Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-03
Differential Equations, Calculus of Variations & Special Functions
Section – A
(Very Short Answers Questions)

1. What do you mean by Homogeneous equation?
A. An equation in which all the terms will be of the same dimension is known as homogeneous equation.
2. Write down the Riccati's equation.
A. $\frac{dy}{dx} = P + Qy + Ry^2$
Where P, Q, and R are functions of x or constants.
3. What is he dimension of the following differential equation.
$$x \; \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 4$$
A. zero
4. The dimension of x is invariably taken as unity. Is it true?
A. True
5. What is dimension of the following differential equation.
$$2x^3 \frac{d^2y}{dx^2} = \left(y - x\,\frac{dy}{dx}\right)^2$$
A. 2
6. Write down a total differential equation in n variables.
A. $\sum_{i=1}^{n} U_i\; dx_i = 0 \; where \; U_i(i = 1, 2\,, \ldots\ldots, n)$ are in functions of some or all of n independent variables $x_1, x_2, \ldots x_n$
7. Write down the necessary and sufficient for integrabil of the total differential equation $Pdx + Qdy + Rdz = 0$
A. $P\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + \; R\;\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = 0$
8. What is the name of the following equation :
$F(x, y, z, p, q, r\;, s, t) = 0$
Where $P = \frac{\partial z}{\partial x}\;, q = \frac{\partial z}{\partial y}\;, r = \frac{\partial z}{\partial x^2}\;, s = \frac{\partial^2 z}{\partial x \partial y}\;, \frac{\partial^2 z}{\partial y^2} = t$
A. General form of a second order partial differential equation in two independent variables $x \; and \; y$.
9. The solution of $r = 2y^2$ is ……………
A. $z = x^2y^2 + x\,F(y) + \; g(y)$

10. The solution of $Log\ s = x + y$ is ....

A. $z = e^{x+y} + x\,F(y) + g(x)$

11. For P.D.E. $Rr + Ss + tT + u\,(rt - s^2) = V$ The Monge's subsidiary equations are .... and ....

A. $R\,dpdy + T\,dqdx + U\,dpdq - V\,dxdy = 0$

and $R\,dy^2 - S\,dxdy + T\,dx^2 + V\,dpdx + U\,dqdy = 0$

12. The λ-equation is Monge's method for solving p.d.e. $2s + (rt + s^2) = 1$ is ....

A. $\lambda^2 + 2\lambda + 1 = 0$

13. The Monge's subsidiary equations for p.d.e.

$pt - qs = q^3$ are .... and ....

A. $q\,dy\,dx + p\,dx^2 = 0$

and $p\,dq\,dx - q^3\,dydx = 0$

14. The Monge's subsidiary equations for p.d.e.

$r = a^2t$ are .... and ....

A. $dpdy - a^2\,dqdx = 0$

and $dy^2 - a^2\,dx^2 = 0$

15. The second order p.d.e. $Rs + Ss + Tt + F(x, y, z, p, q) = 0$ is elliptic if ....

A. $S^2 - 4RT < 0$

16. The p.d.e.

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ is ....

A. Elliptic

17. The p.d.e.

$4\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} = 0$ is ....

A. Hyperbolic

18. Find where the following p.d.e. is parabolic:

$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0$ is ....

A. Parabolic in the region $x^2 = 4y$

19. Write Harmonic equation:

A. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

20. Write two dimensional ware equation:

A. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

21. Write one dimensional diffusion equation:

A. $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$

22. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is two dimensional ...... equation.

A. Laplace

23. Check whether the following boundary value problem:

$y'' + \lambda y = 0\ ;\ y'(-\pi) = 0,\ y'(\pi) = 0$

Strom Liouville problem or not

A. Yes

24. If the following boundary value probem:

$$\frac{d}{dx}\left(e^{2x}\frac{dy}{dx}\right) + (\lambda+1)e^{2x}y = 0$$

$y\,(0) = 0 = Y(\pi)$

Strom – Liouville problem.

A. Yes

25. The values $\lambda's$ for which given boundary value problem has non trival solutions are called of given problem.

A. Eigen values

26. Define eigen function:

A. The non trivial solution of given boundary value problem corresponding to particular eigen values is known as eigen function.

