^2(a^2-c^2\}})}\right)$ with the axis.

A M.T. -04, Pg. 231

51. Prove that at the origin the geodesic curvature of the sections of the surface $2z = aa^2 + by^2$ by the plane $\ln + my + nz = 0$ is $n(be^2 + am^2)/(l^2 + m^2)^{3/2}$

A M.T. -04, Pg. 235

52. State and prove Mapnari – Codazzi equation.

A M.T. -04, Pg. 244

53. For a surface given by $ds^2 = \phi(du^2 + dv^2)$ prove that:

$$l = \frac{\phi_1}{2\phi}, m = \frac{\phi_2}{2\phi}, n = \frac{\phi_1}{2\phi}, \lambda = \frac{\phi_2}{2\phi}, \mu = \frac{\phi_1}{2\phi}, v = \frac{\phi_2}{2\phi}$$

And further show that mainradii-codazzi relation become:

$L_2 - M_1 = \frac{1}{2}\frac{\phi_2}{\phi}(L+N), N_1 - M_2 = \frac{1}{2}\frac{\phi_1}{\phi}(L+N)$

A M.T. -04, Pg. 248

54. State and prove fundamental existence theorem for surfaces.

A M.T. -04, Pg. 249

55. Prove that a entity whose inner product with an arbitrary tension is a tensor is itself a tensor.

A M.T. -04, Pg. 269

56. The fundamental Tensor g_{ij} is a covariant symmetric tensor of the order two.

A M.T. -04, Pg. 277

57. Show that the metric of a Euclidian space referred to cylindrical coordinates is given by :

$$ds^2 = (dr)^2 + (rdQ)^2 + (dz)^2$$

Determine its metric tensor and conjugate metric tensor.

A M.T. -04, Pg. 280

58. Show that the metric of a Euclidean space, reffered to spherical coordinates is given by $ds^2 = (dr)^2 + (rdv)^2 + (v \sin v \, d\phi)^2$
Determine its metric tensor and conjugate metric tensor.

A M.T. -04, Pg. 281

59. The entities defined by (permutation tensor) :

$$\in_{ijk} = \sqrt{g}\; e_{ijk} \;;\; \in^{ijk} = \frac{1}{\sqrt{g}} e_{ijk}$$

Are respectively covariant and contra variant tensor where e_{ijk} is a permutation symbol and g is the determinant of the metric tensor g_{ij}

A M.T. -04, Pg. 284

60. Prove the following :

(i) $[ij, m] = g_{em} \begin{bmatrix} i \\ ij \end{bmatrix}$

(ii) $\frac{\partial g_{ik}}{\partial x^j} = [ij, k] + [kh, i]$

(iii) $\frac{\partial g^{mk}}{\partial x^{l}} = -g^{mi}\begin{Bmatrix} k \\ il \end{Bmatrix} - g^{ki}\begin{Bmatrix} m \\ il \end{Bmatrix}$

A M.T. -04, Pg. 286

61. Surface of sphere is a two dimensional Riemannian space. Complete the christoffel symbols.

A M.T. -04, Pg. 290

62. Prove that the christoffal symbols are not tensor quantities.

A M.T. -04, Pg. 293

63. State and prove Ricci's theorem

A M.T. -04, Pg. 300

64. If A_{ij} is the curl of a covariant vector, prove that $A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$

Show further that this expression is equivalent to L

$$\frac{\partial A_{ij}}{\partial x^k} + \frac{\partial A_{ik}}{\partial x^i} + \frac{\partial A_{ki}}{\partial x^j} = 0 \; if \; A_{ij} = B_{i,j} = B_{j,i}$$

Prove that : $A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$

A M.T. -04, Pg. 308

65. Evaluate $div\, A^j$ in (i) cylindrical polar coordination (ii) Spherical polar coordinates.

A M.T. -04, Pg. 309

66. State and prove Eulers condition of calculus of variation.

A M.T. -04, Pg. 315

67. Assume that we live in a space for which the live element is : $ds^2 = (dx')^2 + [(x')^2 + C^2]\,(dx^2)^2$ which is the surface of a right heloid immeried in a Euclidian three dimensional space determine the diferential equation of a geodesic.

A M.T. -04, Pg. 319

68. Show that on the surface of a sphere, all great circles are geodesic while no other circle is a geodesic.

A M.T. -04, Pg. 322

69. Prove that it is always possible to chose a coordinate system so that all the christofffel symbols vanish at a particular point P_0.

A M.T. -04, Pg. 323

70. Prove the following theorems :
    (i) The magnitude of all vectors of a field of parallel vectors is constant.

(ii) Prove that the geodesic is an auto parallel curve.

A M.T. -04, Pg. 328,329

71. The necessary and sufficient condition for a vector B^i of variable magnitude to suffer a parallel displacement along a curve C that :

$$B^i_j \frac{dx^j}{ds} = B^i f(s)$$

A M.T. -04, Pg. 329

72. The necessary and sufficient condition for a space V_N to be flat is that the Riemann-Christoffel tensor be Identically Zero i.e. $R^{\alpha}_{.ijk} = 0$

A M.T. -04, Pg. 343