Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-04
Differential Geometry & Tensors
Section – A
(Very Short Answers Questions)

1. Write down the equation of tangent line to a curve at a given point.

A $\frac{X-x}{x'} = \frac{Y-y}{y'} = \frac{Z-z}{z'}$

2. Write the condition of simple intersection of a curve and surface.

A If $F\ (t_0) \neq 0$

3. Write the condition for three point contact of a curve and a surface.

A If $F^{\cdot} = 0, F^{\cdot\cdot} = 0\ bur\ F^{\cdot\cdot\cdot} \neq 0$

4. Write the equation of oscillating plane.

A $\left[\vec{R}-\vec{r},\vec{r},\vec{r}\right] = 0$

5. Define the principal Normal.

A The principal normal at any point p of a given curve C is defined as the normal which lies in the oscillating plane at p.

6. Write down the equation of principal normal.

A $\vec{R} = \vec{r} + \lambda\hat{n}$

7. Write down the equation of binormal.

A $\vec{R} = \vec{r} + \mu\ \hat{b}$

8. Define the oscillating circle.

A The circle which has three points of contact with the curve at p is called the oscillating circle at point p on a curve.

9. Define the Bertrand curves.

A Two curves c and c_L are said to be Bertrand curves or conjugate of the principal normal to C are also principal normal to c_L.

10. Write down the formula of Transfer of the incolute.

A $\tau_1 = \pm\frac{(k\tau' - \tau k')}{\mu k\,(\tau^2 + k^2}\ where\ \mu = \ c - s$

11. Define the Evolute?

A If the targets to a curve C are normals to another curve C_L, then c is called an evolute of C_L.

12. Write down the formula to the curvature of evolute.

A $K_1 = \frac{K^3\ Cos^3\ (\Psi + c)}{Kz \sin(\Psi + c) - K^1 \cos(\Psi + c)}$

13. Give the relation between curvature and torsion of the evolute.

A $\frac{\tau_1}{k_1} = -\tan(\Psi + c)$

14. Define Indicatrix.

A The section of the surface by plane Z = h is the same as the section of the conicoid therefore it is a conic given by :

$z = h, 2h = rx^2 + 25xy + ty^2$

And is called the indicatrix

15. Write down the equation of the inflexigonal tangents of a point on given surface.

A $\gamma\ (\varepsilon - x)^2 + 25(\eta - x)(\eta - y) + t(\eta - y)^2 = 0$

16. Write down the equation to find characteristic of a family of surfaces.

A $F\ (x, y, z, \alpha) = 0,\ \frac{\partial F}{\partial \alpha} = 0$

17. How to obtain developable surface.

A Developable surface is obtaining by eliminating the parameter in from the equation $f(m) = 0$ and $F^{\cdot}\ (m) = 0$

18. Define the Ruled surface.

A A ruled surface is a surface which generate by single parameter family of straight lines.

19. Write down the condition of a ruled surface generated by $x = az + \alpha$, $y = bz + \beta$ is devlopable or skew.

A $\alpha' \beta' - \beta' \alpha' = 0 \quad or \quad \neq 0$

20. Write down the equation of tangent plane to a ruled surface in vector notation.

A $(\overrightarrow{R}^* - \overrightarrow{R}).(\overrightarrow{R_1} \times \overrightarrow{R_2}) = 0$

21. Define the regular point.

A The point $P(\overrightarrow{r})$ on the surface is called regular point if : $\overrightarrow{r_1} \times \overrightarrow{r_2} \neq \vec{0}$

22. Write down the equation of sphere in parametric form having radius a and centre at origin.

A $x = a\sin\phi\cos\theta, y = a\sin\phi\sin\theta, z = a\cos\phi$

$where\ \theta and\ \phi are\ parameters.$

23. Define Right helicoid.

A Eight helicold is generated by screw motion of a straight line about the fixed line (axis) such that the straight line meet the axis at right angle.

24. Write down the equation of first fundamental form.

A $(ds)^2 = Edu^2 + 2F\,du\,dv + Gdv^2$

25. Write down the second fundamental form.

A $Ldu^2 + 2M\,du\,dv + Ndv^2$

26. Write down parametric equation of anchor ring.

A $\vec{r} = ((b + a\cos\theta\cos\phi, (b + a\cos\theta)\sin\phi, a\sin\theta$

27. Write down the differential equation of orthogonal Trajctory.

A $(FP - EQ)\partial u + (GP - FQ)\partial v = 0$

28. Define Trajectory.

A A trajectory of given family of curve is a curve which intersects every member of curves by following some definite law.

