M.A./M.Sc. Mathematics

Paper code- MT-01(Advanced Algebra)

Question Bank-2015

Section-A

1. Define isomorphism on groups. Ans.P.16
2. Define solvable group. Ans.P.31
3. Is 5 a unit element in (R,+) but not in (Z,+)? Ans. Yes
4. Define unital module. Ans.P.53
5. Define minimal generating set for submodule N of an R-module M. Ans.P.70
6. Define image of a linear transformation(range space) $t:V \rightarrow V'$. Ans.P.76
7. Is each linear transformation a linear functional? Ans.No.
8. The field C of complex numbers is what type of extension over field R? Ans. Algebraic extension.
9. Define row rank of a matrix. Ans. P.159
10. Define eigen value of a linear transformation. Ans. P.166
11. Define orthogonal set. Ans. P.195
12. Define orthogonal complement of a vector. Ans. P.196
13.

Section-B

1. Let G_1 and G_2 be two groups and $H_1 = \{(a,e_2) \mid a \in G_1\}$, $H_2 = \{(e_1,b) \mid b \in G_2\}$ where e_1nd e_2 are identity of G_1 and G_2 respectively. Then prove that H_1 and H_2 are normal subgroup of $G_1 \times G_2$. Ans.P.5
2. Let (R,+) be the additive group of real number and $(R^+,\cdot)$be a multiplicative group of positive real numbers. Show that a mapping $\phi:R\rightarrow R^+$ defined by $\phi(x) = e^x$ is an isomorphism. Ans.P.17
3. Conjugacy on a group G is an equivalence relation. P.30
4. Let G be a group. Then prove that G is an abelian if and only if $G^{(1)}=\{e\}$, e being the identity of G. P.30
5. Prove that every subgroup of a solvable group is solvable. P.33
6. Prove that every finite group G has a composition series. P.36
7. Let R be a Euclidean ring. Let a and b be two nonzero elements of R such that b is unit in R. Then prove that d(ab)=d(a). P.47

8. If M_1 and M_2 are submodules of an R-module M, then prove that :
   (i) $M_1 \cap M_2$ is a submodule of M and
   (ii) $M_1 + M_2 = \{m_1 + m_2 \mid m_1 \epsilon M_1, m_2 \epsilon M_2\}$ is a submodule of M. P.58

9. Show that the following map is a linear transformation:

   $t: R^3 \rightarrow R^2$ given by $t(x,y,z)=(z,x+y)$ $\forall (x,y,z) \epsilon R^3$. P.76

10. If B= $\{e_1=(1,0), e_2=(0,1)\}$ is the usual basis of R^2. Determine its dual basis. P.91

11. Let K be a field extension of a field F and let $\alpha \epsilon K$ be algebraic over F. Then show that any two minimal monic polynomial for α over F are equal. P.108

12. Let Q be the field of rational numbers, then show that $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$. P.115

14. Prove that an irreducible polynomial f(x) over a field of characteristic p>0 is inseparable if and only if $f(x) \epsilon F[x^p]$. P.123

15. Prove that the order of the Galois group G(K|F) is equal to the degree of K over F. P.135

16. Prove that for any matrix A over field F rank(A) = rank(A^T). P.160

17. Let A be a matrix of order nxn over a field F. Then prove that a scalar $\lambda \epsilon F$ is an eigen value of A iff det(A-λI)=0. P.183

18. Let V be an inner product space. Then for arbitrary vectors u, $v \epsilon V$, prove that $|\langle u,v \rangle| \leq \|u\| \|v\|$. P.192

19. Prove that a linear transformation t from a finite dimensional space V to itself is skew symmetric iff they commute. P.217

Section-C

1. Show that any two subnormal series for the group G have equivalent refinements. P.34
2. Let V and V' be vector spaces over a field F and B = $\{b_1, b_2, \ldots\ldots, b_n\}$ be a basis for V. Then prove that there exists a unique linear transformation t:V→V' for any list $b_1', b_2', \ldots\ldots, b_n'$ of vectors in V' such that $t(b_i) = b_i'$; i= 1, 2, ……….,n.
   P.79
3. Let V be n dimensional vector space over a field F and B = $\{b_1, b_2, \ldots\ldots, b_n\}$ be a basis for V, then prove that for any scalars $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n \in F$ there exists a unique functional $f \in V^*$ such that $f(b_i) = \lambda_i$; i= 1, 2, ……….,n. P.85
4. Let V, V' and V'' be finite dimensional vector spaces over a field F and let B, B' and B'' be there respectively bases. Then show that for linear transformations t:V→V' and s:V'→V'';
   $M_{B''}^{B}(sot) = M_{B''}^{B'}(s) M_{B'}^{B}(t)$. P.155
5. If B = $\{b_1=(-1,1,1);\ b_2=(1,-1,1);\ b_3=(1,1,-1)\}$ is a basis of $V_3(R)$, then find the dual basis to B. P.91