27. Find the eigen values λ_n and eigen functions $y_n(x) for\; y'' + \lambda y = 0$ for the following boundary conditions

$y(0) = 0 = Y(\pi) = 0$

A. $\lambda_n = n^2;\quad n = 1, 2, 3, \ldots ....$

$y_n(x) = \sin n\,x\,;\qquad n = 1, 2\ \ldots \ldots$

28. Write down Euler-Larganje equation:

A. $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

29. Write down Euler-Lagrange equation for the following functional $\int_0^1 \{(y')^2 + 12xy\}^{dx}$

A. $y'' = 6x$

30. Euler-Lagrange equation for the following functional:

$\int_\alpha^\beta (1 + x^2 y')\, y'\, dx$ is …………

A. $1 + 2x^2 y' = c$

31. Euler-Lagrange equation for the following functional:

$\int_{t_0}^{t_1} \{x^2 + (\dot{x})^2 - 2x \sin t\}\, dt$ is ……………..

A. $\frac{d^2x}{dt^2} = x \sin t = 0$

32. The ordinary point of $(1 - x^2)\, y'' + xy' + y = 0$ is …..

A. X = 0

33. The nature of the point $x = 0$ for the equation:

$2x^2 y'' + (2x^2 - x)y' + y = 0$ is………

A. Regular singular point

34. The singular points of the differential equation :

$x(x-1)^2 y'' + 2xy' + (x-1)y = 0$ are ………. And ……..

A. $x = 0, x = 1$

35. The nature of the point $x = 0$ for the equation :

$xy'' + y' - y = 0$ is ………..

A. Regular singular point

36. $2F_1(a, b\,; c\,; o) = \cdots \ldots \ldots$

A. 1

37. $(\alpha)_0 = \cdots \ldots \ldots \ldots \ldots \ldots,\ \ \alpha \neq 0$

A. 1

38. $2F_1(a, b\,; b\,; z) = \cdots \ldots \ldots$

A. $(1-z)^{-a}$

39. $\lim_{b\to 0}\left\{2F_1(1, b\,; 1; \frac{z}{b}\right\} = \cdots \ldots .$

A. e^z

40. For $|z| < 1, \quad 2F_1\left(\frac{1}{2}, 1; \frac{3}{2};\ z^2\right) = \cdots \ldots$

A. $\frac{1}{2z}\log\left(\frac{1+z}{1-z}\right)$

41. $1 + z + z^2 + \cdots \ldots . = \cdots \ldots for\ |z| < 1$

A. $\frac{1}{1-z}$

42. For $|z| < 1, \quad 2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2};\ z^2\right) = \cdots \ldots$

A. $\frac{1}{z}\ sin^{-1} z$

43. For $|z| < 1, \quad 2F_1\left(\frac{1}{2}, 1; \frac{3}{2};\ -z^2\right) = \cdots \ldots$

A. $\frac{1}{z}\ tan^{-1} z$

44. $n\ \geq 1, (\alpha)_n = \cdots ..$

A. α (α+1) (α+2) ……. (α+n-1)

45. Vandermonde's theorem:

$2F_1\ (-n, b\,; c\,; 1) = \cdots \ldots \ldots \ldots$ where n is positive integer.

A. $\frac{(c-b)_n}{(c)_n}$

46. If $|z| < 1\ and\ \left|\frac{z}{1-z}\right| < 1\ then$

$$2F_1\left(a, b; c; \frac{1}{2}\right) = \cdots \ldots$$

A. $2^a 2^{f_1}(a, c-b\,; c\,;\ -1)$

47. $2F_1\left(a, b\,; c\,; \frac{1}{2}\right) = \cdots \ldots ..$

A. $2^a\ 2^{f_1}(a, c-b\,; c\,;\ -1)$

48. $\frac{d^3}{dx^3}[2F_1(a, b\,; c\ \ x) = \cdots \ldots \ldots]$

A. $\frac{(a)_3(b)_3}{(c)_3}\ \ 2F_1\ (a+3, b+3, c+3; x)$

49. $2f_1(a, b\,; c\,; 1) = \cdots ..$

A. $\frac{\Gamma(c)\ \Gamma(c-a-b)}{\Gamma(c-a)\ \Gamma(c-b)}$ where $R(c-a-b) >\ 0$

50. $(a-b)f = \cdots ....$

A. $af\ (a+) - b\ (f\ (b+)$

51. $1F_1\ (a\,; c\,; z) = \cdots ..$

A. $\sum_{r\infty}^{\infty} \frac{(a)_r}{(c)_r} \frac{z^r}{\llcorner r}\ or\ \frac{1}{\Gamma(a)}\int_0^{\infty} e^{-t} t^{a-1} 0F_1(-\,; c\,; zt)dt$