29. Define Normal section and oblique section of a surface.

A If the plane section of the surface if such that it contain normal to the surface at that point, the section is called normal section and which is not normal section is called oblique section.

30. Write down the formulae of the curvature of the normal section.

A $K_u = \frac{Ldu^2 + 2\,M\,du\,dv + Ndu^2}{Edu^2 + 2F\,du\,dv + Gdv^2}$

31. Define radius of Normal curvature.

A Reciprocal of the Normal curvature is called rad. Of normal curvature i.e.

$\delta_n = \frac{1}{k_n}$

32. State Mennier's theorem.

A If k and k_n are the curvature of oblique and normal section through the same tangent line and θ is angle between these sections, then $k_n = k\cos\theta$

33. Define the principal curvature.

A The maximum and minimum curvature of the two principal section of a surface are called the principal curvatures.

34. Define Umbilic.

A A point A(u, v) on the surface $\vec{r} = \vec{r}(u, v)$ is called an umbilic if at the point :

$\frac{E}{L} = \frac{F}{M} = \frac{G}{N} = \frac{\sqrt{EG-F^2}}{\sqrt{LN-M^2}} = \frac{H}{T}$

35. Define Amplitude of Normal curvatures.

A Amplitude of normal curvature is benefied by A is defined as :

$A = \frac{1}{2}(K_a - k_b)$

36. Define the Minimal surface.

A The surface for which the first cuvature is zero of all points, is called minimal surface.

37. Define the lines of curvature.

A A curvature on a surface is called a line of curvature if the tangent at any point of it is along the principal direction of that point.

38. Define Asymptotic directions.

A A self conjugate directions on a surface $\vec{r} = \vec{r}(u, v)$ is called asymptotic direcyion.

39. Write the statement of Beltrammi-Enneper theorem.

A At a point on a surface, where the Ganssian Curvature is negative and equal to K, the torsion of the asymptotic line is $\pm\sqrt{K}$

40. Write the Torsion of an asymptotic line $\vec{r} = \vec{r}(s)$ on the surface $\vec{r} = \vec{r}(u, v)$

A $\tau = [\widehat{N}\,\widehat{N}'\,\vec{r}']$

41. Define Geodesic.

A Geodisc on a surface is defined as the curve of stationary length on a surface between any two points on its plane.

42. Write canwonical equation of a geodisc on the surface $\vec{r} = \vec{r}(u, v)$

A $\frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right) - \frac{\partial T}{-\partial u} = 0 \ and \ \frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right) - \frac{\partial T}{\partial v} = 0$

43. Write the condition the curve u = c is geodisc.

A $GG_1 + FG_2 - 2GF_2 = 0$

44. Define the Normal angle

A The angle between the principal normal $\hat{n}$ and the surface normal $\widehat{N}$ is known as normal angle.

45. Write the statements of Gauss-Bonnet theorem.

A Any curve which encloses a simple connected region R, the excess of closed curve C is equal to the total curvature of R.

46. Define the parallel surfaces.

A A surface S is said to be parallel to another surface S^* if the points of S^* are at constant distance along the normal to S.

47. Write the Weingarten formulae.

A The formulae:

$$H^2 N_1 = (FM - GL)\vec{r}_1 + (FL - EM)\vec{r}_2$$

and

$$H^2 N_2 = (FN - GM)\vec{r}_1 + (FM - EN)\vec{r}_2$$

48. Write the formulae for Ganssian and mean curvature for the parallel surfaces.

A $K^* = \frac{K}{1+2\mu c + kc^2}$

49. Define contrvariant vectors.

A If a set of N quantities A^i in a wordinate system x^i are transformed to the set of another N quantities A^{-j} in the coordinate system x^{-j} by the equation $A^{-j} = \frac{\partial \bar{x} P}{\partial x^2} A^q$ then A^i are said to be component of contranriant vectors.

50. Define Covariant vectors.

A In a set of N quantities A_i in a coordinate system x^i are transformed to a set of another N quantities $\bar{A}_j$ in the coordinate system x^{-j} by equation : $\bar{A}_p = \frac{\partial xq}{\partial \bar{x} P} A_q, then\ A_i$ and said to be compment of covariant vector.

51. Define zero tensor.

A A tensor here components relatively to every coordinate system are at zero is known as zero tensor.

52. Define symmetric tensor.

A A tensor is called symmetric with respect of two contravariant or covariant in Dices if its components remain unaltered upon intercharge of pnedices.

53. Write formuale of Conjugate symmetric tensor of rank two.

A $A^{ij} = \frac{Lofactor\ \ of\ A_{ij}\ in\ |A_{ij}|}{d}$

54. Define relative vector.

A If the rank of relative tensor is one then it is called relative vector i.e.

$$\bar{A}^P = \left|\frac{\partial x}{\partial \bar{x}}\right|^W A^u . \frac{\partial \bar{x} P}{\partial x^u}$$

55. Define permutation symbol in Euclidean three dimensional space V_3.

A $e_{ijk} = \begin{cases} o, is\ any\ two\ of\ i,j,k\ are\ equal \\ +1, if\ i,j,k\ is\ cyclic\ permutation \\ -1, if\ i,j,k, is\ anticyclic\ permutation \end{cases}$

56. Define christoffed symbol of finite kind.

A $[ij,k] = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^e} + \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right)$

57. Define chritoffed symbol of second kind.

A $\{K_{ij}\} = g^{kh}[ij,h]$

58. Write the statement of Ricci's theorem.

A The covariant derivative of the tensors g_{ij}, g^{ij} and $\delta_j^{\ i}$ all vanish pdentically.

59. Define Divergent of a covariant vector.

A The divergence of a covariant vector A_i , λ defined as div A_i $i.e.$

$$div\ A_i = g^{jk}\ A_{j,k}$$

60. Define Gradiant of a scalar.

A If I is calar function of coordinates x^i, the gradient of I, is defined as :

$$grad\ I = I_i = \frac{\partial I}{\partial x^2}$$

61. Define laplacian of scalar.

A If I is a scalar function of coordinate x^p, then the divergence of grad I is defined as the Laplacion of I and denoted by $\nabla^2 I = div\ grad\ I$

62. Define curve of a covariant vector.

A $Curve\ A_i = B^k = \in^{jik}\ \ A_{i,j}$

63. Define Intrinsic derivatives.

A $\frac{\partial A_i}{\partial t} = \frac{\partial A_i}{dt} - \{r_{ij}\} A_r \frac{dx^j}{dt}$

64. Write Buler's differential equation for functional $f\ (x^i, x^p)$

A $\frac{\partial F}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial f}{\partial x^p}\right) = 0$

65. Write the formulae for first curvature of a curve.

A $P^i = \frac{d^2 x^P}{ds^2} + \begin{Bmatrix} i \\ jk \end{Bmatrix} \frac{dx^s}{ds}, \frac{dx^k}{ds}$

66. Define geodesics.

A “A geodicis in a Riemann space V_N is a curve whose length has stationary value with respect to arbitrary small variation of curve.

67. Define Riemann – christoffed tensor.

A $R_{ijk}{}^{\alpha} = \frac{\partial}{\partial x^j}\left\{\begin{matrix}\alpha\\ ik\end{matrix}\right\} - \frac{\partial}{\partial x^k}\left\{\begin{matrix}\alpha\\ ij\end{matrix}\right\} + \left\{\begin{matrix}\alpha\\ Bj\end{matrix}\right\}\left\{\begin{matrix}\beta\\ ik\end{matrix}\right\} - \left\{\begin{matrix}\alpha\\ Bk\end{matrix}\right\}\left\{\begin{matrix}\beta\\ ij\end{matrix}\right\}$

68. Define Einstein tensor.

A It is defined as : $G_j{}^i = g^{il} R_{jl} - \frac{1}{2} R\delta_j{}^i$

69. Define flat space.

A A space for which the Riemann curvature is identically zero at every point of it is called a flat space.

70. Define Isotropic point.

A An isotropic point in Riemann space is a point at which the Riemannian curvature is independent of the vectors A^i and B^i associated to if.

71. Write the statement of Schur’s theorem.

A If a Riemann space V_N $(N > 2)$ is isotropic of each in a region, then the Riemannian curvature is constant through that region.