52. $\frac{d^2}{dx^2}[1F_1\ (a\,; c\,; x)] = \cdots \ldots$

A. $\frac{(a)_2}{(c)_2}\ 1F_1\ (a+2\,;c+2\,;x)$

53. Write Integral representation for confluent hyper geometric functions.

A. If $|z| < 1$ and $Re\ (c)\ >\ Re\ (a)\ >\ 0$ then

$$B(a, c-a)\ 1F_1(a\ ;\ \in ;z) = \int_0^1 t^{a-1}\ (1-t)^{c-a-1} e^{zt} dt$$

OR

$$1F_1(a\,;c\,;z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(c-a)} \int_0^1 t^{a-1}(1-t)^{c-a-1} e^{zt}\ dt$$

54. $\frac{1}{\Gamma(a)}\ \int_0^\infty e^{-t} t^{a-1}\ 0f_1(-\,;b\,;zt)dt = \cdots..$

A. $1F_1(a; b; z)$

55. $a\ F\ (a+) - \ (c-1)F\ (c-) = \cdots ...$

A. $(a-c+1)F$

56. Write down the Legendre equation:

A. $(1-x^2)\ y'' - \ 2xy' + \ n\ (n+1)y = 0$

Where n is a positive integer.

57. $P_1(x) = \cdots ...$

A. x

58. $(n+1)P_{n+1}(x) + \ n\ P_{n-1}(x) = \cdots ....$

A. $(2n+1)x\ P_n\ (x)$

59. $x\ P'_n\ (x) - \ P'_{n-1}\ (x) = \cdots ...$

A. $n\ \ P_n\ (x)$

60. $(2n+1)P_n\ (x) - \ P'_{n+1}(x) + P'_{n-1}(x) = \cdots ..$

A. 0

61. $x\ \ [P_{n-1}\ (x) - \ x\ \ P_n\ (x)] = \cdots$

A. $(1-x^2)\ P'_n\ (x)$

62. Write down Rodrigves formula for $P_n(x)$

A. $P_n(x) = \ \frac{1}{2^n\ \llcorner n}\ \frac{d^n}{dx^n}(x^2-1)^n$

63. $P_n(-1) = \cdots ..$

A. $(-1)^n$

64. $n\ [P_n\ \ Q_{n-1}\ \ - \ \ Q_n\ \ P_{n-1}] = \cdots$

A. 1

65. $\int_x^\infty \frac{dx}{(x^2-1)\ P_n^2}$

A. $\frac{Q_n}{P_n}$

66. $\sum_{m=0}^\infty (2m+1)\ \ P_m(x)\ Q_m\ (y) = \cdots .....$

A. $\frac{1}{y-x}$

67. $Q'_{n+1} - Q'_{n-1} = \cdots ...$

A. $(2n+1)\ Q_n$

68. $\int_{-1}^1 x\ P_n\ \ P_{n-1}\ dx = \cdots ..$

A. $\frac{2n}{4n^2-1}$

69. $J_{\frac{1}{2}}(x) = \cdots ..$

A. $\sqrt{\frac{2}{\pi\, x}} \sin x$

70. $J_0^2 + 2(J_1^2 + J_2^2 + \cdots.) = \cdots.$

A. 1

71. $\int_0^\infty e^{-ax} J_0\,(bx)dx = \cdots \ldots$

A. $\frac{1}{\sqrt{a^2+b^2}}$

72. $H_{2n}(0) = \cdots \ldots ..$

A. $(-1)^n \frac{\llcorner 2n}{\llcorner n}$

73. $H_{2n+1}(0) = \cdots \ldots$

A. 0

74. $L'_n(0) = \cdots ..$

A. $-n$

75. $L''_n(0) = \cdots ..$

A. $\frac{n\,(n-1)}{2}$